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# EFFECTIVENESS OF A MATHEMATICAL MODEL IN SIMULATING NONLINEAR MECHANICAL BEHAVIOUR OF A SEISMIC ISOLATION SYSTEM MADE OF POLYMERIC BEARINGS

# OCENA EFEKTYWNOŚCI MODELU MATEMATYCZNEGO DO OPISU NIELINIOWEGO ZACHOWANIA WIBROIZOLACJI SEJSMICZNEJ W POSTACI ŁOŻYSK POLIMEROWYCH

A b s t r a c t

The present study was focused on determining the effectives of a nonlinear mathematical model in simulating complex mechanical behaviour of a seismic isolation system made of Polymeric Bearings. The proposed mathematical model defines the lateral force as a nonlinear function of the shear displacement and the deformation velocity. The effectiveness of the proposed mathematical model was verified by comparing the seismic response of a 2.30 m high two-storey structure model with the results obtained from the detailed numerical analysis. The results obtained from the numerical investigation using lumped-mass models confirmed that the proposed nonlinear mathematical model can be successfully adopted to simulate the complex mechanical behaviour of the Polymeric Bearings in numerical studies..

*Keywords: Polymeric Bearings, base isolation, mathematical model, shaking table testing, earthquakes, dynamic excitations*

S t r e s z c z e n i e

W pracy dokonano oceny efektywności modelu matematycznego, opisującego nieliniowe zachowanie prototypu wibroizolacji sejsmicznej w postaci Łożysk Polimerowych. W zaproponowanym modelu matematycznym siła pozioma jest nieliniową funkcją przemieszczenia oraz prędkości. Oceny efektywności modelu matematycznego do opisu nieliniowego zachowania łożysk polimerowych dokonano poprzez porównanie wyników badań eksperymentalnych przeprowadzonych na stole sejsmicznym, w których dwupiętrowy model konstrukcji o całkowitej wysokości 2,30 m poddano różnym obciążeniom dynamicznym, z wynikami analiz numerycznych. Duża zgodność wyników analiz numerycznych z wynikami otrzymanymi z badań eksperymentalnych potwierdziła poprawność zaproponowanego modelu matematycznego do symulacji zachowania Łożysk Polimerowych.

*Słowa kluczowe: Łożyska Polimerowe, wibroizolacja sejsmiczna, model matematyczny, stół sejsmiczny, trzęsienia ziemi, wymuszenia dynamiczne*

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## 1. Introduction

Earthquakes are identified among the most severe and unpredictable threats to structures all around the world. Strong ground motions may cause of lot damage in a wide variety of ways, sometimes leaving thousands of casualties in their wake. During the last few years alone, the world has witnessed many major earthquakes, five of which have caused far-reaching consequences of a national scale for Haiti (January 2010), Chile (February 2010), New Zealand (February 2011), Japan (March 2011), and Turkey (October 2011). Damaging earthquakes also take place from time to time in Poland (although their effects are not so dangerous as in other countries.

In spite of a number of available technical solutions of protecting structures against seismic forces, earthquake peril remains the most sinister and unpredictable natural disaster, for which many countries are still not fully prepared. Due to the randomness of occurrence, destructive potential, and insufficient early-warning systems, there is an incessant need of developing and improving earthquake protective systems against seismic loads and their devastating effects in order to minimise the loss of life and property damage.

Base isolation is considered to be one of the most popular and effective structural control methods of protecting structures against seismic forces. Generally, base isolation systems work by separating the building from the horizontal components of the earthquake ground motion by introducing a layer with low horizontal stiffness between the structure and its foundation [3–9]. The main concept of this strategy is to modify structural dynamic properties, such as the fundamental period of vibration and the damping ratio, so that structures can respond more favourably to dynamic excitations. Base isolators, such as Lead-Rubber Bearings, High-Damping Rubber Bearings, and Friction Pendulum Bearings, are commonly used in practice in many earthquake-prone countries in order to enhance structural safety and reliability. However, the past few decades have witnessed a tremendous progress in material engineering. Because of the impressive range of useful and exceptional properties, new materials play an essential and ubiquitous role in many fields of science, also including earthquake engineering. The recent development of polymeric materials has resulted in an increased number of new isolators and modifications of the existing ones.

