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Entrepreneurship Vulnerability to Business Cycle. A New Methodology for Identification Pro-cyclical and Counter-cyclical Patterns of Entrepreneurial Activity

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Abstract

In literature, there is ongoing discussion whether entrepreneurial activity, approximated by, for instance, changes in self-employment, tends to behave pro-cyclically, counter-cyclically or rather is a-cyclical. Thus far, both theoretical and empirical evidence, where various multiple methodological approaches are used, does not provide clear answer to the latter; while widely offered explanations are scattered and lack robustness. This paper is designed to contribute to the present state of the art, by presenting a novel methodological approach to identification of the relationship between the intensity of entrepreneurial activity and business cycle. Put differently, we aim unveil if entrepreneurship (approximated by changes in self-employment) behaves pro-cyclically, counter-cyclically or a-cyclically. To exemplify our new conceptual approach, we use quarterly data on deflated gross domestic product and self-employment. The empirical evidence presents the case of Italy. The period of analysis is restricted to the years 1995-2014. All statistics are extracted from OECD datasets on Annual Labor Force and Gross Domestic Product.

Keywords:	Entrepreneurship; self-employment; vulnerability; small and medium sized enterprises; business cycle; economic growth
JEL codes:	D22, E32, M21, O11, C10

INTRODUCTION

In literature, there is ongoing discussion whether entrepreneurial activity, approximated by, for instance, changes in self-employment, tends to behave pro-cyclically, coun-

ter-cyclically or rather is a-cyclical. Still relatively little efforts have been made to em-pirical verification of entrepreneurial behavior patterns over business cycles. In so far, both theoretical and empirical evidence, where various methodological approaches are used, does not provide clear answer to the latter; while widely offered explanations are scattered and lack robustness. We still lack consistent methodological framework allowing providing clear answer whether entrepreneurship lags or leads business cycles, or whether its behavioral patterns are pro-, or counter-cyclical, hence examining these relationships remains a challenging task.

Regarding the latter, some evidence may be traced in works of Kollinger and Thurik (2012), which using data for 22 OECD countries over the period 1972-2007, use Granger-causality tests to verify if entrepreneur activities are leading or lagging indicator over the business cycles; and their findings they show that entrepreneurship is leading indicator of the business cycle. Rampini (2004), using canonical real business cycle model, finds that entrepreneurship behaves pro-cyclical, which is associated with changes in risk aversion during respective phases of business cycle. Carmona et al. (2010), using quarterly data for self-employment and GDP in Spain and the United States, over the period 1987-2004, adopt the cross-correlations and VAR models to demonstrate that the hypothesis on pro-cyclicality of self-employment cannot be con-firmed. At the same time, they present rather mixed results for various groups of self-employed. Klapper et al. (2014), using data for 109 countries over the period 2002-2012, find that entrepreneurial behavior demonstrates strong pro-cyclical patterns. More recent evidence may be also found in works of, *inter alia*, Parker (2002), Parker et al. (2012a,b), Milan et al. (2012), Baptista and Preto (2011).

This paper is designed to contribute to the present state of the art, by presenting a novel methodological approach to identification of the relationship between the intensity of entrepreneurial activity and business cycle. Put differently, we aim unveil if entrepreneurship (approximated by changes in self-employment) behaves pro-cyclically, counter-cyclically or a-cyclically.

It comprises six logically structured sections. Section first is the introduction. The second section briefly explains motivation and contextual background of our further research; while section third demonstrates literature review regarding empirical research examining existing relationships between entrepreneurial activities and business cycles. Next, section four extensively clarifies novel methodological approach allowing for iden-tification whether entrepreneurship activity – approximated by changes in total self-employment – behaves pro-cyclically, counter-cyclically or rather tends to be a-cyclical. In section fifth, using quarterly data on total gross domestic product and total self-employment in Italy over 1995-2014, we exemplify this new methodology. The period of analysis is restricted to the years 1995-2014. All statistics are extracted from OECD da-tasets on Annual Labor Force and Gross Domestic Product. Finally section six concludes.

MOTIVATION AND CONTEXTUAL BACKGROUND

Both in theoretical and empirical literature, which examines emerging relationships between entrepreneurship and business cycle or economic growth; entrepreneurial activity is approximated by wide variety of measures. Most of studies, bases on the assumption that entrepreneur is simply a business owner; and such approach to defin-

ing an 'entrepreneur' is broadly accepted among scholars, despite the fact that within this numerous group of those who are classified as 'entrepreneurs' , usually only a small selection of them may be classified as 'Schumpeterian entrepreneurs' (Schumpeter 1934, Kirzner 1999, Gick 2002), in its generic sense. However, even though such approach is often an over-simplification, in most of empirical works, each type of business owner is treated as an entrepreneur; henceforth a number of private businesses is an accepted measure of entrepreneurial activity in given economy (Koellinger & Thurick 2009). The latter additionally implies that, across empirical the number of active business / enterprises (where a great majority of them is privately owned-business) is treated equivalently as number of small and medium sized enterprises, number of self-employed persons or – alternatively, number of entrepreneurs. Consequently, changes regarding, *inter alia*, number of start-ups, birth rates, net growth of firm population, business ownership rate (as the percentage of non-agricultural owners incorporated and unincorporated businesses relative to the labour force), in economic literature, are broadly accepted as measures approximating entrepreneurial behaviors and attitudes.

A major disadvantage in running extensive and detailed empirical research on entrepreneurship and SMEs sector is lack of availability of balanced time series allowing for reliable cross-country and cross-time comparisons. In this context, statistical datasets on self-employment are relatively well-balanced and long-time series are available, which allows for reliable comparisons both cross time and space. Bearing in mind the latter, data on self-employment are often considered as good measure of entrepreneurship, especially when one yields for international research in this area of interest. Data on self-employment, as an exclusive alternative, are also broadly applied in short-term analysis, when, for instance, are confronted with business cycle. Hence, despite multiple disadvantages, self-employment rates and business owners rates are commonly used by many researchers as a proxy of entrepreneurship (Iversen et al. 2008, Thurik et al. 2008; Parker 2009; Koellinger, Thurik 2012). Importantly to note, as argued by Congregado et al. (2012), a in so far we lack better alternative to measure entrepreneurial activity, but all limitation associated with data on self-employment as a proxy of the latter, should be borne in mind and when drawing conclusion and formulating recommendations.

Among scholars, treating data on self-employment as a measure of entrepreneurship has gained significant popularity in mid-80s of XX century. Previously, self-employment was considered as 'worse' alternative to being an employee. The opinion that high self-employment rate is a syndrome of economic underdevelopment, and should drop as country develops, was very common. However, over last two decades of XX century, economic situation has changed essentially (Carree et ay. 2007), and self-employment has gained popularity as a 'source' of new jobs creation, contributing effectively to unemployment reduction, and constituting new and highly-demanded alternative to contract work. Since then onward, growing number of self-employment persons was perceived as a manifestation of entrepreneurial spirit, fostering long-term economic growth and development (see Thurik & Wennekers 1999). Despite the previous, it also shall be borne in mind that empirical evidence on the positive role of increasing number of self-employed people is scattered, demonstrates case-wise vulnerability, while according to several research, like for

instance those of Davis et al. (1996), or Blanchflower (2000) do not unequivocally confirm the hypothesis that growing self-employment enhances economic growth[1].

