



Modern Building Materials, Structures and Techniques, MBMST 2016

## Comparison of elastic and viscoelastic analysis of asphalt pavement at high temperature

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### Abstract

There are many various rheological models which are used for modeling of asphalt layers in flexible pavement structures. It can be expected that the use of various models may affect different results of mechanical pavement calculations and analysis. The paper presents comparison of the use of Burgers' and Huet-Sayegh's viscoelastic models and Hooke's elastic model to calculate pavement deflections and strains at the bottom of asphalt layers at high temperatures (from 20°C to 50°C). The authors assumed Polish typical flexible pavement structure and standard wheel loading. The analysis were performed under the central point of pavement loading area. Performed calculations showed that the effect of the use of rheological models may be, from practical point of view, more significant for strains at the bottom of asphalt layers than for the pavement displacements at high temperatures.

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Peer-review under responsibility of the organizing committee of MBMST 2016

*Keywords:* viscoelastic analysis; viscoelasticity; elasticity; Burgers' model; Huet-Sayegh's model; Hooke's model; flexible pavement structure; high temperature; VEROAD.

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### 1. Introduction

The Hooke's linear elastic model is commonly used for mechanistic analysis and design of asphalt pavement structures [4,5,6]. It is considered to be appropriate for determination of response of asphalt layers at lower temperatures. However, Hot-Mix Asphalt (HMA) combines as elastic, as viscous and even plastic properties which are significant especially at high temperatures. The use of elastic model seems not to be sufficient to describe the

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real response of HMA at higher temperatures. Because of that many researchers use different rheological models, for example viscoelastic models [5, 6]. This leads to the aims of the presented analysis which was to assess how the use of various (viscoelastic and elastic) models of asphalt layers affects mechanical pavement response pavement response to the load and how much the choice of the model is significant for pavement structure analysis at high temperatures.

### General approach to analysis

The article presents the analysis of the Polish typical flexible pavement structure for the KR4 traffic at high pavement temperatures from 20°C to 50°C. The asphalt layers were modeled using two approaches: elasticity and viscoelasticity. The Hooke's elastic model, Burgers' and Huet-Sayegh's viscoelastic models were used (Fig. 1).

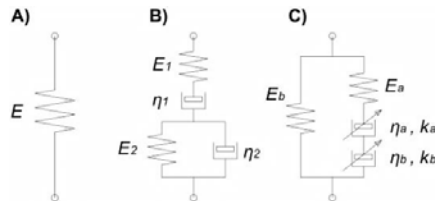


Fig. 1. Rheological models of the materials of asphalt layers: A) Hooke's elastic model, B) Burgers' viscoelastic model, C) Huet-Sayegh's viscoelastic model.

The other layers of the pavement (the base course and the subgrade) were modeled as elastic materials. The analysis were conducted for the temperature range from 20°C to 50°C. To simplify the analysis the same temperature for all asphalt layers was assumed. Such approximation is justified only for the purpose of this study, namely the comparison of viscoelastic and elastic modeling of asphalt layers.

Parameters of rheological models of asphalt layers were determined based on the results of dynamic modulus and phase angle tests, which were conducted at the Gdansk University of Technology but are not a part of this article. The pavement structure was loaded by the typical standard 50 kN single wheel moving at a speed of 60 km/h. Calculations were conducted using VEROAD (Visco-Elastic Road Analysis Delft) program. Surface pavement deflections and strains at the bottom of asphalt layers were computed and analyzed.

## 2. Assumptions for calculations

Firstly, pavement structure, pavement loading and material parameters of pavement layers were assumed.

### 2.1. Pavement structure

According to [1] the following Polish typical flexible pavement construction for traffic KR4 (from 2.5 to 7.4 typical single axis 100 kN during 20 years) was adopted: 4 cm of wearing course (stone matrix asphalt SMA 8 with modified asphalt binder 45/80-55), 6 cm of binder course (asphalt concrete AC 16W with asphalt binder 35/50), 10 cm of asphalt base course (asphalt concrete AC 22P with asphalt binder 35/50), 20 cm of unbound base course (crushed aggregate) and improved subgrade ( $E_2 \geq 100$  MPa).