The present paper aims to verify the effectiveness of a nonlinear mathematical model in simulating complex mechanical behaviour of a prototype seismic isolation system made of Polymeric Bearings (PBs). In order to construct the seismic bearings considered in research, a specially prepared elastomer with improved damping characteristics was employed. The proposed mathematical model was adopted in numerical analysis, where the previously examined two-storey experimental model subjected to various seismic excitations, was idealised as a multi-degree-of-freedom system.

## 2. Polymeric Bearings

In order to construct the seismic isolation bearings considered in this research, a specially prepared flexible polyurethane-based elastomer was employed. It was produced by one of the top international chemical companies specifically for this investigation program. The chemical composition of this polymer includes certain additives in order to improve its damping properties, which are extremely desirable for energy-dissipation devices, particularly for seismic isolation bearings. The basic mechanical properties of this polymer have already been determined and the results obtained presented in previous publications (see, for example, [10–14]). Generally, the analysed polymer is markedly nonlinear and its mechanical behaviour strongly depends on the strain rate, which is typical for viscoelastic materials [12]. More importantly, a relatively high value of the loss factor obtained from the Dynamic Mechanical Analysis (DMA) confirms its high damping and energy-dissipation properties [13].

PB is made up of a polymer cylinder (28 mm in diameter and 28 mm high) with a centrally located hole (14 mm in diameter) into which a pin-ended steel core (6 mm in diameter) is inserted. Two additional steel anchor plates (30 mm in diameter and 15 mm high) are mounted at both ends of the polymer cylinder. The total height of the PB is 58 mm. The basic components of the PB are shown in Fig. 1.

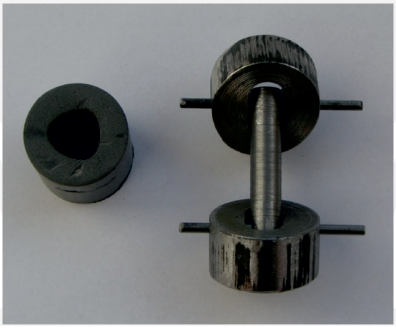

Fig. 1. Components of the PB: a polymer cylinder with a centrally located hole (left) and pin-ended steel core with the anchor plates (right)

The proposed prototype solution ensures that the steel core sustains vertical forces, while the polymer cylinder is subjected to shearing. This way, the steel core prevents the polymer cylinder from carrying vertical loads, which could possibly lead to undesirable bulging of the bearing. It should be underlined, however, that due to uniaxial (horizontal) shaking table used in the investigation, the PBs were intentionally constructed as one-dimensional devices. A multi-dimensional version of the PB has also been proposed and already submitted to the Polish Patent Department [15].

### 3. Shaking table tests

In order to conduct the experimental investigation, a two-storey structure model was firstly prepared. The welded steel frames were constructed using rectangular hollow section elements (RHS 15×15×1.5 mm). The columns were arranged in a rectangular pattern with spacing of 0.465 m along the longitudinal direction and 0.556 m along the transverse one. Additionally, diagonal bracing was used in the sidewall planes to counteract transverse and torsional vibrations. Moreover, three concrete plates (50×50×7 cm) were employed in order to simulate the weight of floors and a foundation slab. The welded steel frames were clamped together using M10 bolts. Eventually, the two-storey structure model is 2.30 m high and weighs 148 kg. The experimental model, with and without the PBs, is shown in Fig. 2.