When treating self-employment (absolute level, in-time variations or time-dynamics) as a measure of entrepreneurship, it shall be noted that this solution also has several limitations and disadvantages. Mueller and Arum (2004) emphasize extreme heterogeneity of this group of people, which results from several aspects. First, only a small share of persons classified as self-employed are those who offer contract work. Second, by some people being self-employed is treated as permanent work status; while for the rest of them being self-employed is only casual. Third, as claimed by Audretsch (2002), Baumol et al. (2009), or Congregado et al.(2012) most of self-employed persons are labeled as 'replicative' or 'me too', and only a small part of them is innovators. Fourth, among self-employed persons may be identified owners of 'old' firms and owners of newly born businesses (Koellinger, Thurik 2009); while both of them impact economic growth differently. Fifth, growing number of self-employed persons may be determined by legal regulations, like for instance, emerging new flexible forms of employment. Very frequently, occupational choices are not results of independent individual decisions, but are caused by pressure of former employers, who forced employees to establish their own businesses, if they want to continue their jobs. The latter usually negatively affects low-skilled persons, and may be identified as a 'dark side of flexible production' (Mueller & Arum 2004, p.12). All these aspects mentioned above may, in effect, generate inconclusive and hard to interpret results. For instance, Spain and Italy where self-employment rates are relatively high, if compared with other developed economies, demonstrate more intensive entrepreneurial attitudes than, for instance, the United States where self-employment rates are essentially lower (Iversen et al. 2008; Congregado et al. 2012). The fact, that Spain and Italy have relatively high self-employment rates does not mean that these economies are more entrepreneurial than economy of the USA where self-employment rates are essentially lower. In the same line, Portuguese economic development should be identified as more entrepreneurial-led than for instance German economy. Belso (2005), Parker (2005) or Carree et al. (2007) argue that each economy is characterized by entrepreneurship 'equilibrium' rate, which may be pre-conditioned by cultural background, social norms and attitudes, history or structure of the economy. Parker (2004) stated that however rates of entrepreneurship vary strongly between countries they exhibit 'a fairly high degree of temporal stability'. According to Parker (2005) stability of these rates in given regions (countries) and stable differences between them may be explained by self-perpetuating occupational choices within regions (countries) affecting payoffs in entrepreneurship and in-paid employment. On the individual level it is highly probable that a person who is currently self-employed, in the future will be classified as such. Such kind of 'inertia'[2] on regional and national levels was observed in broad variety of research and reported in seminal papers of, inter alia, Parker (1996), Cowling & Mitchell (1997), Parker & Robson (2004), Bruce & Mohsin (2006), Fritsch & Mueller (2007). Risk and sunk costs related to change of occupation

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[1] "Probably the greatest interest in entrepreneurship springs from a belief that small businesses are essential to the growth of a capitalist economy. While the view that small businesses are responsible for a disproportionate share of job creation and innovation is disputed, this view is a common one". (Blanchflower 2000 p. 473)

[2] Some authors (ex. Dixit, Rob 1994) called such inertia "hysteresis".

reduce the total amount of entries and exits (Dixit and Rob 1994). "Only when average incomes in entrepreneurship reach some upper 'trigger point' will people become entrepreneurs. And the will leave entrepreneurship in the presence of adjustment costs if incomes drop to some lower trigger point (Congregado et al. 2012, p.1243). Hence business cycles affect entrepreneurship (self-employment) fluctuating around long-term trend determined by natural rate of self-employment. Congregado et al. (2012) demonstrate that various kinds of self-employed people exhibit different reaction to business cycle. Analyzing quarterly self-employment changes in Spain, between 1987 and 2008, he found employer self-employment rates tend to behave pro-cyclically, while own-account self-employment rates evolve rather counter-cyclically. However the same authors in another paper they did not find such relationship in case of the USA economy.

When analyzing the nature of the emerging relationships between entrepreneurship and business cycle, one should pay special attention to discrimination between the so called 'pull-factors' and 'push-factors', which heavily determine entries and exits from 'self-employment. The 'pull-factors' and 'push-factors' Parker (2009) labeled as 'opportunity pull' and 'recession push' respectively, emphasizing that during expansion phase of business cycle people are more toward self-employment-oriented, while during recession phase choosing self-employment – as an alternative, is often perceived as the only opportunity to work and exclusive source of personal income. At the same time, it appears to be extremely difficult to provide exact calculations the number of those entrepreneurs for whose the 'pull-factors' were decisive, and – those for whom the 'push-factors' determined their decision to enter self-employment. However, it may be argued that those for whom the 'pull-factors' were decisive, are more active entrepreneurs, more success-oriented and are more likely to create contracted work posts. Conversely, the entrepreneurs motivated by 'push-factors' probably consider their self-employment status as casual, and once the recession is over are more likely to become contracted workers. Notably, the following scenario is also probable: during the recession phase of business cycle, the impact of 'pull-factors' is much stronger than during expansion phase; which implies that a significant number of entrepreneurs, bearing in mind relatively low production costs and high rate of bankruptcies, should enhance them to set up their own business and create innovation. Providing empirical proofs in for or against of one these scenarios, still remains a challenging task. In this context, empirical evidence and research provided by GEM, are considered to be exclusive as those discriminating entrepreneurs when their own motivation to set-up own business is considered (Koellinger & Thurik 2009, p.10).

When discussing empirical research on entrepreneurship, the great majority of empirical evidence is based on self-employment data (total number of self-employed persons and/or self-employment rate). In here, it also should be noted that the total number of self-employed person reflects the net effects of inflows and outflows; and for this reason it should be carefully interpreted in terms of economic changes. Moreover, it is hardly possible to state whether self-employment entries may be treated as flow from unemployment and/or economically inactive group, and/or group of contracted workers. Analogously, we cannot identify the self-employment outflows.

In literature, there may be found empirically tractable arguments speaking in support of three different hypothesis regarding relationships between entrepreneurship and business cycle. These hypotheses are (Table 1):

1. Entrepreneurship behaves pro-cyclically;
2. Entrepreneurship behaves counter-cyclically;
3. Entrepreneurship behaves a-cyclically.

Arguably, if we observe that growing/falling number of self-employed persons, increasing/dropping rate of self-employment (*ceteris paribus*) or increasing/decreasing number of firm entries is accompanied by negative/positive changes in gross domestic product that approximates business cycle, then we state that entrepreneurship behave counter-cyclically. However, if both self-employment and gross domestic product commove in the same direction, that is to say – we observe simultaneous growths in self-employment and GDP, or drops both in self-employment and GDP; hence we claim that entrepreneurship behaves pro-cyclically. If, self-employment and GDP fluctuations around long-term trends are random, we may state that entrepreneurship behaves a-cyclically. Many claim that pro-cyclical entrepreneurship patterns are far better explainable by 'pull-factors' compared to 'push-factors'; which suggests that growing demand, prices and profits (during expansion phase of business cycle) attracts people to establish their own businesses, more effectively than low costs of entry and production during recession. During recession, reported low costs and barriers of entry, along with low costs of production may attract potential future entrepreneurs, unemployed persons or contracted workers, enhancing them to set up their own business. Additionally, a selection of 'push-factors' (recession push) may determine flows from employees to own account workers. There are several studies concluding on positive correlation between the short-term economic growth and entrepreneurship (self-employment). Such claims may be found in works of Shleifer (1986), Audretsch, Acs (1996), Grant (1996) or Rampini (2004). Carmona et al. (2010), using quarterly data on self-employment and GDP, over 1980-2009, in the United States and Spain were considered, did not confirm the hypothesis on pro-cyclicality of self-employment. In turn, empirical evidence of Congregado et al. (2012), demonstrated for the same countries and time period as this of Carmona et al. (2010), allows confirming the hypothesis on pro-cyclical behavior of self-employment. Klapper et al. (2014), using panel data for 109 countries over the period 2002-2012, displayed that entrepreneurial behavior demonstrates strong pro-cyclical patterns. Reversely to the previous, counter-cyclical character of entrepreneurship (self-employment) may be concluded from such studies like: Caballer & Hammou (1994), Blanchflower (2000), Francois & Lloyd-Ellis (2003), Perotin (2006), or Millan et al. (2015). In previously cited works for Spain and USA, Congregado et al. (2012) have shown counter-cyclical changes of self-employment are reported only for own-account self-employed (not employers self-employed) and exclusively in Spain; while for the United States similar relationship was not confirmed. More evidence (see also Table 1) on the emerging relationships between entrepreneurship and business cycle, may be traced in works of, Bernanke & Gertler (1989) or Carlstrom & Fuerst (1997) who find that self-employment is rather a-cyclical versus GDP fluctuation; according to the research of Koellinger and Thurik (2009), deploying panel data for 22 OECD countries between 1972 and 2007, both hypotheses on counter-, or pro-cyclical behavior of entrepreneurship may not be confirmed – see also – Bernanke & Gertler (1989), Carlstrom & Fuerst (1997) or Rampini (2004).

