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# ANALYSIS OF THE IMPACT OF MASS FLOW EXTRACTION ON THE CHANGE OF PARAMETERS IN A LABYRINTH SEAL USING THE STODOLA METHOD

**Paweł Kaszowski, Marek Dzida**

*Gdańsk University of Technology*
*Ul. Narutowicza 11/12, 80-233 Gdańsk, Poland*
*Phone: +48 58 347 1842,*
*e-mail: pawkaszo1@pg.gda.pl*
*dzida@pg.gda.pl*

**Abstract**

*Labyrinth seals are an important element of a steam turbine set design. The use of diagnostic extraction makes it possible to control the operation of a seal by providing information on the thermodynamic parameters along the length of the seal. Diagnostic extraction has a considerable impact on the change of the parameters, the amount of the extracted mass. This article describes the dependence of pressure in the clearance downstream of the extraction and in the chamber in which the extraction point is located, on the amount of the extracted steam. Relation between pressure and nominal seal clearance is discussed, which enables the control of the seal operation. The calculations of the seal operation parameters were performed using a method proposed by Stodola.*

**Keywords:** labyrinth seals, Fanno curve, diagnostic extraction, Stodola

## 1. Introduction

Clearances between the elements of the rotor and the frame are associated with the leaks of the working medium. The resultant losses may be reduced using labyrinth seals that contain a number of cross-section constrictions where the flow rate significantly increases. The kinetic energy of the stream flowing out of a constriction is converted into heat as a result of whirls taking place in the chamber between the constrictions. The seal design should ensure that the flow rate upstream of the next clearance is completely decelerated.

A diagram of a labyrinth seal is given in Figure 1. When analysing flow through such a seal it should be assumed that isentropic expansion takes place in each clearance, while in the chamber kinetic energy is converted into heat as a result of an isobaric process. Generally, it is assumed that the process of kinetic energy conversion into heat is adiabatic, with no heat exchange between the stream and the seal walls.

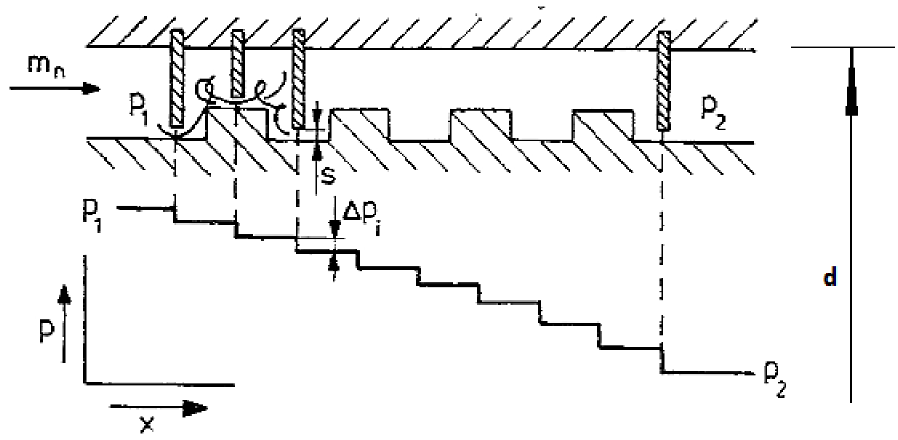


*Fig. 1 Labyrinth seal diagram*

## 2. Stodola method

The labyrinth seal calculation method proposed by Stodola is described below.
The rate at which the working medium flows out of a clearance is expressed as follows:

$$c = \sqrt{2h_s} \tag{1}$$

where $h_s$ – isentropic enthalpy reduction in the analysed clearance.

Assuming that the flow constriction in each clearance is identical and that it is a steady flow, the equation of continuity for the given case looks as follows:

$$m_n v = Ac \tag{2}$$

In equation (2) $A$ stands for an effective cross-section expressed with the following formula:

$$A = \propto A_n \tag{3}$$

where:

$$A_n = \pi d s \tag{4}$$

$A_n$ stands for a geometrical cross-section, while $\alpha$ is the liquid flow coefficient of contraction.