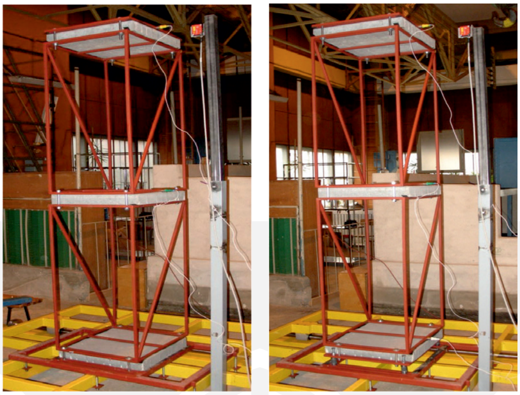

Fig. 2. Fixed-base (left) and base-isolated (right) two-storey structure model

The dynamic properties of the analysed two-storey structure model, fixed-base and base-isolated, were evaluated by conducting the free vibration and sine sweep tests. The results obtained were used to determine the natural frequencies as well as the damping ratios of the experimental model with and without the PBs. In the next stage, the seismic response of the analysed two-storey structure model, fixed-base and base-isolated, during various earthquake ground motions, was extensively studied. The investigation was carried out using the middle-sized shaking table located at the Gdańsk University of Technology (GUT), Poland. The effectiveness of the PBs in reducing structural vibrations was verified by comparing the peak accelerations experienced by the tested model with and without the base isolation system. The results obtained from the comprehensive shaking table study, which explicitly confirm the effectiveness of the PBs as a prototype base isolation system, have already been presented in previous publications [16–18].

## 4. Mathematical model

In nonlinear structural dynamics, there are many different approaches, which can be applied to simulate the mechanical behaviour of base isolation devices. Most of the frequently adopted models, however, are limited to those exhibiting general hysteretic behaviour and cannot be used for an accurate simulation of the force-displacement characteristics of the seismic bearings considered in this study. On the other hand, more complex mathematical models for base isolation devices [19–23] were evaluated for different base isolators and are not appropriate for numerical analysis of the PBs. Therefore, in the present study, a new nonlinear mathematical model to simulate the complex behaviour of the PBs under dynamic excitations is proposed, as defined by Equations 1–3. The proposed model defines the lateral force as a nonlinear function of the shear displacement $u_b(t)$, and the deformation velocity $\dot{u}_b(t)$. In this approach, function parameters $a_1 \div a_5$ were estimated by curve fitting the Equation 1 to the experimentally obtained hysteresis loops using the least squares optimisation method. In order to obtain hysteresis loops, four PBs, supporting a concrete slab (50×50×7 cm), were mounted on the shaking table platform and subjected to dynamic oscillatory tests. The experimental testing was carried out with the excitation frequency of 2 Hz for different shear strain levels. The evaluated set of the function parameters $a_1 \div a_5$ is summarised in Table 1 and the results of the curve fitting algorithm are presented in Fig. 3. The normalised mean square error calculated for the presented fit was calculated to be 4.44%. The results clearly demonstrate that the proposed mathematical model with the evaluated set of function parameters fits the experimental results quite accurately, and therefore can be used for further numerical simulations.

$$F_b(u_b(t),\dot{u}_b(t)) = K_b(u_b(t),\dot{u}_b(t)) \cdot u_b(t) + C_b \cdot \dot{u}_b(t) \tag{1}$$

$$K_b(u_b(t),\dot{u}_b(t)) = a_1 + \frac{a_2}{\cos h[a_3\dot{u}_b(t)] \cdot \cos h[a_4 u_b(t)]} \tag{2}$$

$$C_b = a_5 \tag{3}$$

Table 1

**Evaluated set of the function parameters**

| Parameter | Estimated value |
|---|---|
| $a_1$ | 38.657 [N/m] |
| $a_2$ | 36.300 [N/m] |
| $a_3$ | 23.253 [s/m] |
| $a_4$ | 331.626 [1/m] |
| $a_5$ | 2.193 [kg/s] |

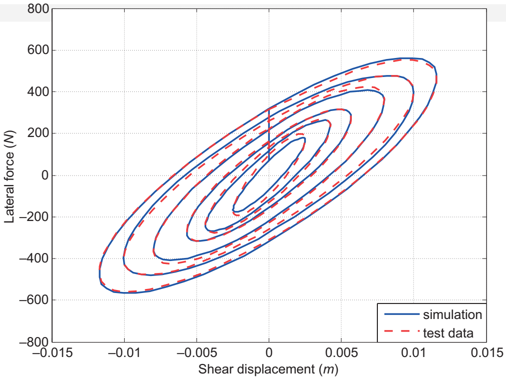


Fig. 3. Results of the curve fitting of the proposed nonlinear mathematical model into the experimentally obtained hysteresis loops