Wearing course, binder course and asphalt base course were modeled as elastic or viscoelastic materials, whereas unbound base course and subgrade were modeled only as elastic materials. Analyzed pavement structure is presented in Figure 2.

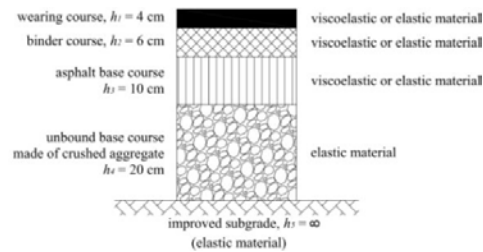


Fig. 2. The analyzed pavement structure [1].

The authors performed analysis for temperatures from 20°C to 50°C at every 10°C. It was assumed that temperature is constant for all asphalt layers. For each of analyzed temperature three cases were analyzed: (1) asphalt layers are modeled using elastic Hooke's model; (2) asphalt layers modeled using viscoelastic Burgers' model and (3) asphalt layers modeled using viscoelastic Huet-Sayegh's model. Unbound base course and improved subgrade were modeled as elastic materials using Hooke's model. Following parameters were assumed: for unbound base course – elastic modulus  $E = 400$  MPa, Poisson's ratio  $\nu = 0.30$  and for improved subgrade – elastic modulus  $E = 100$  MPa, Poisson's ratio  $\nu = 0.35$ .

## 2.2. Pavement loading

For calculations the typical Polish standard wheel loading was assumed, (see Fig. 3). The parameters of pavement loading were assumed as: wheel weight  $P = 50$  kN, circular contact area - diameter  $2r = 0.274$  m, contact pressure  $q = 850$  kPa and speed of moving wheel  $v = 60$  km/h.

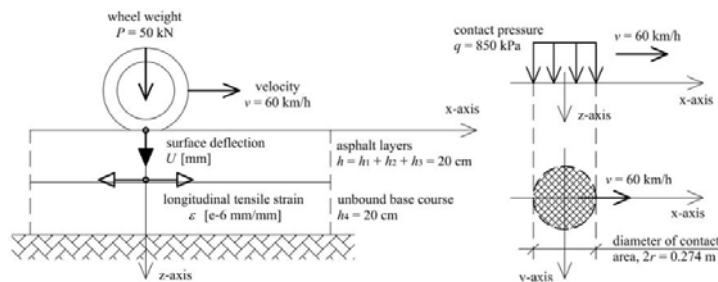


Fig. 3. Assumed pavement loading.

## 2.3. Asphalt layers parameters

Asphalt layers were characterized using parameters of rheological models and Poisson's ratios. In this section, parameters of viscoelastic models (Burgers' and Huet-Sayegh's) and Hooke's elastic model are described in detail. Values of Poisson's ratios of the asphalt layers were adopted according to [9] and shown in Table 1.

Table 1. Poisson's ratio vs. asphalt layer temperature [9].

Temperature $T$ [deg. C]	Poisson's ratio $\nu$ [-]
20	0.385
30	0.436
40	0.472
50	0.495



### Parameters of viscoelastic models of asphalt layers

In the case of Burgers' and Huet-Sayegh's viscoelastic models, properties of asphalt layers are described by elastic moduli and viscous coefficients. Therefore, parameters of viscoelastic models were assumed for each asphalt layer and each temperature, separately. Firstly, parameters of Burgers' and Huet-Sayegh's models were determined based on the results of dynamic modulus and phase angle tests [5] using VEROAD software. Table 2 presents parameters of Burgers' and Huet-Sayegh's models depending on the temperature.

Table 2. Burgers' and Huet-Sayegh's models parameters of asphalt layers.