Considering experimental and computational data, an average coefficient of contraction amounts to:

$$\alpha \approx 0{,}8 \qquad [6]$$

Combined equations (1) and (2) result in:

$$\frac{m_n}{A} = \frac{c}{v} = \frac{\sqrt{2h_s}}{v} = \text{const} \tag{5}$$

Equation (5) on an i – s diagram is represented by a Fanno curve that constitutes the geometric locus of the end points of all expansion lines in constrictions, as shown in Figure 2. The diagram shows thermodynamic changes taking place during a liquid flow through a labyrinth seal. There is a change reflecting the isentropic enthalpy reduction during a liquid flow through a seal clearance between points $1_1$ and $2_1$, and a change in isobaric expansion during the outflow of the liquid from the seal clearance between points $2_1$ and $1_2$.

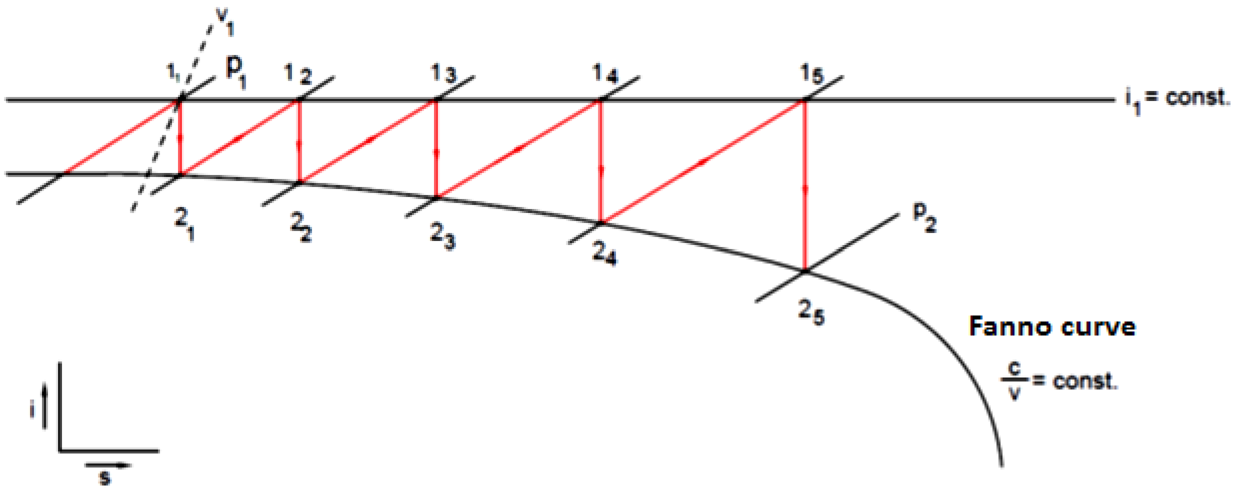


*Fig. 2 Change in a labyrinth seal, $1_1$, $1_2$, $1_3$ – states upstream of clearances, $2_1$, $2_2$, $2_3$ – states downstream of the seal clearance*

The Fanno curve may be established in the following manner. Assuming that the effective dimensions of clearance $A$ cross-section are known. Assuming that the value of leak $m_n$ is known using formula (5)

$$\frac{c}{v} = \frac{m_n}{A} = K \tag{6}$$

the following is determined:

$$v = \frac{\sqrt{2h_s}}{K} \tag{7}$$

Starting from the first clearance, knowing the initial state in point $1_1$ ($p_1$,$v_1$) upstream of the seal, such value of $h_1$ is selected using the trial method that it satisfies equation

$$v_{2_1} = \frac{\sqrt{2h_1}}{K} \tag{8}$$

That way point $2_1$ and pressure downstream of the first clearance are determined. The state upstream of the second clearance $1_2$ is expressed using values $i_{1,2} = i_1$, $p_{1,2} = p_{2,1}$. Further points are determined using the given formula. In practice such a procedure would be tedious, which is why Stodola provided simple, approximate formulas. In case of a large number of fins $z \gg 1$ the pressure drop in the clearance $\Delta p$ is small, therefore enthalpy reduction may be calculated using formula

$$h_s = v\Delta p \tag{9}$$

hence

$$c = \sqrt{2v\Delta p} \tag{10}$$

for the states upstream of clearances

$$i = \frac{\kappa}{\kappa - 1} pv = const \tag{11}$$

Using this dependence for any given clearance the following is correct:

$$p_i v_i = p_1 v_1 \tag{12}$$

If the following is assumed:

$$\kappa = const$$

considering equation (10) and equation (11) in formula (5) the following is obtained