Table 1. Relationships between entrepreneurship and business cycle

Research	Problem and research targets	Data	Country	Period	Method	Conclusion
Pro-cyclicality						
Shleifer 1986	How entrepreneurs' commonly shared expectations and their independent investment decisions influence the cyclical behavior of macroeconomic variables.	None	None	None	Theoretical quantitative model	Pro-cyclicality of entrepreneurship emerging as an effect of entrepreneurs' decision not to internalize external effects of their decisions to innovate and invest.
Audretsch, Acs 1994	Examination of determinants of new firm startups.	New entries in 117 industries, macroeconomic growth rate, cost of capital, unemployment rate.	USA	1976-1986	Pooled cross-section regression model	New firm startups are positively correlated with economic growth, pursuing of innovative activities and university-based research
Rampini 2004	Relationships between entrepreneurial activity, risk aversion, costs of agency.	None	None	None	Theoretical model of the optimal contracting	Entrepreneurial activity is pro-cyclical due to the risk associated with it.
Klapper at . 2014	New firm registration along business cycle	Numbers of newly registered private limited liability companies per year	109 countries	2002-2012	Pooled OLS	Strong pro-cyclical pattern of new firm registration
Counter-cyclicality						
Caballero, Hamour 1991	The response of industries to cyclical variations in demand.	Davis and Haltiwanger's data on jobs flow (creation and destruction) in manufacturing	USA	1972-1986	A vintage model of creative destruction.	Productivity improving activities are undertaken during recessions when opportunity costs are temporarily low.
Francois, Lloyd-Ellis 2003	Entrepreneurs' decisions about sales and production along business cycle.	None	None	None	Theoretical; quantitative model.	Entrepreneurs do innovations and produce when costs are low (during recessions) and sale during booms when demand is high.
Blanchflower 2000	The role of self-employment in economies of OECD count	Self-employment rates and unemployment rates.	OECD countries	1966-1996	Pooled OLS, fixed effects models, probit models	Negative relationships between self-employment rates and unemployment rates for most OECD countries. Exceptions were Italy and Iceland.
Perotin 1996	Social entrepreneurship - why there are so few labor-managed firms (ex. cooperatives) in modern economies.	Entries and exits flows of cooperatives.	France	1979-2002	Poisson Maximum Likelihood (ML) estimations	Cooperative creations tend to be more counter-cyclical than conventional firms.
Millan et al. 2014	The role of the business cycle in the individual decision of own-account workers to hire employees.	Panel data from the European Community Household Panel (henceforth ECHP)	EU-15	1994-2001	Random effects binary logit models	Own-account workers are less likely to hire employees during recessions.
A-cyclicality						
Bernanke, Gertler 1993	The influence of entrepreneur's net worth on borrowing conditions and as result on investment fluctuations.	None	None	None	Theoretical neoclassical model of real business cycle.	Agency costs of investing are inversely related to entrepreneurs' net worth; emergence of accelerator effect during expansion phase of business cycle; asymmetric shocks on productivity
Carlstrom, Fuerst 1997	Development of quantitative model capturing the propagation of productivity shocks through agency costs.	None	None	None	General equilibrium model.	Assumption that that share of entrepreneurs in population is constant and does not fluctuate along business cycle.
Leading indicator						
Koellinger, Thurik 2009	Testing the theoretical predictions found in the literature with real data.	Real GDP, standardized unemployment rates and the shares of business owners in total labor force, share of nascent entrepreneurs in the adult population.	22 OECD countries	1972-2007 (OECD data); 2001-2006 (GEM)	Bivariate correlation, regression analysis.	Entrepreneurship is a leading indicator of business cycles.

Source: authors' compilation.

NEW METHODOLOGICAL APPROACH. CONCEPT CLARIFICATION

As argued in previous section, answering the question whether changes in entrepreneurial activity – approximated by, for instance, changes in total self-employment[3] tend to behave pro-cyclically, counter-cyclically or maybe are contemporaneous over business cycle, still remains a challenging task. Some attempts of identification of the relationships between these two variables encompass adoption of various statistical and econometric techniques; however still, all these methods seem to be conclusive and interpretive enough to provide clear answer to the latter.

This section presents newly developed methodological approach, which was designed to examine how entrepreneurial dynamics changes over business cycle. We have intended to keep this method simple, conclusive and interpretive, so it may be adopted in broad variety of studies. However, at the heart of our considerations is whether entrepreneurship, measured by total self-employment, demonstrates pro-cyclical, counter-cyclical or contemporaneous behavior over business cycle.

Conceptualization

Our methodology developed to measure the entrepreneurial behavior over business cycle, is based on the following assumption:

- Business cycle is measured as deviations from long-term trends in value of total gross domestic product expressed in real terms (corrected for inflation);
- Entrepreneurial activity is measured by total self-employment (despite the fact, that self-employment does not perfectly measure entrepreneurship, it is broadly accepted by scholars, due to several important merits, like for instance – inclusiveness and convenience (Congredago et al. 2012) as self-employment time series are available for of long-run periods and for multiple countries, which enables cross-country comparisons; and as argued by Iversen et al. (2008) self-employed person are those which tend to bear risk, and thus demonstrate strong pro-entrepreneurial activities);
- Self-employment data may be used both including, or alternatively, excluding self-employment in agricultural sector (depending on time series availability);
- Self-employment and gross domestic product time series are seasonally adjusted;
- Preferably, quarterly time series data are applied.

Suppose that $Self_{i,t}$ stand for the number of people defined as self-employed[4] in national economy, where *i* denotes country and *t* – time; while $GDP_{i,t}$ expresses the value of total gross domestic product (in real terms), with similar notations. By convention, we target to unveil whether $Self_{i,t}$ as a proxy of entrepreneurial intensity behaves *pro-cyclically*, *counter-cyclically* or *contemporaneously* over business cycle.

Defining business cycles, approximated by $GDP_{i,t}$, as deviations from long-term trends $GDP_{i,t}$, yields decomposing the time series into trend and the cycle. As the major aim of the proposed specification is to determine whether the variable $Self_{i,t}$ demonstrates *pro-cyclical*, *counter-cyclical or contemporaneous* behavior, our primary interest

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[3] In the paper terms 'entrepreneurship' and 'sel-employment' are used interchangeably.

[4] According to national concept.

turns into examining the comovements of cycle components $(c_t) \rightarrow c_t\_Self_{i,t}$ and $c_t\_GDP_{i,t}$, which have been taken out of the original time series after its detrending.

Therefore, the original time series must be decomposed into two components that may be additively separable, as proposed by Hodrick and Prescott (1997):

$$y_t = \tau_t + c_t + \varepsilon_t \tag{1}$$

where:

y_t - is the time series;

τ_t - is trend component;

c_t - stands for the stationary cyclical component that is determined by the stochastic cycles across multiple periods (Cogley and Nason, 1995);

ε_t - may be defined as unobserved random component.

Following the Eq.(1), it may be argued that the trend component may be calculated by simply extracting:

$$\tau_t = y_t - c_t - \varepsilon_t \tag{2}$$

and so the cycle components follows as:

$$c_t = y_t - \tau_t - \varepsilon_t \tag{3}$$

Original time series may be easily decomposed into long-term trend and cyclical components by the use of various filtering techniques, like for instance, Baxter-King (King et al. 1995, Murray 2003), Butterworth (Selesnick & Burrus 1998), Christiano-Fitzgerald (Christiano & Fitzgerald 2003), or Hodrick-Prescott (Hodrick & Prescott 1997) filters.

In what follows, we demonstrate this new methodology, which combines two different approaches to identify how entrepreneurship behaves with business cycle.

First Approach

The first approach to identification whether entrepreneurship demonstrates *pro-cyclical*, *counter-cyclical or contemporaneous* behavior with the business cycle, involves standard time-series detrending procedures, which allow extracting the cyclical components from original time series.

Therefore, our time series on $Self_{i,t}$ and $GDP_{i,t}$ are filtered and decomposed into long-term trends and cyclical components.

Henceforth we obtain two long-term trends:

$$y_t\_Self_{i,t} = \tau_t\_Self_{i,t} + c_t\_Self_{i,t} \tag{4}$$

and

$$y_t\_GDP_{i,t} = \tau_t\_GDP_{i,t} + c_t\_GDP_{i,t} \tag{5}$$

with standard notation.

We argue that $c_t\_Self_{i,t}$ – as self-employment cyclical components may be labeled as 'entrepreneurship cycle', while $c_t\_GDP_{i,t}$ – as GDP cyclical components may be labeled as 'GDP cycle'.

To observe the entrepreneurial activity over business cycle, on standard coordinate system we plot a number of observations (while the number of observation refers to the number of periods considered for analysis) – points; while each observation is defined by two coordinates determined by the value of cyclical components – $c_t\_Self_{i,t}$ and $c_t\_GDP_{i,t}$, at exact time period – t (see Fig.1).