Temp. $T$ [deg. C]	Burgers' model parameters				Huet-Sayegh's model parameters					
	$E_1$ [MPa]	$E_2$ [MPa]	$\eta_1$ [MPa.s]	$\eta_2$ [MPa.s]	$E_a$ [MPa]	$E_b$ [MPa]	$\eta_a$ [MPa.s]	$\eta_b$ [MPa.s]	$k_a$ [-]	$k_b$ [-]
SMA8 45/80-55										
20	8,731	2,645	869	599	59	20 041	1.841E+05	1.841E+07	0.35	0.7
30	4,947	713	334	335	59	20 041	1.023E+04	1.023E+06	0.35	0.7
40	2,803	192	128	187	59	20 041	8.484E+02	8.484E+04	0.35	0.7
50	1,588	52	49	104	59	20 041	1.049E+02	1.049E+04	0.35	0.7
AC16W 35/50										
20	18,327	5,973	2,710	1,391	70	33 930	1.882E+06	1.882E+08	0.33	0.7
30	10,402	1,541	983	626	70	33 930	1.122E+05	1.122E+07	0.33	0.7
40	5,904	398	357	282	70	33 930	8.167E+03	8.167E+05	0.33	0.7
50	3 351	103	130	127	70	33 930	7.262E+02	7.262E+04	0.33	0.7
AC22P 35/50										
20	23,172	10,730	4,313	2,457	70	36 930	1.708E+07	1.708E+09	0.31	0.6
30	15,102	2,471	1,743	1,035	70	36 930	1.060E+06	1.060E+08	0.31	0.6
40	8,582	569	705	436	70	36 930	5.383E+04	5.383E+06	0.31	0.6
50	5,223	131	285	184	70	36 930	2.239E+03	2.239E+05	0.31	0.6

### Parameters of elastic models of asphalt layers

Properties of asphalt layers, modeled as Hooke's materials, are described by elastic moduli. It was assumed that elastic moduli are equal to the stiffness moduli, determined on the basis of dynamic modulus and phase angle test results [5]. Figure 4 presents stiffness modulus master curves at reference temperature  $T_{ref} = 20^\circ\text{C}$  and time-temperature shift factor ( $\alpha_T$ ) vs. temperature.

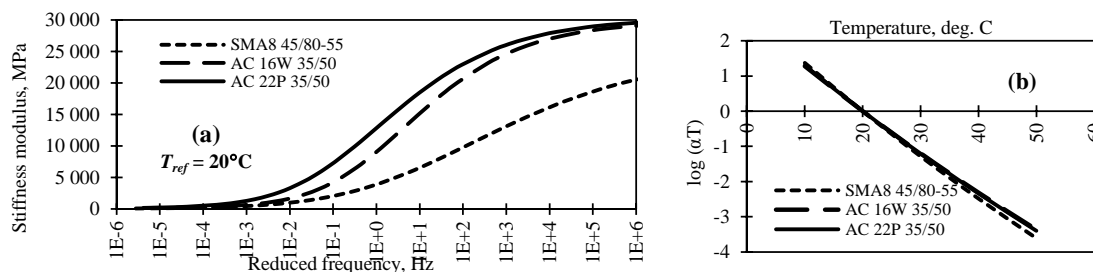


Fig. 4. Master curves (a) and logarithm of time-temperature shift factor  $\alpha T$  (b) for analyzed HMA [5].

To determine the stiffness moduli from master curve specific temperature and loading frequency must be known. According to the authors' simplified assumption, temperature is independent of the depth, what means that the same temperature occurs in all asphalt layers. Whereas, the loading frequency was assumed as independent of the depth (the greater depth, the longer loading time and the smaller loading frequency). Because there are few different asphalt layers (different material properties), the loading distribution is not the same at any depth. Consequently, frequency does not vary with the depth in the same way in each asphalt layer. The authors calculated loading frequencies at the middle of the thickness of each asphalt layer. For these calculations the method described in the *Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures* [2] was used. The method consists of transformation multilayered pavement structure (with asphalt layers described by various elastic moduli  $E^{(n)}$  and Poisson's ratios  $\nu^{(n)}$ ) to a new one-layer structure which is described by elastic modulus  $E^{(SG)}$  and Poisson's ratio  $\nu^{(SG)}$  of the subgrade. The method assumes that the angle of load distribution in the subgrade is equal to  $45^\circ$ . Based on that, it is possible to determine a special depth  $Z_{eff}$ , called effective depth, in transformed pavement structure where the loading frequency is exactly the same as loading frequency at the depth of interest in multilayered pavement structure – equation (1).