## 5. Numerical analysis

In order to perform the numerical evaluation of seismic response of the previously examined experimental model, lumped-mass systems were adopted [24, 25]. The fixed-base two-storey structure model was idealised as a two-degree-of-freedom (2-DOF) system, whereas for the base-isolated one, a three-degree-of-freedom (3-DOF) system was employed. The numerical simulation of seismic response of the structure model with and without the PBs was performed with the use of the proposed mathematical model (Equations 1–3). The analysed model was subjected to the El Centro earthquake of 1940, and Northridge earthquake of 1994. In order to conduct the numerical evaluation of dynamic response, the unconditionally stable Newmark's average acceleration method was applied [26]. The results obtained from the investigation are presented in Fig. 4–11. Additionally, the Fast Fourier Transform (FFT) functions are also shown in Fig. 12–15. The peak accelerations computed at the top of the analysed two-storey structure model with and without the PBs during different seismic excitations are briefly reported in Table 2.

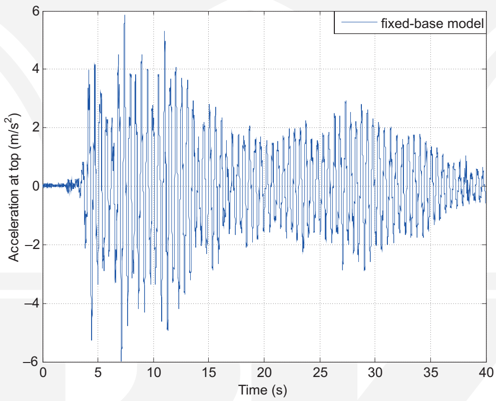


Fig. 7. Time-acceleration history plot for the base-isolated model during the 1940 El Centro earthquake (numerical analysis)

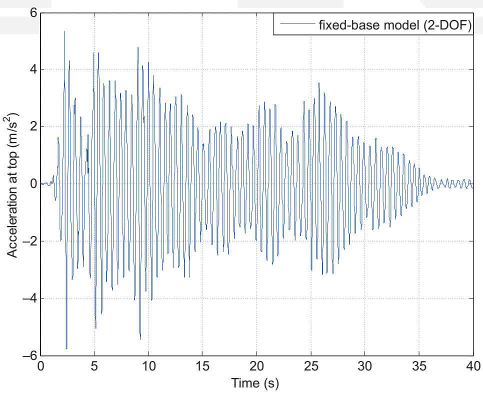


Fig. 8. Time-acceleration history plot for the fixed-base model during the 1994 Northridge earthquake (shaking table test)

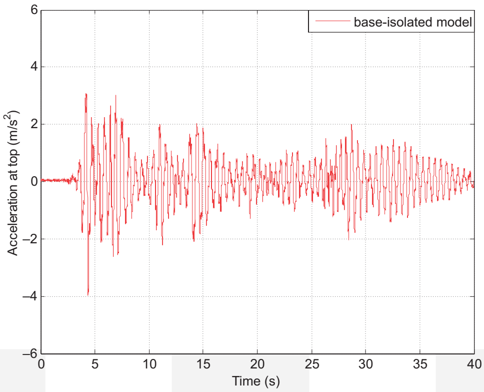


Fig. 4. Time-acceleration history plot for the fixed-base model during the 1940 El Centro earthquake (shaking table test)