We assume that changes in total self-employment are rather determined changes in gross domestic product than vice-versa, henceforth we plot $c_t\_Self_{i,t}$ as depended variable, while $c_t\_GDP_{i,t}$ is preferably treated as explanatory variable. We additionally plot two 'zero-lines' as two-dimensional reference grid, where one vertical (red) line indicates long-term GDP trend ($y_t\_GDP_{i,t}$); and – horizontal (blue) line indicates long-term self-employment trend ($y_t\_Self_{i,t}$). Drawing two 'zero-lines' on the original coordinate system allows defining four distinct quarters – Q1, Q2, Q3 and Q4, which is essential for our further analysis. We argue that all observations falling into 1Q and 3Q present pro-cyclical behavior of $Self_{i,t}$ versus $GDP_{i,t}$; while observation in 1Q present the present pro-cyclical behavior of $Self_{i,t}$ versus $GDP_{i,t}$ during the expansion phase of business cycle, and the observation in 3Q present the present pro-cyclical behavior of $Self_{i,t}$ versus $GDP_{i,t}$ during the recession phase of business cycle.

Contrary to the previous, we claim that observations falling into 2Q and 4Q present counter-cyclical behavior of $Self_{i,t}$ versus $GDP_{i,t}$; while the observation in 2Q present the present counter-cyclical behavior of $Self_{i,t}$ versus $GDP_{i,t}$ during the recession phase of business cycle, and the observation in 4Q present the present counter-cyclical behavior of $Self_{i,t}$ versus $GDP_{i,t}$ during the expansion phase of business cycle.
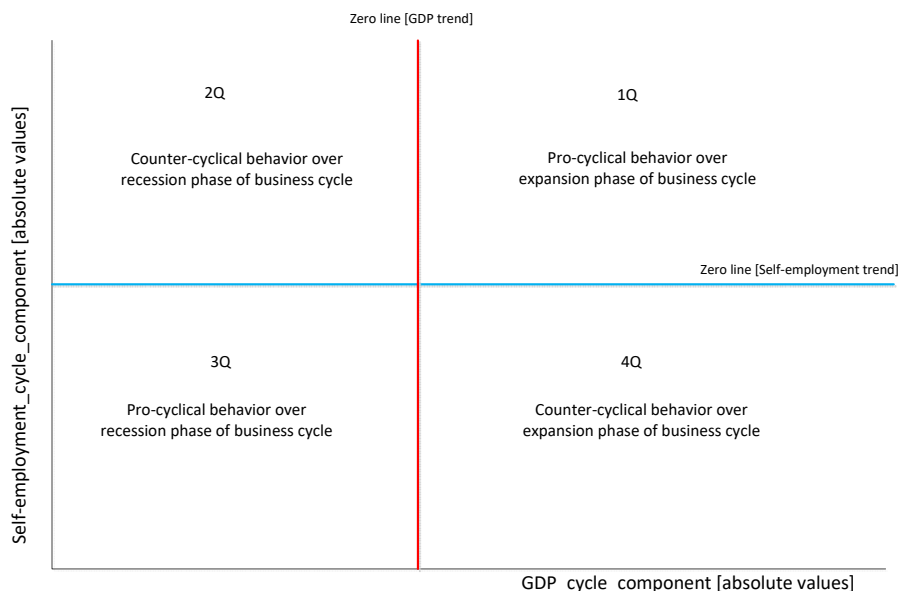

Figure 1. $Self_{i,t}$ **versus** $GDP_{i,t}$ **– detecting pro-cyclical, counter-cyclical or contemporaneous behavior of self-employment with the business cycle**
Source: authors` elaboration.

The first step of the analysis involves calculations of correlation coefficients between $c_t\_Self_{i,t}$ and $c_t\_GDP_{i,t}$ for the number of observations falling into respective quarters – 1Q, 2Q, 3Q and 4Q, which allows for preliminary identification of the strength of pro-cyclical or counter-cyclical behavior of $Self_{i,t}$ versus $GDP_{i,t}$ (see Table 2). However, importantly to note that to draw qualitative conclusion, total number of observation in

each quarter (1Q, 2Q, 3Q and 4Q) is essential, as concluding on very limited number of observations may be misleading and lack representativeness.

Table 2, summarizes expected statistical relationships between $c_t\_Self_{i,t}$ and $c_t\_GDP_{i,t}$ over respective phases of business cycle. Suppose, we are in the first or third quarter (Q1 or Q3), which shows that self-employment demonstrates rather pro-cyclical behavior during expansion/recession phase of business cycle. In this case, we expect that $c_t\_Self_{i,t} > 0$ and $c_t\_GDP_{i,t} > 0$ (during expansion phase), or $c_t\_Self_{i,t} < 0$ and $c_t\_GDP_{i,t} < 0$ (during recession phase). Importantly to note, that – regardless the quarter – calculated correlation coefficient may be both positive and negative. In Q1, positive correlation coefficient indicates that the higher changes in $|c_t\_GDP_{i,t}|$ the higher changes in $c_t\_Self_{i,t}$, and vice versa; while if correlation coefficient results to be negative – the higher changes in $|c_t\_GDP_{i,t}|$ the lower changes in $|c_t\_Self_{i,t}|$, and vice versa. In Q3, positive correlation coefficient indicates that the higher changes in $|c_t\_GDP_{i,t}|$ the higher changes in $|c_t\_Self_{i,t}|$ and vice versa; while if correlation coefficient results to be negative – the higher changes in $|c_t\_GDP_{i,t}|$ the lower changes in $|c_t\_Self_{i,t}|$, and vice versa. Similar calculation, however for Q2 and Q4, may suggest that self-employment demonstrates rather counter-cyclical behavior during expansion/recession phase of business cycle. In this case, we expect that $c_t\_Self_{i,t} < 0$ and $c_t\_GDP_{i,t} > 0$ (during expansion phase), or $c_t\_Self_{i,t} > 0$ and $c_t\_GDP_{i,t} < 0$ (during recession phase). In Q2, positive correlation coefficient indicates that the higher changes in $|c_t\_GDP_{i,t}|$ the higher changes in $|c_t\_Self_{i,t}|$, and vice versa; while if correlation coefficient results to be negative – the higher changes in $|c_t\_GDP_{i,t}|$ the lower changes in $|c_t\_Self_{i,t}|$, and vice versa. Analogous relationships are reported for Q4.

Table 2. Identification of pro-cyclical or counter-cyclical behavior of $Self_{i,t}$ with the business cycle

2Q	1Q
Counter-cyclical behavior of $Self_{i,t}$ over the recession phase of business cycle (below GDP_trend line) If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²<0 – relatively high/low changes in var1 (modulus) are accompanied by relatively low/high changes in var2 (modulus). If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²>0 – relatively high/low changes in var1 (modulus) are accompanied by relatively high/low changes in var2 (modulus).	**Pro-cyclical** behavior of $Self_{i,t}$ over the expansion phase of business cycle (above GDP_trend line) If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²<0 – relatively high/low changes in var1 (modulus) are accompanied by relatively low/high changes in var2 (modulus). If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²>0 – relatively high/low changes in var1 (modulus) are accompanied by relatively high/low changes in var2 (modulus).
3Q	4Q
Pro-cyclical behavior of $Self_{i,t}$ over the recession phase of business cycle (below GDP_trend line) If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²<0 – relatively high/low changes in var1 (modulus) are accompanied by relatively low/high changes in var2 (modulus). If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²>0 – relatively high/low changes in var1 (modulus) are accompanied by relatively high/low changes in var2 (modulus).	**Counter-cyclical** behavior of $Self_{i,t}$ over the expansion phase of business cycle (above GDP_trend line) If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²<0 – relatively high/low changes in var1 (modulus) are accompanies by relatively low/high changes in var2 (modulus). If *r-squared* of $c_t\_Self_{i,t}$ *versus* $c_t\_GDP_{i,t}$ – r²>0 – relatively high/low changes in var1 (modulus) are accompanies by relatively high/low changes in var2 (modulus).

Source: authors` elaboration.

Next, we propose to develop a new coefficient allowing identifying the **'entrepreneurship vulnerability'** to business cycle. The Entrepreneurship Vulnerability Coefficients is as:

$$EVC_{i,t,q}^{self\leftrightarrow gdp} = \frac{c_t\_Self_{i,t}}{c_t\_GDP_{i,t}} \qquad (6)$$

where:

$c_t\_Self_{i,t}$ - represents self-employment deviation (%) from long-term trends;

$c_t\_GDP_{i,t}$ - stands for gross domestic product deviations (%) from long-term trends;

i - is country;

t - time period and q stands respectively for Q1, Q2, Q3 and Q4 (as defined in Fig.1).