$$Z_{eff}^{(n)} = \sum_{i=1}^{i=n-1} \left( h_i \times \sqrt[3]{\frac{E^{(i)}}{E^{(SG)}} \times \frac{1-\nu_{SG}^2}{1-\nu_i^2}} \right) + 0.5 h_n \times \sqrt[3]{\frac{E^{(n)}}{E^{(SG)}} \times \frac{1-\nu_{SG}^2}{1-\nu_i^2}} \quad (1)$$

where:  $n$  - the number of the asphalt layer of interest,  $h_i$  - the thicknesses of asphalt layer  $i$  and other symbols as above. To calculate loading frequencies equation (2) was used.

$$f^{(n)} = \frac{v}{2r + 2Z_{eff}^{(n)}} \quad (2)$$

where:  $f^{(n)}$  – loading frequency [Hz] in the middle of layer  $n$ ,  $2r$  – diameter [m] of contact area;  $v$  – speed [m/s];  $Z_{eff}^{(n)}$  – effective depth for the middle of asphalt layer  $n$  (from equation (1)).

To summarize, there are four general steps to determine the elastic modulus  $E^{(n)}$  for asphalt layer  $n$  at temperature  $T$ :

- 1) Determination of moduli  $E^{(i)}$  and Poisson's ratios  $\nu^{(i)}$  for asphalt layers from  $i = 1$  to  $i = n$  (depending on their temperatures) and elastic modulus  $E^{(SG)}$  and Poisson's ratio  $\nu^{(SG)}$  for the subgrade.
- 2) Determination of effective depth  $Z_{eff}^{(n)}$ , which corresponds to the depth of the middle of asphalt layer  $n$  – equation (1).
- 3) Calculation of loading frequency  $f^{(n)}$  at the effective depth  $Z_{eff}^{(n)}$  for assumed loading parameters – equation (2).
- 4) The use of master curve for material of asphalt layer  $n$  to determine stiffness modulus  $|E^*|$  depending on  $f^{(n)}$  and  $T$ . The final assumption:  $E^{(n)} = |E^*|$ .

For example, determination of elastic modulus  $E^{(3)}$  for asphalt base course at temperature  $T = 30^\circ\text{C}$  is following:

Ad. 1) The parameters of asphalt layers (from Tables 1 and 2) at temperature  $T = 30^\circ\text{C}$  are:  $E^{(1)} = 4,947$  MPa;  $E^{(2)} = 10,402$  MPa;  $E^{(3)} = 15,102$  MPa and  $\nu^{(1)} = \nu^{(2)} = \nu^{(3)} = 0.436$ . The parameters of the subgrade (from the section 2.1) are:  $E^{(SG)} = 100$  MPa and  $\nu^{(SG)} = 0.35$ .

Ad. 2) The effective depth, calculated from equation (1), is equal to  $Z_{eff}^{(3)} = 0.217$  m.

Ad. 3) For assumed loading parameters ( $2r = 0.274$  m;  $v = 60$  km/h) from the section 2.2, the loading frequency, calculated from equation (2), is equal to  $f^{(3)} = 21.6$  Hz.

Ad. 4) Using the master curve for AC 22P 35/50 (Fig. 4) and  $f^{(3)} = 21.6$  Hz,  $T = 30^\circ\text{C}$ , the stiffness modulus is equal to  $|E^*| = 13,580$  MPa. Therefore, the final result is  $E^{(3)} = |E^*| = 13,580$  MPa.

Elastic moduli for the other cases were calculated using the same procedure. Results are presented in Table 3.

Table 3. Loading frequencies and elastic moduli (parameters of Hooke's elastic model) of asphalt layers.