$$\begin{gathered} \frac{m_n}{A} = \sqrt{2\frac{\Delta p}{v}} \quad or \\ \left(\frac{m_n}{A}\right)^2 = 2\frac{\Delta p}{v} = 2\frac{p\Delta p}{pv} \\ p\Delta p = \frac{p_1 v_1}{2}\left(\frac{m_n}{A}\right)^2 \end{gathered} \tag{13}$$

expression (13) is true for any clearance.
The $p\Delta p$ product is identical for each clearance; by summing up equation (13) for all of the clearances the following is obtained:

$$\sum p\Delta p = z\frac{p_1 v_1}{2}\left(\frac{m_n}{A}\right)^2 \tag{14}$$

If drop $\Delta p$ is small, the summation sign may be replaced with integration

$$\int_{p_2}^{p_1} pdp = \frac{{p_1}^2 - {p_2}^2}{2} = \frac{zp_1 v_1}{2}\left(\frac{m_n}{A}\right)^2 \tag{15}$$

In this case expression (14) enables the calculation of leak $m_n$ with the assumed number of teeth $z$

$$m_n = A\sqrt{\frac{{p_1}^2 - {p_2}^2}{z p_1 v_1}} = \alpha A_n \frac{1}{\sqrt{z}}\sqrt{\frac{p_1}{v_1}\left[1 - \left(\frac{p_2}{p_1}\right)^2\right]} \tag{16}$$

Leak $m_n$ is proportional to the reciprocal of the $z$ seal number root

$$m_n \sim \frac{1}{\sqrt{z}} \tag{17}$$

Formula (16) may be simplified

$$m_n = A_n \Phi_s \sqrt{\frac{p_1}{v_1}} \tag{18}$$

where

$$\Phi_s = \alpha\sqrt{\frac{1}{z}\left[1 - \left(\frac{p_2}{p_1}\right)^2\right]} \tag{19}$$

Using property (16) it is possible to solve a reverse problem where the size of leak $m_n$ is assumed by searching for the number of constrictions of a flow through the seal.

$$z = \left(\frac{\alpha A_n}{m_n}\right)^2\left[1 - \left(\frac{p_2}{p_1}\right)^2\right]\frac{p_1}{v_1} \tag{20}$$

The problem presented in this way can sometimes be more difficult to solve, in case of minor leaks $m_n$ leading to a large number of teeth $z$.

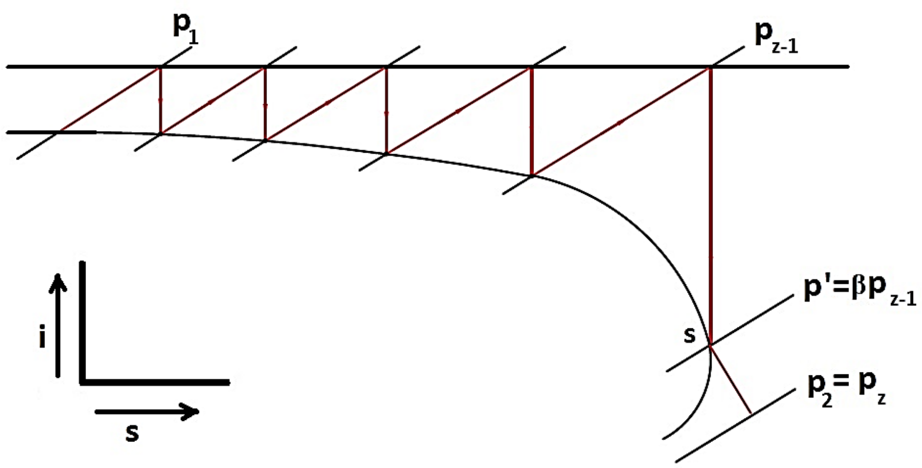


*Fig. 3 Expansion curve for a labyrinth seal*

An increase in specific volume along the Fanno curve results in a constant increase in velocity and enthalpy reduction in the subsequent clearances, as shown in Figure 2. Critical velocity $c=a$ may be reached in the ultimate clearance, therefore further expansion to ultimate pressure $p_2$ takes place downstream of the clearance, Figure 3. In such a case the described procedure is applied until and including the penultimate clearance. Critical pressure occurs in the ultimate clearance.

