Nonlinear Modeling in Time Domain Numerical Analysis of Stringed Instrument Dynamics

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Abstract. Musical instruments are very various in terms of sound quality with their timbre shaped by materials and geometry. Materials' impact is commonly treated as dominant one by musicians, while it is unclear whether it is true or not. The research proposed in the study focuses on determining influence of both these factors on sound quality based on their impact on harmonic composition. Numerical approach has been chosen to allowed independent manipulation of geometrical and material parameters as opposed to experimental study subjected to natural randomness of instrument construction. Distinctive element of this research is precise modeling of whole instrument and treating it as one big vibrating system instead of performing modal analysis on an isolated part. Finite elements model of a stringed instrument has been built and a series of nonlinear time-domain dynamic analyses were executed to obtain displacement signals and perform subsequent spectral analysis. Precision of computations seems sufficient to determine the influence of instrument's macroscopic mechanical parameters on timbre. Further research should focus on implementation of acoustic medium in attempt to include dissipation and synchronization mechanisms. Outside the musical field this kind of research could be potentially useful in noise reduction problems.

INTRODUCTION

There is large variety concerning sound quality of musical instruments. This quality is often referred to as *tone* by musicians. In scientific world it is synonymous to timbre or tone color, not actual tone. Informal *tone* covers description of brightness, thickness, projection and many more imprecise attributes of sound. Most of them are heavily subjective both in perception and definition. Nevertheless, most musicians within single musical genre seem compatible in what they perceive as good and bad *tone*.

Sound quality is linked with materials used to build an instrument. Wood type is usually the defining factor in setting a stringed instrument's market value. Different wood genres are subjectively associated with specific timbre characteristics. However, many of these associations carry resemblance to physical and visual aspects of materials. For example, maple and spruce are said to sound bright, while mahogany and rosewood sound - and look - darker. Graphite nut seems to induce mild attack, while steel one would create a harsher, colder sound.

Most of these characteristics are connected with presence of upper harmonics in an acoustic wave. Their composition, proportions and fluctuation in time domain (Fig. 1) is what makes up for an impression of *tone* mentioned before. It is possible to capture these characteristics and represent them in a form of harmonic spectrum. It would be interesting to measure actual impact of various materials on harmonic composition and associate them with the impressions of *tone*.

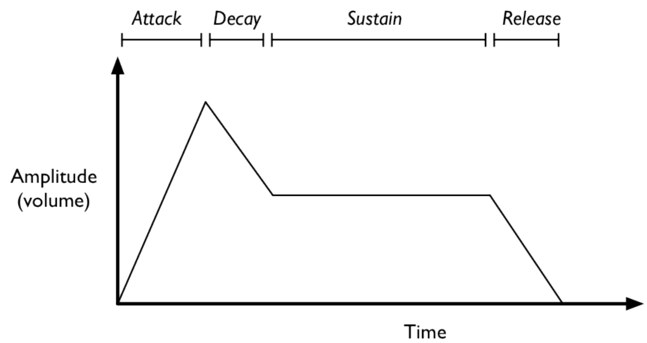

FIGURE 1. Attack-Decay-Sustain-Release (ADSR) stages of sound projection in time

The aim of this study is to tell whether it is possible to use finite elements method for this task. Guitar has been chosen for the research due to its availability and relative simplicity. What has been done is creating a detailed FEM model and performing nonlinear static and dynamic analyses in order to acquire instrument's frequency response. Later on, introducing material and geometry changes to the model could help getting a general idea on their impact on timbre.

Comparative experimental study could be misleading because of large sound variation between instruments of the same type. Two supposedly identical copies can sound differently due to subtle differences in construction and execution. It would be doable, but not cost-effective to examine large number of instruments and then try to work with the data. Numerical analysis allows changing individual parts of the model independently, thus making it possible to monitor influence of used materials on instrument's frequency response.

The most important and innovative aspect of this study is treating a guitar as a whole in the numerical analysis. Musical instrument is a very complex and sensitive vibrating system. It should not be analysed partially by isolating a small region of the system. It is impossible to apply boundary conditions reflecting properties of the rest of the instrument, as these parts influence each other during the dynamic process. Modeling a full guitar allowed for more realistic interaction between its vibrating elements.

Some earlier works have already taken up the problem. Zoran[1] have created a benchmark guitar body and executed experimental research on top plate influence on guitar sound. Ezcurra[2] examined behavior of guitar affected by different surrounding gas conditions. Torres[3] researched influence of bridge location on guitar sound. Becache[4] have performed advanced numerical simulation of guitar, taking up heavily analytical approach. Bretos[5] examined charasteristics of wooden xylophone bars.