Layer	Asphalt layer temperature $T$ [°C]	Effective depth $Z_{eff}^{(n)}$ [m]	Loading frequency $f^{(n)}$ [Hz]	Elastic modulus $E^{(n)}$ [MPa]
$n = 1$ wearing course (SMA8 45/80-55)	20	0.030	49.8	8,769
	30	0.027	51.0	4,932
	40	0.023	52.2	2,418
	50	0.019	53.3	1,098
$n = 2$ binder course (AC 16W 35/50)	20	0.119	32.5	18,174
	30	0.104	34.6	11,376
	40	0.090	36.8	5,606
	50	0.076	39.1	2,318
$n = 3$ asphalt base course (AC 22P 35/50)	20	0.283	19.8	19,942
	30	0.249	21.6	13,580
	40	0.217	23.6	7,474
	50	0.186	25.8	3,378

### 3. Calculations results and their analysis

In this article two parameters are presented and analyzed: pavement deflections  $U$  [mm] at the depth  $z = 0$  cm and horizontal tensile strain  $\varepsilon$  [10<sup>-6</sup> mm/mm] at the bottom of asphalt layers (at the depth  $z = 20$  cm). Both quantities were calculated under the center of the pavement loading area ( $x = y = 0$ ). Additionally, the relative differences between results obtained for various models were calculated: (1) Burgers' vs. Hooke's models in relation to Hooke's model; (2) Huet-Sayegh's vs. Hooke's models in relation to Hooke's model; (3) Huet-Sayegh's vs. Burgers' models in relation to Burgers' model.

#### 3.1. Pavement deflections

Figures 5 present calculated deflections  $U$  [mm] on the pavement surface and relative percentage differences of deflections vs. pavement temperature  $T$  [°C], respectively.

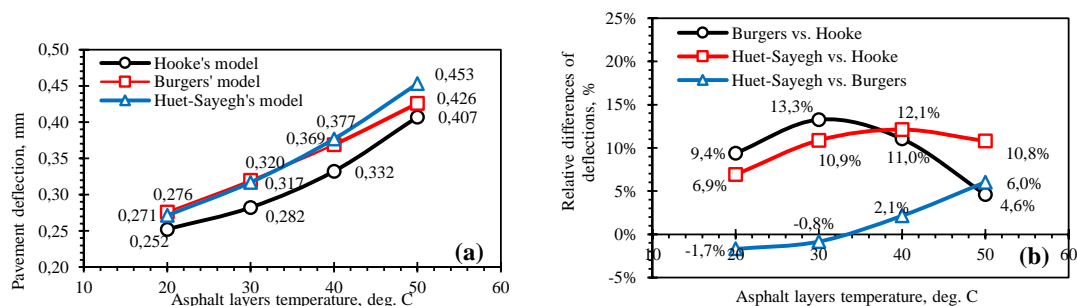


Fig. 5. Pavement deflection (a) and relative differences between them (b) vs. pavement temperature.

The higher temperature causes greater deflection on the pavement surface. The calculations showed that deflection at temperature 50°C is 50-70% greater than the deflection at temperature 20°C. The reasons of that are: the increase of importance of viscous properties, smaller elastic moduli and smaller viscous coefficients at high temperatures (see Tables 2 and 3).

However, the differences between pavement deflections calculated under moving load for various asphalt models at the temperature range from 20°C to 50°C are not greater than 0.05 mm, which corresponds to relative differences

not greater than 14%. This is not very significant value from the practical point of view. However, taking into account irreversible (permanent) part of displacements accumulated for a long period of time, the use of various models could lead to significant differences between displacements on the pavement surface. It is worth to note that vertical displacements (and also strains) in the case of Hooke's model and Huet-Sayegh's model are fully reversible and do not lead to permanent pavement deformations in opposition to Burgers' model.