Put simply, the $EVC_{i,t,q}^{self\leftrightarrow gdp}$ expresses the ratio between $c_t\_Self_{i,t}$ and $c_t\_GDP_{i,t}$, indicating the difference in strength of simultaneous changes of self-employment (measured as $c_t\_Self_{i,t}$) respective to GDP (measured as $c_t\_GDP_{i,t}$) at given period t. By definition the entrepreneurship vulnerability coefficient – $EVC_{i,t,q}^{self\leftrightarrow gdp}$ value ranges from $(-\infty)$ to $(+\infty)$, but differs from zero ($EVC_{i,t,q}^{self\leftrightarrow gdp} \neq 0$). Put different-ly, the Entrepreneurship Vulnerability Coefficients, measures strength and direction of entrepreneurship (measures as total self-employment) reaction to changes in gross domestic product in i-country and t, q – time period.

As claimed, entrepreneurship may demonstrate pro-cyclical or counter-cyclical behavior over respective phases of business cycle. In this line, arguably when $EVC_{i,t,q}^{self\leftrightarrow gdp} > 0$ it shows that *self-employment* (entrepreneurship) *behaves pro-cyclically* regardless whether expansion or recession phase of business cycle is considered. And, reversely, when $EVC_{i,t,q}^{self\leftrightarrow gdp} < 0$ it shows that *self-employment* (entrepreneurship) *behaves counter-cyclically* regardless whether expansion or recession phase of business cycle is considered.

Considering the fact that ($EVC_{i,t,q}^{self\leftrightarrow gdp}$) coefficient represents the ratio between $c_t\_Self_{i,t}$ and $c_t\_GDP_{i,t}$, for the in-depth analysis of entrepreneurship vulnerability to changes in gross domestic product over business cycle. We propose to discriminate be-tween 3 different categories of entrepreneurship vulnerability:

– *Strong Entrepreneurship Vulnerability* (hereafter – $SEV_{i,t,q}^{self\leftrightarrow gdp}$) – indicates that en-trepreneurship may demonstrate strong pro-cyclical or strong counter-cyclical behav-ior both during expansion or recession phase of business cycle;

– *Weak Entrepreneurship Vulnerability* (hereafter – $WEV_{i,t,q}^{self\leftrightarrow gdp}$) – indicates that en-trepreneurship may demonstrate weak pro-cyclical or strong counter-cyclical behavior both during expansion or recession phase of business cycle;

– *Neutral Entrepreneurship Vulnerability* (hereafter – $NEV_{i,t,q}^{self\leftrightarrow gdp}$) – indicates that entrepreneurship may demonstrate neutral pro-cyclical or strong counter-cyclical be-havior both during expansion or recession phase of business cycle.

$SEV_{i,t,q}^{self\leftrightarrow gdp}$, $WEV_{i,t,q}^{self\leftrightarrow gdp}$ and $NEV_{i,t,q}^{self\leftrightarrow gdp}$ maybe identified, over respective phases of business cycle, regardless whether entrepreneurship patterns are reported as pro-cyclical or counter-cyclical. Arguably, entrepreneurship unveils strong vulner-ability ($SEV_{i,t,q}^{self\leftrightarrow gdp}$) to business cycle when $\left|EVC_{i,t,q}^{self\leftrightarrow gdp}\right| > 1$; weak vulnerability

$(WEV_{i,t,q}^{self \leftrightarrow gdp})$ to business cycle when $\left| EVC_{i,t,q}^{self \leftrightarrow gdp} \right| < 1$; and − neutral vulnerability $(NEV_{i,t,q}^{self \leftrightarrow gdp})$ to business cycle when $\left| EVC_{i,t,q}^{self \leftrightarrow gdp} \right| = 1$ (also see − Fig.2).
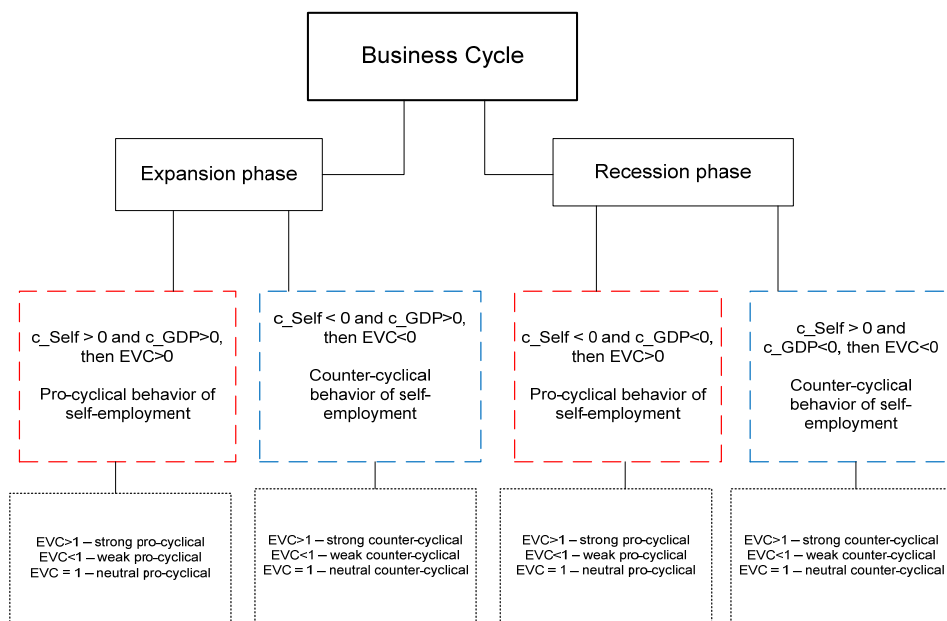

Figure 2. Categories of entrepreneurship vulnerability
Source: Authors` elaboration.

Second Approach

Second approach to identification if entrepreneurship demonstrates *pro-cyclical*, *counter-cyclical or contemporaneous* behavior over business cycle, encompasses deployment of original time series on self-employment and GDP, which are not decomposed into long-term trends and cyclical components. In this approach we rather target to conclude on pro-cyclical or counter-cyclical behavior of self-employment versus gross domestic product basing on detailed analysis of year-to-year dynamics of examined variables.

To this aim we define 4 variables:

− $GDP_{i,t_{dynamic}}$;
− $GDP_{i,[t;(t+n)]\_}aver_{dynamic}$;
− $Self_{i,t_{dynamic}}$;
− $Self_{i,(t;(t+n))\_}aver_{dynamic}$.

where:

i - denotes country

t - year, and the total period of analysis is given by $[t + n]$.

Moreover, we assume that over analyzed period $(t + n)$, $GDP_{i,t_{dynamic}} > 0$ and $Self_{i,t_{dynamic}} > 0$.

We also assume that the following is true (see also Fig.3):

- $GDP_{i,t_{dynamic}} > 1 \rightarrow$ expansion phase of business cycle at certain t-period;
- $GDP_{i,t_{dynamic}} < 1 \rightarrow$ recession phase of business cycle at certain t-period;
- $GDP_{i,[t;(t+n)]_{aver_{dynamic}}} > 1\ 1 \rightarrow$ long-term growth cycle during the (t+n) period;
- $GDP_{i,[t;(t+n)]_{aver_{dynamic}}} < 1 \rightarrow$ long-term recession cycle during the (t+n) period.

<div align="center">

Business cycle

Expansion phase   Recession phase

Self-employment dynamic >1   Selfemployment dynamic <1   Selfemployment dynamic > 1   Selfemployment dynamic < 1
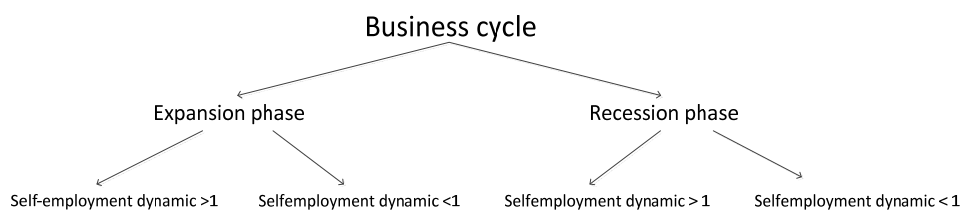
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Figure 3. Business cycle phases versus self-employment
Source: authors` elaboration.

The second approach allows defining 4 different cases, which may be easily distinguished regarding long-term trends in self-employment and gross domestic product. These are:

CASE_1: Original time series data on self-employment and gross domestic product demonstrate average year-to-year dynamics higher than 1, over analyzed period → ($Self_{i,[t;(t+n)]_{aver_{dynamic}}} > 1$ and $GDP_{i,[t;(t+n)]_{aver_{dynamic}}} > 1$).