Most of these works were focused on experimental research, treating numerical analysis as a support tool and often limiting itself to modal analysis. This particular study is weighed towards advanced numerical methods and is supposed to recreate the process of instrument excitation using nonlinear time-domain explicit analysis. What has been done by Zoran's team experimentally[1] here is done using FEM. Subject matter - materials and geometry impact on instrument's timbre - remains similar, as does the main data evaluation tool - response spectrum.

Theoretical side of this research is based on famous books by Helmholtz[6] and Chladni[7] as well as Wolfe's massive online repository on music theory and instruments acoustics[8].

MATERIALS AND METHODS

Overview

Guitar used for research was LAG Tramontane T200D. It is a standard Dreadnought acoustic guitar. Most of the data on construction and materials was available by direct examination and measurement. Some of the information about interior parts of the guitar was supplied by instrument's manufacturer on demand. A general idea on inner bracing and joints was easily accessible from typical Dreadnought construction drafts.

After building 3D numerical model of the guitar, computations have been performed in two subsequent steps. First one was aimed towards reflecting the guitar's initial state. It included introducing proper string tension and neck curve, often referred to as neck relief by guitarists. It has been executed as nonlinear static analysis in an iterative process. Second one was focused on making the instrument vibrate in a natural way by pulling the strings.

It has been carried away as an explicit dynamic analysis. Arbitrarily selected points on the top plate's surface then served as sources of displacement discrete signals in time domain. Converting these with Fast Fourier Transform algorithm resulted in creating response spectrum of guitar's top plate.

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Numerical Model

Geometry and Materials

Full 3D guitar geometry has been created in CAD environment. It was then imported to *SIMULIA Abaqus* engine in separate parts. Complexity of guitar geometry required usage of solid, shell and beam elements. Guitar parts with their respective types have been listed in Tab. 1.

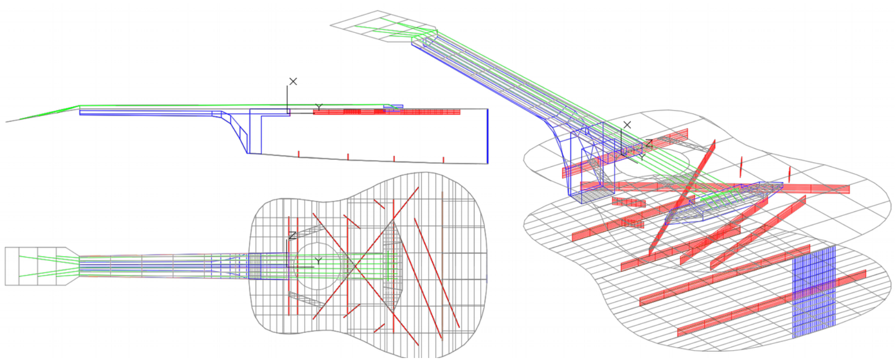


FIGURE 2. Guitar geometry created in CAD environment

Almost all wood types have been implemented with orthotropic material model. The only exception is rosewood, for which no information on its orthotropic properties have been found. Most of the data on material properties have been obtained from USDA Forest Products Laboratory[9]. Several parameters missing in this repository have been found in Eric Meier's Wood Database[10]. All of the values were specified for casual conditions, i.e. room temperature and 12% moistue content. Graphite properties were hard to specify, as there are many subtypes of this material and manufacturer lacked detailed data about it. As graphite parts are of marginal meaning in this model, approximate average values have been used without further research. Part-material pairs are listed in Tab. 1. Material properties can be found in Tab. 2.

TABLE 1. FEM model parts sorted by element and material properties

Part	Geometry	Element	Material	Material model
top plate	shell	S4R	western red cedar	orthotropic
back	shell	S4R	Honduran mahogany	orthotropic
sides	shell	S4R	Honduran mahogany	orthotropic
neck	solid	C3D4	Honduran mahogany	orthotropic
head	shell	S4R	Honduran mahogany	orthotropic
fretboard	shell	S4R	East Indian rosewood	isotropic
bracing	shell	S4R	sitka spruce	orthotropic
neck block	solid	C3D4	sitka spruce	orthotropic
end block	shell	S4R	Honduran mahogany	orthotropic
bridge	shell	S4R	graphite	isotropic
nut	shell	S4R	graphite	isotropic
truss rod	beam	B31	steel	isotropic
truss rod anchors	solid	C3D8R	steel	isotropic
strings	beam	B31	steel	isotropic