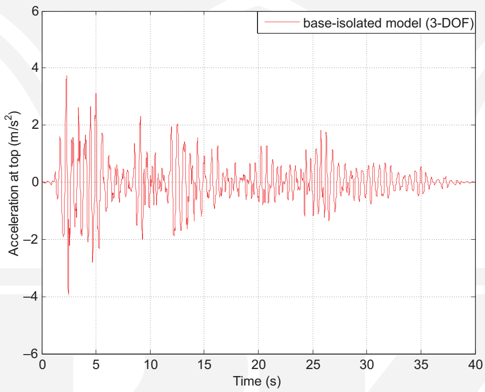


Fig. 5. Time-acceleration history plot for the fixed-base model during the 1940 El Centro earthquake (numerical analysis)

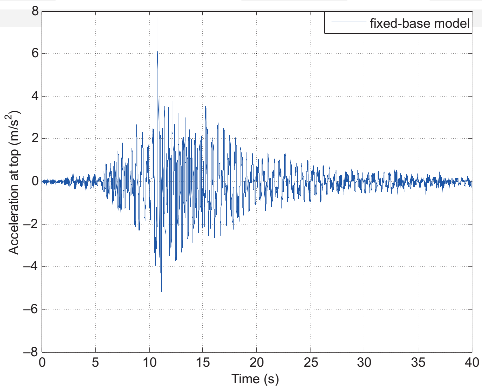


Fig. 6. Time-acceleration history plot for the base-isolated model during the 1940 El Centro earthquake (shaking table test)

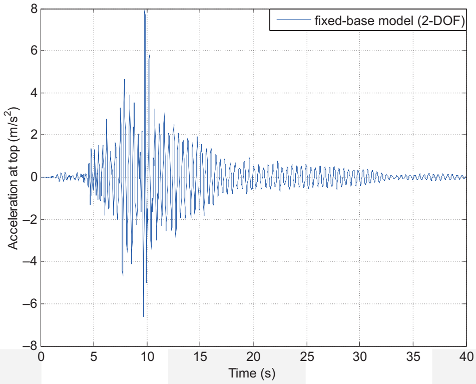


Fig. 9. Time-acceleration history plot for the fixed-base model during the 1994 Northridge earthquake (numerical analysis)

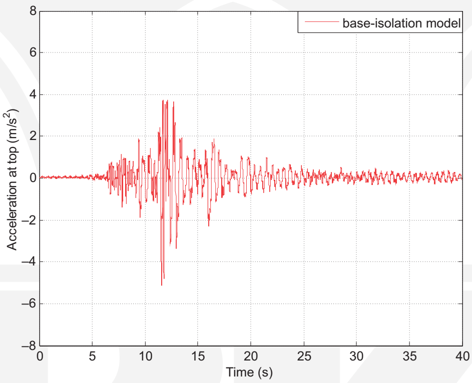


Fig. 10. Time-acceleration history plot for the base-isolated model during the 1994 Northridge earthquake (shaking table test)

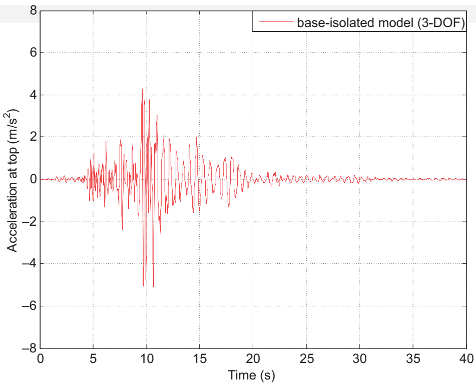


Fig. 11. Time-acceleration history plot for the base-isolated model during the 1994 Northridge earthquake (numerical analysis)

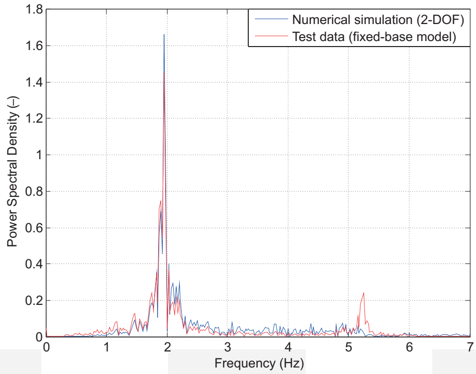


Fig. 12. Comparison between the FFT functions for the fixe-base model under the 1940 El Centro earthquake