CASE_2: Original time series data on self-employment and gross domestic product demonstrate average year-to-year dynamics lower than 1, over analyzed period → ($Self_{i,[t;(t+n)]_{aver_{dynamic}}} < 1$ and $GDP_{i,[t;(t+n)]_{aver_{dynamic}}} < 1$).

CASE_3: Original time series data on self-employment demonstrate average year-to-year dynamics higher than 1, over analyzed period ($Self_{i,[t;(t+n)]_{aver_{dynamic}}} > 1$); while data on gross domestic product demonstrate average year-to-year dynamics lower than 1 ($GDP_{i,[t;(t+n)]_{aver_{dynamic}}} < 1$).

CASE_4: Original time series data on self-employment demonstrate average year-to-year dynamics lower than 1, over analyzed period ($Self_{i,[t;(t+n)]_{aver_{dynamic}}} < 1$); while data on gross domestic product demonstrate average year-to-year dynamics higher than 1 ($GDP_{i,[t;(t+n)]_{aver_{dynamic}}} > 1$).

In what follows we explain

a. Entrepreneurship behavior in expansion phase of business cycle

(CASE_1 and CASE_4)

As claimed in previous paragraphs, during expansion phase of business cycle, entrepreneurship may demonstrate pro-cyclical or counter-cyclical behavior.

a.1. CASE_1: ($Self_{i,[t;(t+n)]_{aver_{dynamic}}} > 1$ and $GDP_{i,[t;(t+n)]_{aver_{dynamic}}} > 1$).

Self-employment behavior in expansion phase of business cycle.
Self-employment average year-to-year dynamic > 1.

Self-employment dynamic

| 1 | 2 | 3 |

Average self-employment dynamic

| 4 | 5 | 6 |

1

| 7 | 8 | 9 |

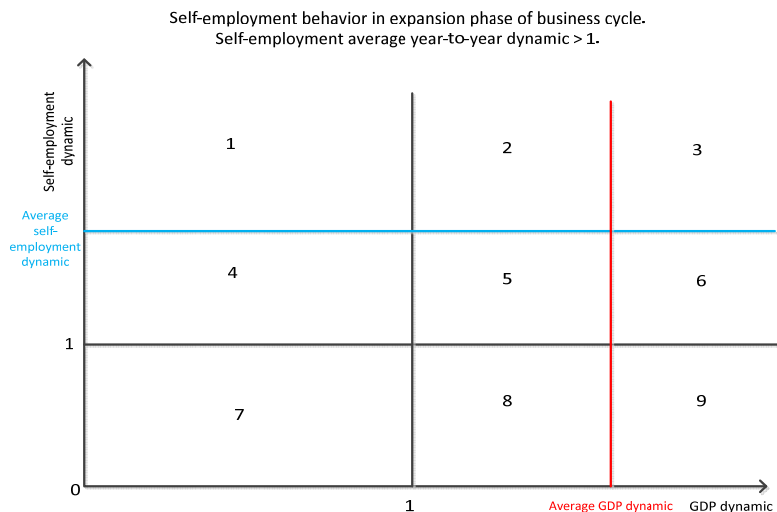0   1   Average GDP dynamic   GDP dynamic

**Figure 4. $Self_{i,t}$ versus $GDP_{i,t}$ – detecting pro-cyclical
or counter-cyclical behavior of Self-employment**
Note: this specification assumes no time series detrending procedures,
but it bases on the $Self_{i,t}$ and $GDP_{i,t}$ year-to-year dynamics over analyzed period.
Source: authors`elaboration.

Table 3.

	Type of behavior $Self_{i,t}$ over business cycle	Formal condition
1P	Strong counter-cyclical	$Self_{i,t_{dynamic}} > Self_{i,[t;(t+n)]_{aver_{dynamic}}}$; $GDP_{i,t_{dynamic}} < 1$
2P	Strong pro-cyclical	$Self_{i,t_{dynamic}} > Self_{i,[t;(t+n)]_{aver_{dynamic}}}$; $1 < GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
3P	Average pro-cyclical	$Self_{i,t_{dynamic}} > Self_{i,[t;(t+n)]_{aver_{dynamic}}}$; $GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
4P	Average counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}} > 1$; $GDP_{i,t_{dynamic}} < 1$
5P	Average pro-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}} > 1$; $1 < GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
6P	Weak pro-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}} > 1$; $GDP_{i,t_{dynamic}} > 1$
7P	Pro-cyclical in recession phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} < 1$; $GDP_{i,t_{dynamic}} < 1$
8P	Counter-cyclical in expansion phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} < 1$; $1 < GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
9P	Counter-cyclical in expansion phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} < 1$; $GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$

Source: authors' study.

a.1. CASE_4: $(Self_{i,[t-(t+n)]_{aver_{dynamic}}} < 1)$ and $(GDP_{i,[t-(t+n)]_{aver_{dynamic}}} > 1)$.

Self-employment behavior in expansion phase of business cycle.
Self-employment average year-to-year dynamic < 1.

Figure 5. $Self_{i,t}$ versus $GDP_{i,t}$ – detecting pro-cyclical
or counter-cyclical behavior of self-employment
Note: this specification assumes no time series detrending procedures,
but it bases on the $Self_{i,t}$ and $GDP_{i,t}$ year-to-year dynamics over analyzed period.
Source: authors`elaboration.

Table 4.

	Type of behavior $Self_{i,t}$ over business cycle	Formal condition
1P	Counter-cyclical in recession phase - no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} > 1; GDP_{i,t_{dynamic}} < 1$
2P	Pro-cyclical in expansion phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} > 1$ $1 < GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
3P	Pro-cyclical in expansion phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} > 1$ $GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
4P	Average pro-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} < Self_{i,t_{dynamic}} < 1$ $GDP_{i,t_{dynamic}} < 1$
5P	Average counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} < Self_{i,t_{dynamic}} < 1$ $1 < GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
6P	Weak counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} < Self_{i,t_{dynamic}} < 1$ $GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
7P	Strong pro-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}}$ $GDP_{i,t_{dynamic}} < 1$
8P	Strong counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}}$ $1 < GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
9P	Average counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}}$ $GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$

Source: authors' study.

b. Entrepeneurship behavior in recession phase of business cycle
(CASE_2 and CASE_3)

b.1. CASE_2: ($Self_{i,[t-(t+n)]_{aver_{dynamic}}} < 1$ and $GDP_{i,[t-(t+n)]_{aver_{dynamic}}} < 1$).
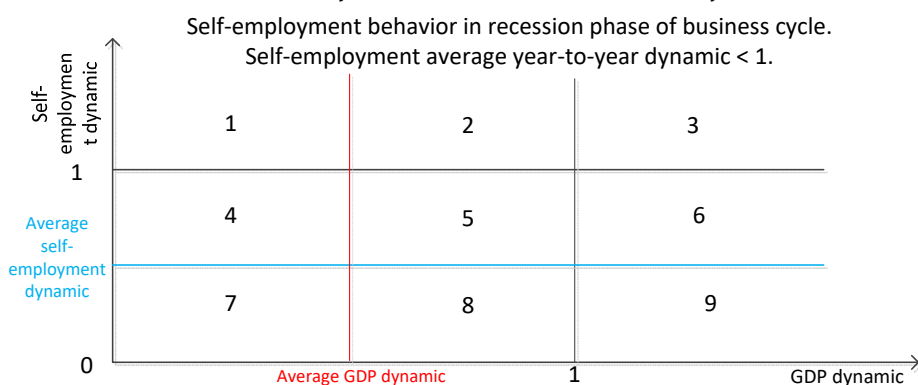

Figure 6. $Self_{i,t}$ *versus* $GDP_{i,t}$– detecting pro-cyclical
or counter-cyclical behavior of self-employment

Note: this specification assumes no time series detrending procedures, but it bases on the $Self_{i,t}$ and $GDP_{i,t}$
year-to-year dynamics over analyzed period.
Source: authors`elaboration.

Table 5.