## 3. Calculations

The amount of steam extracted as a result of diagnostic extraction (Fig. 4) has a significant impact on the pressure distribution in a labyrinth seal and the enthalpy change curve (Fanno curve). For this purpose calculations were performed that show the dependence of differential pressure of a labyrinth without diagnostic extraction $\Delta m_0 = 0.0\ kg/s$ and a labyrinth with mass extraction that amounts to, respectively:

- $\Delta m_1 = 0.2\ kg/s$
- $\Delta m_2 = 0.4\ kg/s$
- $\Delta m_3 = 0.6\ kg/s$

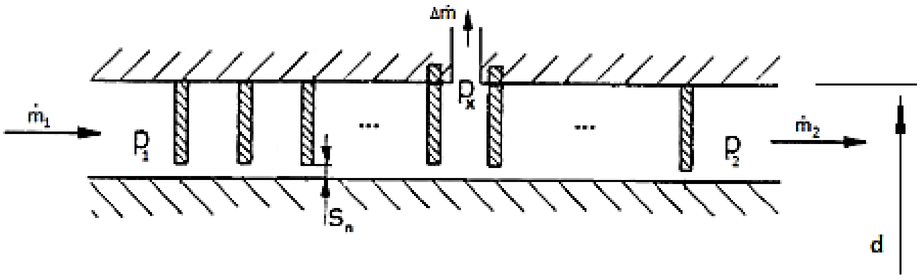


*Fig. 4 Diagram of a labyrinth seal with diagnostic extraction*

Parameters of operation of a high pressure (HP) turbine interframe seal listed in Table 1 were assumed for the calculations.

*Table 1. Parameters of an HP turbine interframe seal*

| | | | |
|---|---|---|---|
| Pressure upstream of seal | $p_1$ | MPa | 9.861 |
| Steam temperature upstream of seal | $t_1$ | °C | 522.5 |
| Enthalpy upstream of seal | $h_1$ | kJ/kg | 3433 |
| Steam pressure downstream of seal | $p_2$ | MPa | 4.204 |
| Steam flow through seal | $m_n$ | kg/s | 2.228 |
| Shaft diameter | $d$ | mm | 475 |
| Nominal seal clearance | $s_n$ | mm | 1.0 |
| Number of seal teeth | $z$ | | 80 |

Further analysis is based on a labyrinth seal calculation method proposed by Stodola. In accordance with this method, a mass flow through a labyrinth is determined with the following formula:

$$m_n = \alpha A_n \frac{1}{\sqrt{z}} \sqrt{\frac{p_1}{v_1}\left[1 - \left(\frac{p_2}{p_1}\right)^2\right]} \tag{21}$$

It is proportional to the reciprocal of the z seal teeth number root in accordance with the following dependence:

$$m_n \sim \frac{1}{\sqrt{z}} \tag{22}$$

Expression (21) was used to derive a formula for the coefficient of contraction of a flow through the clearance of a labyrinth seal:

$$\alpha = \frac{m_n\sqrt{z}}{A_n\sqrt{\frac{p_1}{v_1}\left[1-\left(\frac{p_2}{p_1}\right)^2\right]}} \tag{23}$$

where:

$m_n$ – mass flow through the seal,
$\alpha$ – flow coefficient of contraction,
$A_n$ – clearance effective cross-section,
$z$ – number of the seal teeth,
$p_1$ – pressure upstream of the seal,
$p_2$ – pressure downstream of the seal,
$v_1$ – specific volume of steam.

The rearrangement of formula (21) provided an expression for the value of pressure $p_x$ in the n clearance of the seal:

$$p_x = \sqrt{p_1^2 - \frac{m_1^2 p_1 z_1 v_1}{\alpha A_n}} \tag{24}$$

## 4. Results

The coefficient of flow contraction calculated using equation (3) equals $\alpha = 0.904$. The value of pressure in clearance $z_{70} = 70$ is $p_{70} = 52.48\,bar$. Pressure in this clearance was chosen because diagnostic extraction was used downstream of the 70th tooth of the seal. Pressure value in clearance $z = 70$ remains constant irrespective of whether there is diagnostic extraction or not. A pressure change may be observed in the next clearance $z = 71$ located downstream of the mass extraction. Below is a list of pressure values in the clearance just downstream of the diagnostic extraction for $z = 71$ at different values of the extracted mass $\Delta m$, respectively for:

- $p_{x_0} = 52.06\,bar$, for $\Delta m_0 = 0.0 kg/s$ (no extraction)