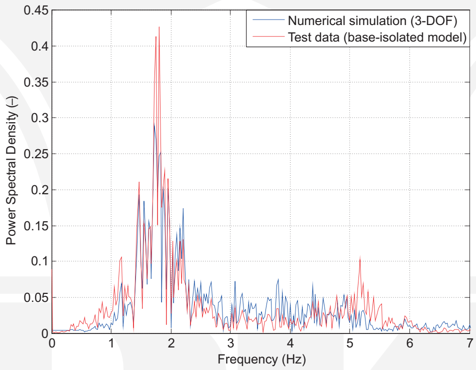


Fig. 13. Comparison between the FFT functions for the base-isolated model under the 1940 El Centro earthquake

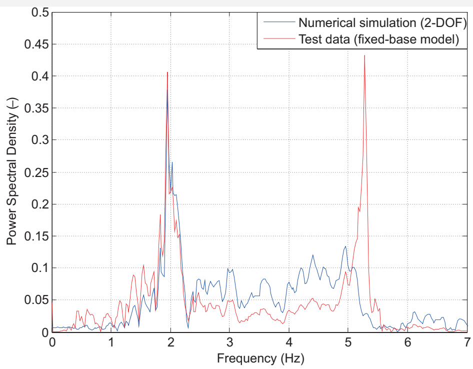


Fig. 14. Comparison between the FFT functions for the fixed-base model under the 1994 Northridge earthquake

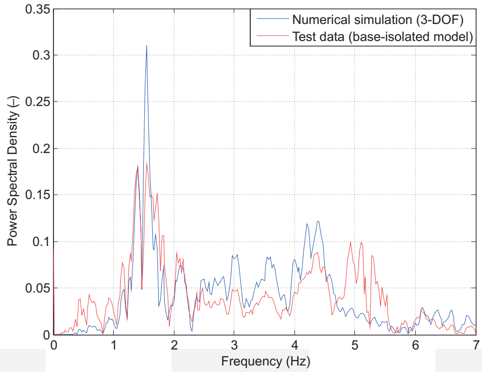


Fig. 15. Comparison between the FFT functions for the base-isolated model under the 1994 Northridge earthquake

Table 2

**Results of the numerical investigation with the use of lumped-parameter models**

| Dynamic excitation | Peak acceleration computed at the top of the numerical model (m/s$^2$) | | Reduction (numerical analysis) | Reduction (shaking table study) |
|---|---|---|---|---|
| | Fixed-base model (2-DOF) | Model isolated with the PBs (3-DOF) | | |
| El Centro earthquake, 18.05.1940 (NS component, PGA = 1.535 m/s$^2$) | 5.77 | 3.89 | 33% | 33% |
| Northridge earthquake, 17.01.1994 (Santa Monica station, EW component, PGA = 4.332 m/s$^2$) | 7.85 | 5.10 | 35% | 33% |

where PGA denotes the *Peak Ground Acceleration*.

## 6. Results and conclusions

The results obtained from the numerical analysis confirm that the proposed nonlinear mathematical model (Equations 1–3) can be successfully adopted for numerical simulations of the PBs. Inspection of Table 2 indicates that the reduction in the computed peak accelerations at the top of the analysed numerical model due to installation of the PBs is very similar to that observed during shaking table investigation. Also, the experimentally obtained results confirm the high effectiveness of the PBs in reducing structural vibrations due to earthquake ground motions. Additionally, the time-acceleration history plots and FFT functions obtained for the fixed-base structure modelled as a 2-DOF system are consistent with the results of the experimental investigation, which confirms the accuracy in assuming lumped parameters to characterize the analyzed two-storey structure model.

The results of the present study have confirmed both the high effectiveness of the PBs as a prototype base isolation system and the proposed mathematical model (both in time and frequency domain). Nevertheless, further development of the PBs is planned. Subsequent studies will include numerical simulations with the use of Finite Element Analysis as well as the experimental tests of a full-scale multi-dimensional PBs.

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