TABLE 2. Mechanical properties of materials: density ρ[kg/dm^3], Young's modulus E[MPa], shear modulus G[MPa]

Material	ρ	E_1	E_2	E_3	υ_1	υ_2	υ_3	G_{12}	G_{13}	G_{23}
cedar	0.32	7700	624	424	0.378	0.296	0.484	670	662	39
mahogany	0.45	10300	1102	659	0.314	0.533	0.600	680	886	288
spruce	0.36	9900	772	426	0.372	0.467	0.435	634	603	30
rosewood	0.75	12300	-	-	0.330	-	-	4624	-	-
graphite	2.00	20000	-	-	0.200	-	-	8300	-	-
steel	7.80	210000	-	-	0.300	-	-	81000	-	-

Boundary Conditions

Boundary conditions were applied in the places where strap buttons are located on real guitar (Fig. 3}). It constrained the system just enough to keep it stable during excitation. In the same time it did not affect the guitar body movement significantly, allowing it to deform and vibrate freely. All six degrees of freedom have been constrained on indicated surfaces.

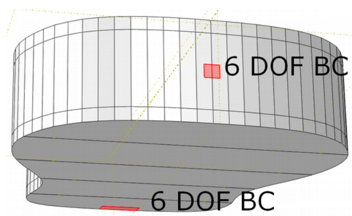
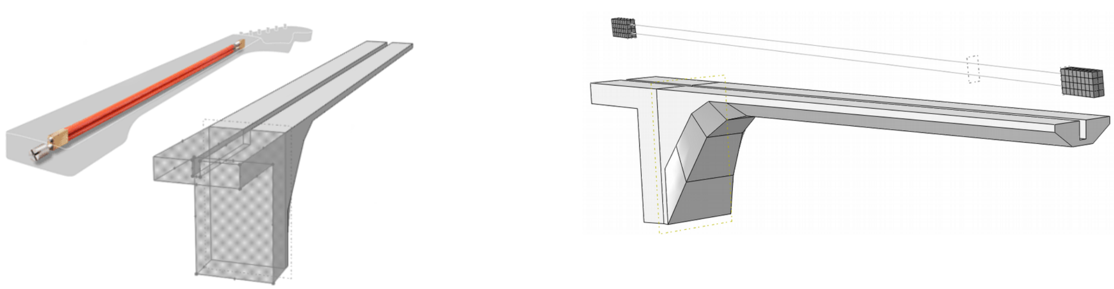


FIGURE 3. Surfaces constrained by boundary conditions in the model

Proper truss rod modeling was important to retain realistic neck stiffness. In real guitar this part is fixed on the body end and free to slide on the head end (Fig. 4a). T200D is equipped with a double action truss rod, consisting of passive bar (closer to the strings) and active one (deeper in the neck). By turning the hex key on the nut located on the body side, guitar player shortens the active bar and applies tension to it. As an effect, truss rod bends down and pulls the neck against the strings. However, due to slide feature of truss rod's opposite end, no additional compression is introduced to the neck apart from the one triggered by bending moment. Including this feature in the model was accomplished by creating a detailed contact definition between truss rod and neck/fretboard surfaces limiting its movement.



(a) (b)

FIGURE 4. Typical truss rod placement (a) and its realization in the model (b)

Each string was divided in three parts - one active and two passive. First part's sound generation purpose is obvious. Passive anchors were preserved to join the active part with guitar head and bridge plate. These short sections must not have been omitted, as they provide balance and symmetry of nut and bridge load. Otherwise there would be too much bending applied to bridge and nut, resulting in top plate warping. To avoid undesirable local effects and stress concentration, strings have been attached to guitar body using kinematic coupling distributed on a surface.

Initial State - Nonlinear Static Analysis

Setting up a real guitar is an iterative process. It consists of two repeated steps - tightening/loosening the strings and adjusting tension in a truss rod. It is done in order to maintain a proper neck curvature when having the strings tuned to pitch. Neck should be bowed away from the strings by a fraction of millimeter, enough to let them vibrate freely without hitting the frets. Too much of a back-bow will make the guitar uncomfortable to play, so a proper balance must be found. It is accomplished by trial and error in real life and it has been done the same way in the analysis.