And for extraction values they amount to, respectively

- $p_{x_1} = 52.09\,bar$, for $\Delta m_1 = 0.2\,kg/s$
- $p_{x_2} = 52.13\,bar$, for $\Delta m_2 = 0.4\,kg/s$
- $p_{x_3} = 52.17\,bar$, for $\Delta m_3 = 0.6\,kg/s$

The absolute differential pressure between clearance $z = 70$ and $z = 71$ for the analysed clearance looks as follows:

- $\Delta p_0 = 0.42\ bar$, for $\Delta m_0 = 0.0 kg/s$ (no extraction)
- $\Delta p_1 = 0.39\ bar$, for $\Delta m_1 = 0.2\ kg/s$
- $\Delta p_2 = 0.35\ bar$, for $\Delta m_2 = 0.4\ kg/s$
- $\Delta p_3 = 0.31\ bar$, for $\Delta m_3 = 0.6\ kg/s$

The value of steam pressure in clearance $z = 71$ downstream of the extraction point increases proportionally to the increase in the value of steam $\Delta m$ extracted through the diagnostic extraction point. The dependence of absolute differential pressure $\Delta p_i$ between clearance $z = 70$ and $z = 71$ for a seal with and without extraction is shown in Figure 5.

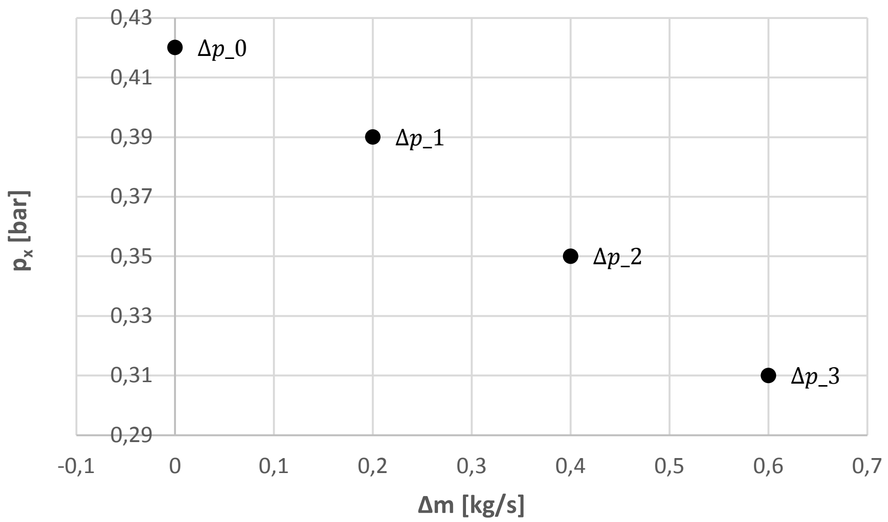


*Fig. 5 Dependence of pressure increase $\Delta p_x$ between a seal with and without extraction for clearance $z = 71$ for different values of mass extraction $\Delta m$.*
*$\Delta p_0$- no extraction, $\Delta p_1$ for $\Delta m_1 = 0.2\ kg/s$, $\Delta p_2$ for $\Delta m_2 = 0.4\ kg/s$, $\Delta p_3$ for $\Delta m_3 = 0.6\ kg/s$*

The diagram shows that the change in pressure between the 70th and the 71st tooth of the seal drops with an increase in the value of the steam jet extracted at the diagnostic extraction point. The observed falling tendency is caused by the fact that the steam jet flowing through the seal downstream of the extraction point is smaller than upstream of it, which with the assumed isentropic expansion implies a smaller pressure drop in this clearance.

A change in pressure in the chamber where the diagnostic extraction point is located can also be observed. Equation (25) specifies the mass flow through a labyrinth seal:

$$m_{70} = m_{71} + \Delta m \tag{25}$$

where:

$m_{70}$ – mass flow into a labyrinth with diagnostic extraction
$m_{71}$ – mass flow out of a labyrinth with diagnostic extraction
$\Delta m$ – steam jet value at the diagnostic extraction point

Using formula (21) equations determining the leakages stream in the individual sections of a labyrinth seal were established:

$$m_{70} = \alpha A_n \frac{1}{\sqrt{z_{70}}} \sqrt{\frac{p_1}{v_1}\left[1 - \left(\frac{p_i}{p_1}\right)^2\right]} \tag{26}$$

and

$$m_{71} = \alpha A_n \frac{1}{\sqrt{z_{71}}} \sqrt{\frac{p_i}{v_i}\left[1 - \left(\frac{p_2}{p_i}\right)^2\right]} \tag{27}$$