The differences between Burgers' and Huet-Sayegh's viscoelastic models are small and not greater than 0.03 mm (relatively 6%) at any pavement temperature. The biggest difference was obtained at the highest temperature 50°C. Except that one specific extremely high temperature, the differences are not greater than 0.008 mm (relatively about only 2%) what it means that there are no significant differences between deflections for Burgers' and Huet-Sayegh's models and both models can be used interchangeably for calculations of pavement deflections.

Analyzing the differences between elastic and viscoelastic models, it can be seen that the smallest pavement deflections were obtained for Hooke's elastic model at each of analyzed temperatures. The highest differences reach up to 0.05 mm (relatively about 12%) for Huet-Sayegh's model and up to 0.04 mm (relatively about 13%) for Burgers' model. In the case of elastic model the elapsed time affect only the increase of deformations caused by approaching wheel (increase of loading). However, viscoelastic models include viscous dampers in which deformations are dependent on the loading time. Because of that deformation of viscoelastic material is dependent not only on the increase of loading but also on the loading time which takes part in causing deformations. In opposite to the case of elastic model, the total effect is a sum of both factors.

### 3.2. Strains at the bottom of asphalt layers

Figures 6 present calculated tensile strains  $\varepsilon$  [ $10^{-6}$  mm/mm] at the bottom of asphalt layers and relative differences of strains depending on pavement temperature  $T$  [°C], respectively.

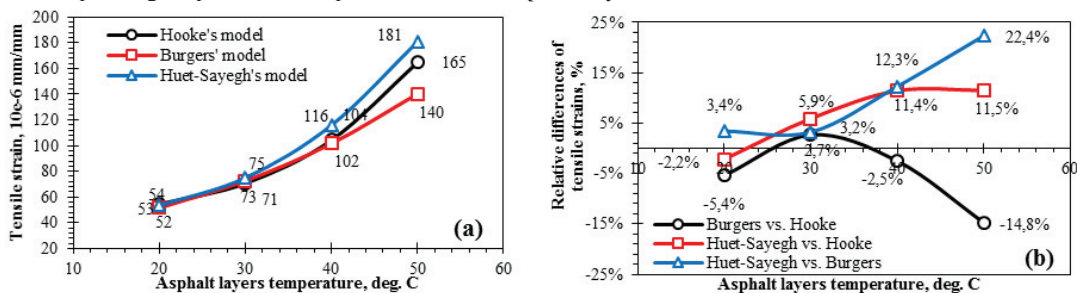


Fig. 6. Strains at the bottom of asphalt layers (a) and relative differences between them (b) vs. pavement temperature.

The increase of pavement temperature causes relatively greater increase of tensile strains at the bottom of asphalt layers than the increase of pavement surface deflections. The calculated tensile strains at temperature 50°C are several times greater than strains at temperature 20°C. In general, the highest values of strains were obtained for Huet-Sayegh's viscoelastic model and the lowest values were obtained for Burgers' model.

In the case of strains at the bottom of asphalt layers the differences between Burgers' and Huet-Sayegh's viscoelastic models are insignificant at temperatures from 20°C to 30°C (relative differences are not greater than 3.5%), so the use of each of the viscoelastic models gives almost the same results in strains at the bottom of asphalt layers. The higher pavement temperature the greater difference between viscoelastic models in considered context. The smallest strains were obtained for Burgers' viscoelastic model (about 22% smaller than for Huet-Sayegh's viscoelastic model at temperature 50°C). In the terms of strains at the bottom of asphalt layers at extremely high pavement temperatures, the differences may be significant, so the choice of type of viscoelastic model may be relevant.

Quite similar values of strains were obtained for all models of asphalt layers at pavement temperature 20-30°C. The differences between them are not greater than  $6 \times 10^{-6}$  mm/mm, what corresponds to relative differences not

greater than 6%. At temperature higher than 30°C the differences become greater and thus the more significant. In general, the highest strains were obtained for Huet-Sayegh's model and the lowest for Burgers' model. Relative differences between strains for viscoelastic and elastic models are not greater than 15% what corresponds to strain difference not more than  $25 \times 10^{-6}$  mm/mm at the highest analyzed temperature (50°C). In general, the higher temperature the greater differences between strains.