First, strings have been loaded with appropriate stresses using bolt load in order to tune them to their respective fundamental tones. Stresses were easy to calculate using common formulas for the speed of wave. Guitar neck displacement was measured. Then the strings have been taken off completely and an arbitrary stress has been introduced to truss rod. Guitar neck bent in the opposite direction and displacement was measured again. Finally both strings and truss rod have been loaded. After several attempts desired curvature was obtained with strings tuned to adequate fundamental tones. Initial stress in active truss rod's bar ended up to be around 200 MPa.

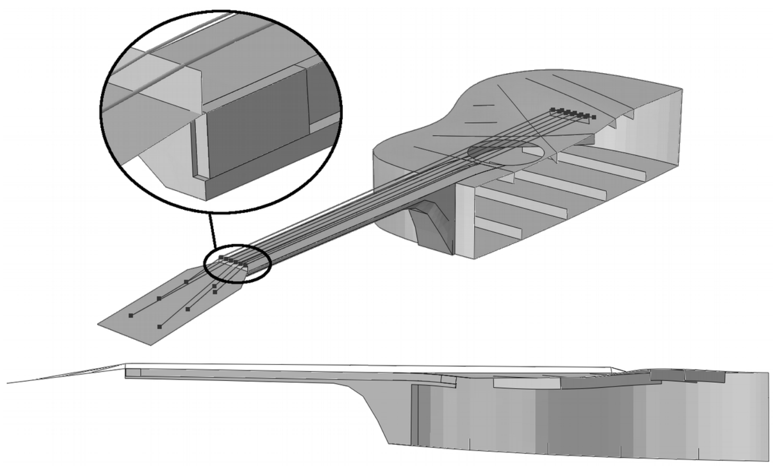


FIGURE 5. Prestressed model in initial state for dynamic analysis (1:10 scale). Truss rod displacement caused by its contraction (up) and realistic behavior of guitar body with apparent humps at sensitive areas due to string tension (down)

Vibration Measurement - Explicit Dynamic Analysis

Second phase of analysis simulated making the guitar vibrate by pulling the strings. It was accomplished by forcing displacement using ramp function and releasing the constraint afterwards. It allowed to distribute string vibrations to whole guitar body via the bridge. Each string resonated in its own harmonic modes, creating a wide spectrum of frequencies transferred to the system.

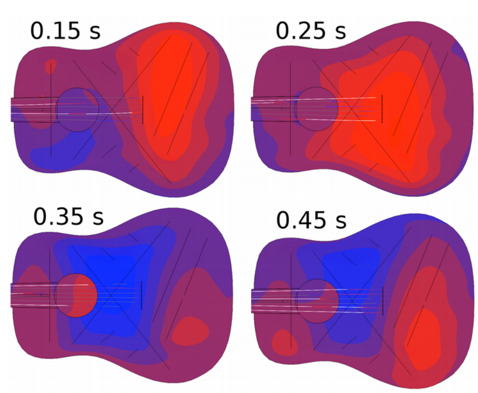


FIGURE 6. Top plate displacement map at fixed time points of explicit dynamic analysis

Analysis has been performed with a 0.00025 s write step, resulting in a 4000 Hz sample frequency of time-displacement signals. Due to explicit analysis nature time step was much smaller, around 10^{-8} second, and has been calculated automatically during the analysis. Write step has been set manually before the analysis to reduce output data file size. Total simulated time was 0.5 second and it resulted in 2.5 GB output file. Calculation time was around 9 hours on a moderately powerful home PC with 6-core 4.2 GHz processor and 8 GB of RAM.

Signal Processing

Time-displacement signals have been obtained from previous step of analysis. Two arbitrary points (Fig. 7) on the top plate have been selected as sources of these signals. Point A has been chosen because of its symmetric position and proximity of the bridge. Point B, as opposite to previous one, was placed in a peripheral, asymmetric position. Maximum diversification of harmonic spectra were intended and expected.