The consideration of equality $p_1 v_1 = p_i v_i$ and the substitution of equation (26) and (27) for formula (25) result in a 4-degree polynomial equation, the solution for which is the value of pressure $p_i$ in the chamber downstream of the 70$^{th}$ tooth of the seal.
The following results are obtained:

- $p_{x_0} = 52.55\ bar$, for $\Delta m_0 = 0.0 kg/s$ without diagnostic extraction
- $p_{x_1} = 54.06\ bar$, for $\Delta m_1 = 0.2\ kg/s$
- $p_{x_2} = 55.61\ bar$, for $\Delta m_2 = 0.4\ kg/s$
- $p_{x_3} = 57.22\ bar$, for $\Delta m_3 = 0.6\ kg/s$

Figure 6 shows the dependence of the value of differential pressure $p_i$ in the chamber where the diagnostic extraction point is located between a seal without extraction and a seal with extraction, calculated using the following formula:

$$\Delta p = |p_i - p_{i+1}| \tag{28}$$

That amount to, respectively:

- $\Delta p_1 = 1.51\ bar$, for $\Delta m_1 = 0.2\ kg/s$
- $\Delta p_2 = 3.09\ bar$, for $\Delta m_2 = 0.4\ kg/s$
- $\Delta p_3 = 4.67\ bar$, for $\Delta m_3 = 0.6\ kg/s$

in the function of the extracted mass $\Delta m_i$ in the seal.
An increase in pressure in the chamber downstream of the 70$^{th}$ tooth of the seal, progressing with the increase in the value of the extracted steam, can be observed in comparison with a seal without extraction.

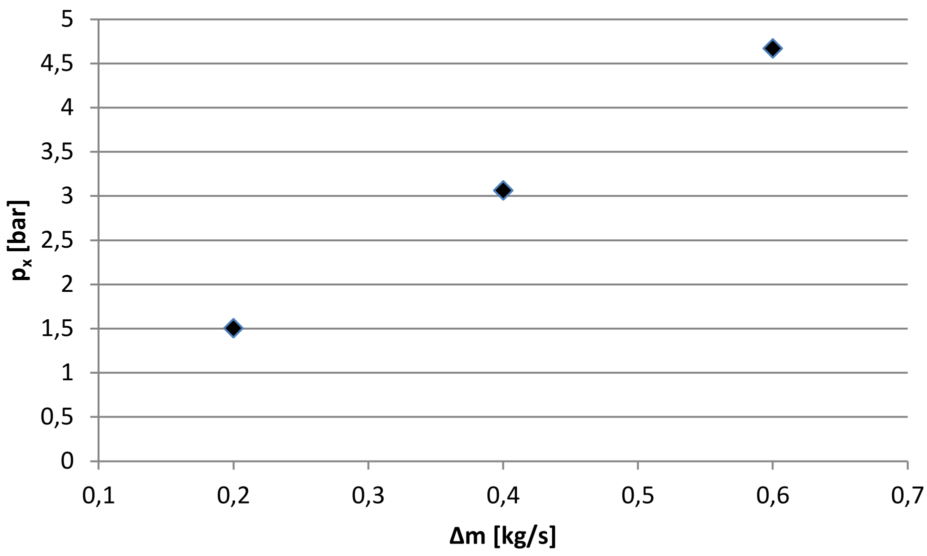


*Fig. 6 Dependence of differential pressure $\Delta p_x$ in a chamber with diagnostic extraction on the mass extraction value*

When analysing the obtained results shown in the form of a diagram in Figures 6 and 7, an increase in pressure $p_i$ in the chamber between the 70$^{th}$ and the 71$^{st}$ tooth of the seal where a diagnostic extraction point is located may be observed that depends on the size of the extracted mass. A rapid increase in the enthalpy value will be visible in the location of the diagnostic extraction point in diagram $h-s$. In the remaining section, from the chamber with an extraction point up to the end of the seal, the resistance of the flow through the clearances of the seal for a decreased leak will be lower. This is proven by the occurrence of lower flow rate values in clearances and smoother curves of a pressure drop in the enthalpy/entropy dependence diagram.