	Type of behavior $Self_{i,t}$ over business cycle	Formal condition
1P	Counter-cyclical in recession phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} > 1$ $GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
2P	Counter-cyclical in recession phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} > 1$ $1 < GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
3P	Pro-cyclical in expansion phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} > 1$ $1 < GDP_{i,t_{dynamic}}$
4P	Weak counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} < Self_{i,t_{dynamic}} < 1$ $GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
5P	Average counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} < Self_{i,t_{dynamic}} < 1$ $1 < GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
6P	Average counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} < Self_{i,t_{dynamic}} < 1$ $1 < GDP_{i,t_{dynamic}}$
7P	Average counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}}$ $GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
8P	Strong counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}}$ $1 < GDP_{i,t_{dynamic}} > GDP_{i,[t;(t+n)]_{aver_{dynamic}}}$
9P	Strong counter-cyclical	$Self_{i,[t;(t+n)]_{aver_{dynamic}}} > Self_{i,t_{dynamic}}$ $1 < GDP_{i,t_{dynamic}}$

Source: authors' study.

b.2. CASE_3: ($Self_{i,[t-(t+n)]aver_{dynamic}} > 1$ and $GDP_{i,[t-(t+n)]aver_{dynamic}} < 1$).
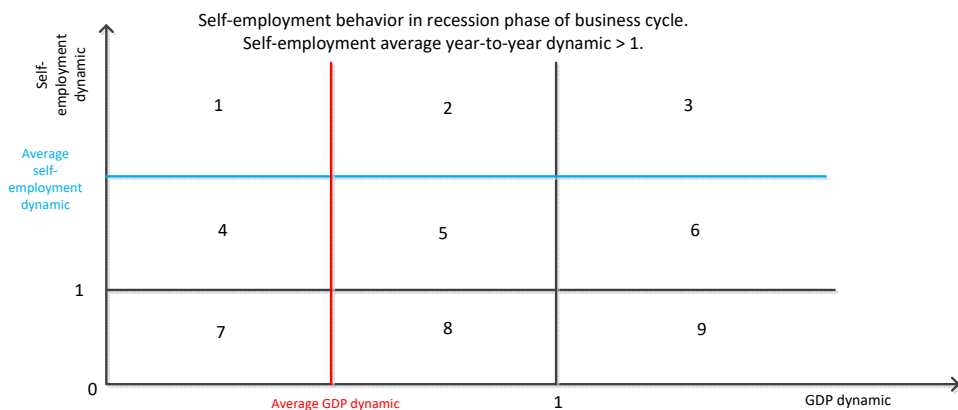

Figure 7. $Self_{i,t}$ versus $GDP_{i,t}$ – detecting pro-cyclical
or counter-cyclical behavior of self-employment
Note: this specification assumes no time series detrending procedures, but it bases on the $Self_{i,t}$ and $GDP_{i,t}$
year-to-year dynamics over analyzed period.
Source: authors`elaboration.

Table 6.

	Type of behavior $Self_{i,t}$ over business cycle	Formal condition
1P	Average counter-cyclical	$Self_{i,[t;(t+n)]aver_{dynamic}} < Self_{i,t_{dynamic}}$ $GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]aver_{dynamic}}$
2P	Strong counter-cyclical	$Self_{i,[t;(t+n)]aver_{dynamic}} < Self_{i,t_{dynamic}}$ $GDP_{i,[t;(t+n)]aver_{dynamic}} < GDP_{i,t_{dynamic}} < 1$
3P	Strong pro-cyclical	$Self_{i,[t;(t+n)]aver_{dynamic}} < Self_{i,t_{dynamic}}$ $GDP_{i,t_{dynamic}} > 1$
4P	Weak counter-cyclical	$Self_{i,[t;(t+n)]aver_{dynamic}} > Self_{i,t_{dynamic}} > 1$ $GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]aver_{dynamic}}$
5P	Average counter-cyclical	$Self_{i,[t;(t+n)]aver_{dynamic}} > Self_{i,t_{dynamic}} > 1$ $GDP_{i,[t;(t+n)]aver_{dynamic}} < GDP_{i,t_{dynamic}} < 1$
6P	Average pro-cyclical	$Self_{i,[t;(t+n)]aver_{dynamic}} > Self_{i,t_{dynamic}} > 1$ $GDP_{i,t_{dynamic}} > 1$
7P	Pro-cyclical in recession phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} < 1$ $GDP_{i,t_{dynamic}} < GDP_{i,[t;(t+n)]aver_{dynamic}}$
8P	Pro-cyclical in recession phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} < 1$ $GDP_{i,[t;(t+n)]aver_{dynamic}} < GDP_{i,t_{dynamic}} < 1$
9P	Counter-cyclical in expansion phase – no discrimination among Strong/Average/Weak behavior	$Self_{i,t_{dynamic}} < 1$ $GDP_{i,t_{dynamic}} > 1$

Source: authors' study.

ENTREPRENEURIAL PATTERNS IN ITALY. PRO-CYCLICAL, COUNTER-CYCLICAL OR RANDOM WALK?

This section encompasses three consecutive parts. Section 5.1 briefly explains statistical data on self-employment and gross domestic product in Italy, which have been used in our empirical research. Next, Section 5.2 presents summary statistics on self-employment and gross domestic product in Italy over the period 1995-2015. In this section we also demonstrate self-employment and GDP time series decomposition, and investigate comovements between examined variables by using cross-correlation methodological framework (Burns & Mitchell, 1946). Finally, section 5.3 exemplifies adoption of our newly developed methodological approach to identification whether self-employment behaves pro-cyclically or counter-cyclically.

Data

To demonstrate whether entrepreneurial activity demonstrates pro-cyclical or rather counter-cyclical behavior over business cycle, we consider the case of Italy. To this aim we use exclusively two types of economic time series: quarterly data on total self-employment (hereafter – $Self_{i,t}$) – to measure the intensity of entrepreneurship; and quarterly data on total gross domestic product (hereafter – $GDP_{i,t}$) – to measure the business cycle. The sample period is set for 1995q1-2015q2. Data on self-employment and gross domestic product have been seasonally adjusted, and additionally GDP data has been corrected for inflation using seasonally adjusted quarterly deflators. Data on self-employment has been extracted from OECD Annual Labor Force Statistics, while data on GDP and deflators – from OECD Statistical database (accessed: December 2015).

Descriptive statistics and time-series properties.
Measuring entrepreneurship and business cycle co-movements – a standard approach

This section briefly discusses changes in self-employment ($Self_{It,1995-2015}$) and gross domestic product ($GDP_{It,1995-2015}$) in Italy between years 1995-2015. Additionally, it preliminary investigates whether self-employment 'movements' are rather pro-cyclical or counter-cyclical over business cycle over examined period in Italy. To this aim, using statistical analysis of cross-correlations (Burns & Mitchell 1946), it shortly demonstrates results of comovements analysis between pair of series – $Self_{It,1995-2015}$ and $GDP_{It,1995-2015}$.

In our research, we follow the general convention and define the business cycle fluctuation as deviations from long-run trend in total gross domestic product time series data. In this line, we need to decompose the original time series into trend and its component. In literature there are several time series filters used allowing for separating trend and business-cycle components over various economic data; these are, for instance Hodrick-Prescot, Baxter-King or Christiano-Fitzgerald filters. In here we propose to adopt of the square-wave high-pass Butterworth filter, for time series detrending procedure, which originally was developed by Butterworth (1930), and then used in multiple empirical researches, see for instance works of Kaiser & Maravall (1999, 2012), Gomez (2001), Harvey & Trimbur (2003). As suggested by Stove (1986) and Pollock (2000), the use of low pass Butterworth filter, also referred as 'maximally flat', allows for flexible smoothing data time series, and hence trends and cyclical components estimation from

economic data. Also as claimed by Pollock et al. (1999) or Gomez (2001), the mechanical application of Hodrick-Prescott filter for economic time series detrending procedures, often leads to obtaining spurious results, and thus other band-pass filters or Butterworth filter is more suitable for extracting smooth cycles from economic time series. Consider that given time series may be decomposed into two additively separable components as proposed by Hodrick and Prescott (1997):

$$y_t = \tau_t + c_t + \varepsilon_t \tag{7}$$

where:

y_t - is the time series;

τ_t - is trend component;

c_t - stands for the stationary cyclical component that is determined by the stochastic cycles across multiple periods (Cogley and Nason, 1995);

ε_t - may be defined as unpredictable random component.

Following the Eq.(7), it may be argued that the trend component may be calculated by simply extracting:

$$\tau_t = y_t - c_t - \varepsilon_t \tag{8}$$

and so the cycle components follows as:

$$c_t = y_t - \tau_t - \varepsilon_t \tag{9}$$

Despite the fact, that the unobserved random component - ε_t – may affect the business cycle, most of standard detrending filters simply take out the trend out of original time series, while the sum of cyclical and unobserved random component ($c_t + \varepsilon_t$) are treated comprehensively as a measure of a cycle (Carmona et al. 2010).