It can be noted that the highest sensitivity of strains to pavement temperature were obtained for Huet-Sayegh's viscoelastic model and Hooke's elastic model, whereas the lowest sensitivity were obtained for Burgers' viscoelastic model (see graphs slopes in Figure 6). It means that the increase of pavement temperature causes greater increase of strains in the case of Huet-Sayegh's or Hooke's model than Burgers' model.

#### 4. Conclusions

- (1) The increase of pavement temperature from 20°C to 50°C may cause increase of pavement deflections of tens of percent (depending on the rheological model) and several greater strains at the bottom of asphalt layers under center of loading.
- (2) The sensitivity of deflections to temperature is the greatest for Hooke's elastic model and the smallest for Burgers' viscoelastic model. In the case of strains, the smallest strain sensitivity to pavement temperature were obtained for Burgers' model and similar sensitivity in the cases of Hooke's and Huet-Sayegh's models.
- (3) The differences between deflections obtained for both viscoelastic models (Burgers' and Huet-Sayegh's) are very similar at each of analyzed temperatures and not significant from the practical point of view. Because of that, in analyzed cases, both rheological models can be used interchangeably at high temperatures for the calculations of pavement deflections. The use of each of viscoelastic models leads to obtaining greater deflections than the use of Hooke's elastic model.
- (4) The use of each of analyzed rheological models leads to obtaining quite similar strains at the bottom of asphalt layers at temperatures 20-30°C. However, in the case of extremely high pavement temperatures (higher than 30°C) the differences between strains obtained for various rheological models increase and may be significant. In that case, strains at the bottom of asphalt layers are significantly dependent on the choice of rheological model. Consequently, different strains obtained for various asphalt models may cause significantly different results of pavement fatigue life calculations.

It should be clearly noted that obtained results and drawn conclusions shouldn't be treated as the rules for other cases. For different pavement structures, layers parameters, temperatures and load conditions the results may be different. The calculations and analysis were conducted for the speed  $v = 60$  km/h, which is typical and commonly used value in the analysis and design of pavement structures. However, for lower speeds (in the cases of slow-moving traffic lanes, truck lanes, intersections, parking places, etc.) viscous effects may be more significant and differences between obtained results for various rheological models of asphalt layers may be greater. In the article only an example of flexible pavement structure analysis with asphalt layers modeled with use of various rheological models was presented.

#### References

- [1] General Directorate of National Roads and Motorways, Catalogue of typical flexible and semi-rigid pavement structures, GDDKiA, Warsaw, 2013 (in Polish).
- [2] Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures. Final Document. Appendix CC-3: Updated Traffic Frequency Calculation For Asphalt Layers, National Cooperative Highway Research Program, Transportation Research Board, National Research Council, Illinois 2003.
- [3] P.C. Hopman, The Visco-Elastic Multilayer Program VEROAD, Heron 41(1) (1996).
- [4] J. Judycki et al., Analizy i projektowanie konstrukcji nawierzchni podatnych i półsztywnych, WKŁ, Warszawa, 2014 (in Polish).
- [5] J. Judycki et al., Badanie wpływu zastosowania warstw betonu asfaltowego o wysokim module sztywności (AC-WMS) w konstrukcjach nawierzchni na spękania niskotemperaturowe i na zmniejszenie powstawania deformacji trwałych, Raport końcowy, Gdańsk 2014 (in Polish).
- [6] R. Nagórski et al., Mechanika nawierzchni drogowych w zarysie, PWN, Warszawa, 2014 (in Polish).
- [7] A.C. Pronk, Revival of the Huet-Sayegh Response Model, DWW-2003-29, RHED, Delft 2003.
- [8] VEROAD: User manual, Version 2000 April. Appendix B: Additional Theory, NPC bv, Utrecht, Netherlands 2000.
- [9] E.J. Yoder, M.W. Witzczak, Principles of Pavement Design, 2nd Edition, John Wiley & Son Inc., New York 1975.