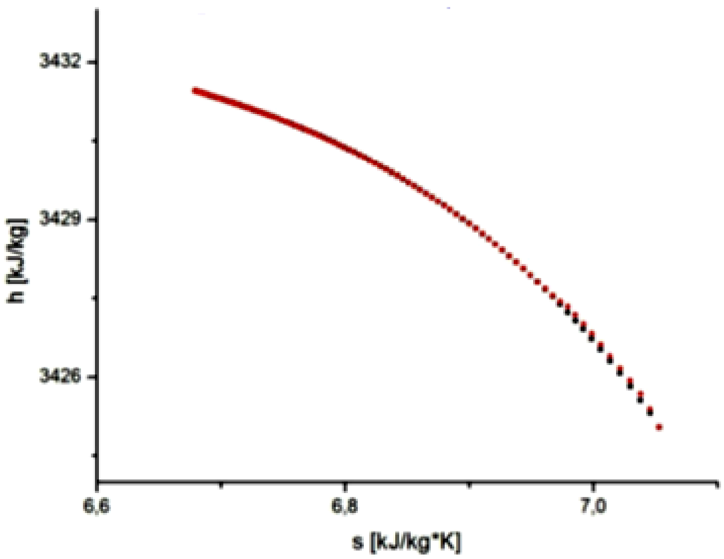


*Fig. 7 Fanno curve for a labyrinth seal without extraction (black) and with diagnostic extraction (red)*

Nominal seal clearance $s_n$ also affects the change in thermodynamic parameters in the seal, especially pressure. Given fixed mass extraction amounting to $\Delta m = 0.4 kg/s$, calculations were performed aimed at finding the value of pressure in a chamber with diagnostic extraction at the change of parameter $s_n$. The difference between the resultant pressure values and the pressure value for a seal without extraction is shown in Figure 8.

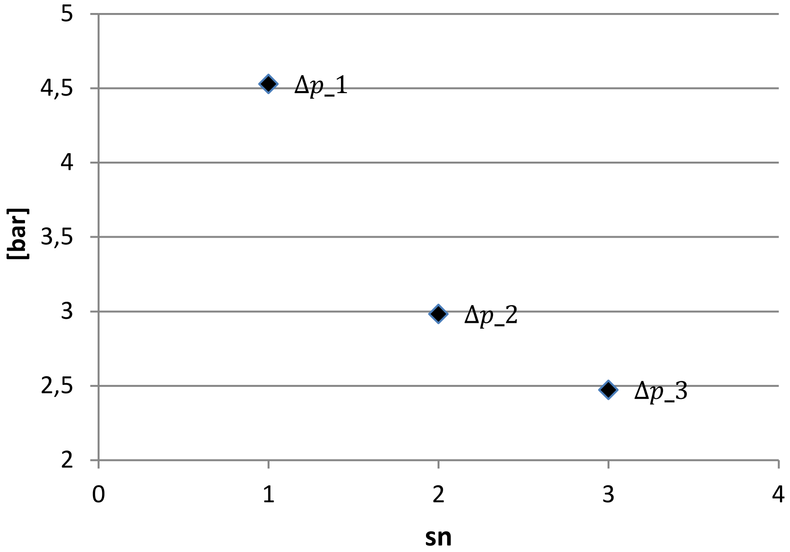


*Fig. 8 Dependence of differential pressure in a chamber with and without diagnostic extraction in function $s_n$ of nominal seal clearance for $\Delta m = 0.4 kg/s$*

On the basis of the obtained results it may be concluded that the change in differential pressure $\Delta p_1$, $\Delta p_2$, $\Delta p_3$ between a seal with and without extraction is related with increasing nominal seal clearance and a reduction in the rate of mass flow through a labyrinth seal associated with a change in the clearance height.

## 5. Final conclusions

Labyrinth seals are an important element of a steam turbine set design. The use of diagnostic extraction makes it possible to control the operation of a seal by providing information on the thermodynamic parameters along the length of the seal. Diagnostic extraction has an impact on the change of the parameters, the amount of the extracted mass. This article described the dependence of pressure in the clearance downstream of the extraction and in the chamber in which the extraction point is located, on the amount of the extracted steam. Relation between pressure and nominal seal clearance was discussed, which enables the control of the seal operation. All the dependences were derived in accordance with the method proposed by Stodola. The results of calculations given above clearly show that the thermodynamic parameters in a labyrinth seal (in the location of a diagnostic extraction point to be precise) depend on the amount of the extracted mass and nominal seal clearance. The presented calculations are the first attempt at applying the Stodola method. Further analyses will focus on confirming the obtained results using CFD modelling methods.

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