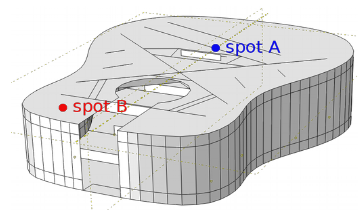


FIGURE 7. Points with their displacements measured during time domain analysis

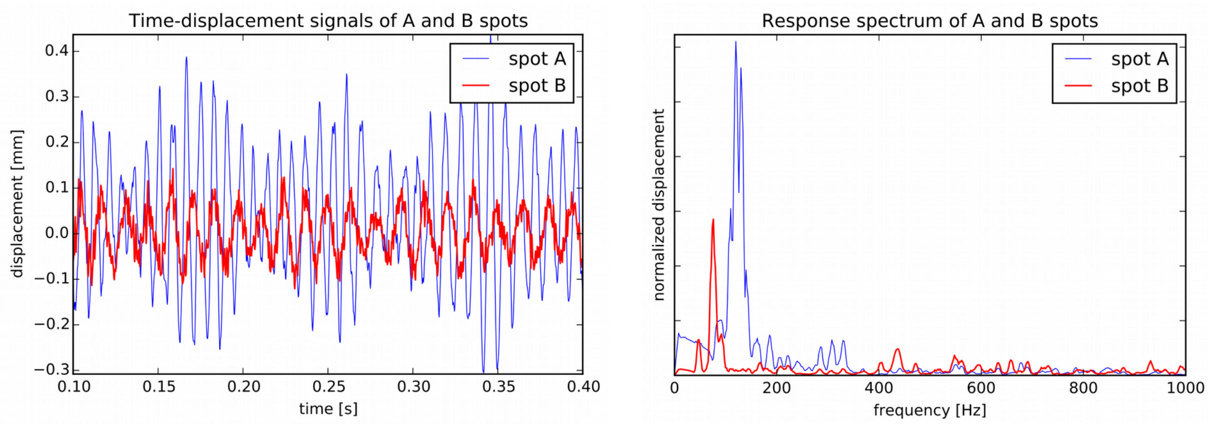
These signals have then been converted into frequency response spectra using Fast Fourier Transform algorithm written in *Scilab*. A mix of built-in functions and script programming was used. Given a 4000 Hz input signal, FFT was able to build a frequency domain ranging between 0 and 2000 Hz. It was enough to capture up to 5 overtones of the highest pitched E string with its 330 Hz fundamental tone.

RESULTS AND DISCUSSION

Frequency response spectrum (Fig. 8b) shows massive presence of local resonant frequencies in selected spots. Higher register, although much weaker, is still present and varies between two signals. It is stronger with spot B located in a stiffer place, which is satisfactory. However, resulting graphs have continuous character, while more discrete one was expected due to nature of strings' harmonic modes. This could be due to relatively short time of analysis. Half a second covers mainly initial stage of sound projection, called attack (Fig. 1). Better results could be obtained by focusing on stabilized stages such as sustain. This would result in capturing more steady state of vibrations. Initial stage might include too much perturbation and unwanted interference.

Disparity between resonant frequency and harmonic series amplitudes seems exaggerated. If this was the case with real guitar, the same resonant hum would be always audible regardless of the notes played. The reason for this phenomenon could be insufficient stiffness of the system, as well as lack of dissipation mechanism draining energy from the resonant modes. Both of these issues can be fixed by introducing an acoustic medium, such as air, to the numerical model.

Implementing air would result in a very important advantage. Real guitar body works as an air pump, sucking it and pushing away as the top plate vibrates. This is directly what produces the sound. Having this feature in the model would allow to measure pressure change in time domain outside the guitar body. This is exactly what human ear or a microphone does. It would make it possible to measure whole instrument's spectral characteristic at once, instead of limiting oneself to several arbitrary points. Similar analysis is already common in computations of speaker performance, eg. [11].



(a) (b)

FIGURE 8. Displacement signals (a) and Fourier transforms (b) obtained from the analysis; selected ranges visible

CONCLUSIONS

While pure solid body analysis gives reasonable output data, introducing acoustic medium to the model would greatly improve the outcome. It would provide damping, synchronization of guitar's top and bottom plates, realistic sound generation mechanism and an easy way to capture data by measuring air pressure. It should be the next step in developing the analysis regardless of effort required to implement it. FFT should then be performed on signal obtained during sustain stage.

Including air in the model could also be useful in prediction of instrument's sustain time. It would require a long and computationally costly analysis. However, sustain should not be ignored, as for many guitar players it is the main reason to choose one instrument over another. While precise computation is very hard, it should still be possible to estimate various factors' impact on sound duration.

Some experimental study would still be of use to serve as a verification tool for numerical analysis. Empirical Fourier transforms could be easily compared with the ones obtained from the simulations. Concerning solid body model, it would be necessary to install accelerometers on guitar's top plate and measure the signals while exciting the strings. In case of acoustic medium model it would be enough to record a real guitar with a microphone and preform FFT algorithm on recorded signal. This is yet another argument to follow this direction in future analyses.

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