Table 7 presents summary statistics on self-employment and GDP seasonally adjusted quarterly data in Italy over the period 1995q1-2015q2. In Fig.8 original time series on self-employment and GDP, including long-term trends ($y\_Self_{It,1995-2015}$; $y\_GDP_{It,1995-2015}$) generated from Butterworth filter, are plotted (left-hand graph). Additionally, Fig.8 unveils the statistical relationship between $Self_{It,1995-2015}$ and $GDP_{It,1995-2015}$ original time series over examined period (right-hand graph).

Table 7. Summary statistics – Self-employment and GDP in Italy. Quarterly levels, data seasonally adjusted. 1995q1-2015q2

	Obs.	value in 1995q1	value in 2015q2	Average	Min. value	Max. value	Average quarterly growth rate (%)
Gross Domestic product (total, millions, euro)	82	351785.9	386272.7	391139.2	351785.9 (1995q1)	425053.6 (2008q1)	0.11% per quarter
Self-employment (total, thousands, persons)	82	6024.5	6233.8	6378.9	6024.5 (1995q1)	6744 (2004q4)	0.042 % per quarter

Source: authors study.

As presented in Fig.8, in Italy over the period 1995q1-2015q2, at first look regarding self-employment (entrepreneurship) and GDP trends, both upward and downward trends are easily observable and distinguishable. Over analyzed years in Italy, considering original GDP time series the upward trend spans from first quarter of 1995 till first quarter of 2008, when GDP reached its maximum at $GDP_{It,2008q1}$ = 425053,6 (mln Euro) (the trend peak is reported for 2007q1 and corresponds to $y\_GDP_{It,2007q1}$=414925,6 mln

Euro); and since 2008q1 sharp downward trend of GDP is clearly visible. Regarding time series on self-employment similar observations with respect to in-time changes are reported. Preliminary analysis of self-employment time series, over the period 1995-2015, also allows distinguishing both upward and downward 'parts' of long-run trend. Regarding original sell-employment time series, the total number of self-employed persons was steadily growing reaching its maximum in fourth quarter of 2004, $SELF_{It,2004q4}$=6774 thousands of persons (the trend peak is reported for 2005q3 and corresponds to $y\_Self_{It,2005q3}$= 6628 thousands of persons); while since then onward rapid decreases in total number of self-employed persons are noted.
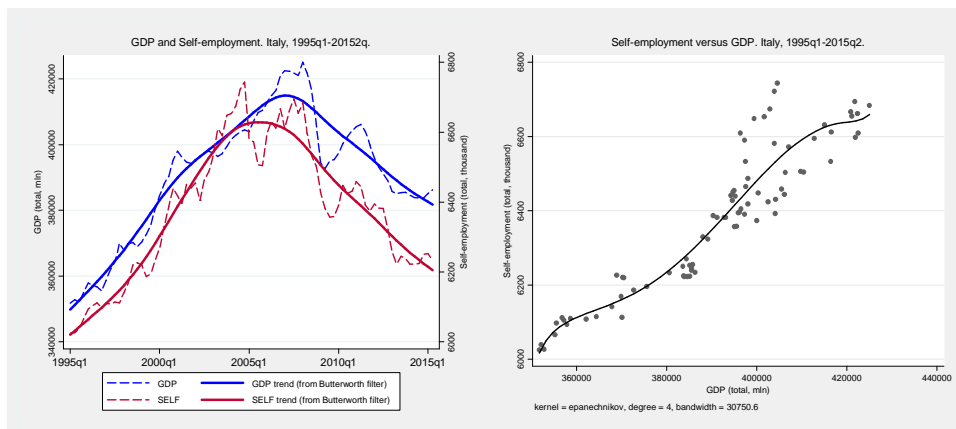

**Figure 8. Self-employment and GDP in Italy. Quarterly levels,
data seasonally adjusted. 1995q1-2015q2**
Source: suthors` elaboration. Note: right-hand graph – non-parametric approximation
(4-degree local polynomial smoothing applied).

Additionally, the left-hand graph shows considerably high and possibly positive relationship between self-employment and GDP trends. Similar claims may be raised when looking at the right-hand graph where self-employment original time series are plotted versus GDP data. These may suggest that total number of self-employed persons and value of total gross domestic product are highly correlated over analyzed time periods, and they tend to 'move' in the same direction. This supposition may be also supported by calculated pairwise correlation coefficients for consecutive pairs of variables: $Self_{It,1995-2015}$ and $GDP_{It,1995-2015}$, which is 0.92 (statistically significant at 5% level of significance); and $y\_Self_{It,1995-2015}$ and $y\_GDP_{It,1995-2015}$, which is 0.95 (statistically significant at 5% level of significance).

Next, Fig.9 shows self-employment ($c\_Self_{It,1995-2015}$) and GDP ($c\_GDP_{It,1995-2015}$) cyclical components fluctuating around the trends, in Italy over the period 1995q1-2015q2. By convention, cyclical components – $c\_Self_{It,1995-2015}$ and $c\_GDP_{It,1995-2015}$, are expressed as deviations from long-term trends (%); while changes of $c\_Self_{It,1995-2015}$ approximate 'entrepreneurship cycle' and changes of $c\_GDP_{It,1995-2015}$ approximate 'business cycle'. Casual analysis and observation of displayed in Fig.9 entrepreneurship cycle and business cycle may suggest that self-employment unveils rather pro-cyclical

tendencies, instead of behaving counter-cyclically or contemporaneously. Interestingly, what may be concluded from $c\_Self_{It,1995-2015}$ and $c_{GDP_{It,1995-2015}}$ density plots (see Fig. 10 below), and descriptive statistics summarized in Table 4, values of $c\_Self_{It,1995-2015}$ and $c_{GDP_{It,1995-2015}}$ vary between <-1.8%; 1.8%> and <-3.5; 2,8%> respectively, however a great majority of observations ranges from (-2%) to (2%) regarding both $c\_Self_{It,1995-2015}$ and $c_{GDP_{It,1995-2015}}$. This again supports the supposition that entrepreneurship cycle and business cycle are positively correlated (see also correlation coefficient between $c\_Self_{It,1995-2015}$ and $c_{GDP_{It,1995-2015}}$ that is as 0.56 – see results in Table 8)

Table 8. Self-employment and GDP. Pairwise correlations. Italy, 1995q1-2015q2

	$Self_{i,1995-20}$	$GDP_{i,1995-20}$	$c\_Self_{It,1995}$	$c\_GDP_{It,1995}$	$y\_Self_{It,1995}$	$y\_GDP_{It,1995-2015}$
$Self_{i,1995-2015}$	1,00 (82)					
$GDP_{i,1995-2015}$	0,92* (82)	1,00 (82)				
$c\_Self_{It,1995-2015}$	0,32* (82)	0,17 (82)	1,00 (82)			
$c\_GDP_{It,1995-2015}$	0,18 (82)	0,30* (82)	0,56* (82)	1,00 (82)		
$y\_Self_{It,1995-2015}$	0,96* (82)	0,92* (82)	0,05 (82)	0,03 (82)	1,00 (82)	
$y\_GDP_{It,1995-2015}$	0,92* (82)	0,96* (82)	0,03 (82)	0,05 (82)	0,95* (82)	1,00 (82)

Source: authors` calculations. Note: in parenthesis – number of observations. Coefficients with (*) – statistically significant at 5%.
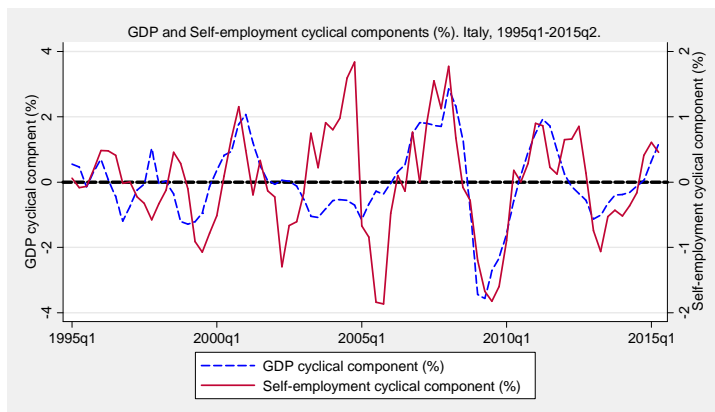

Figure 9. Self-employment and GDP cyclical components (%). Italy, 1995q1-2015q2
Source: authors' elaboration.

Next, deploying traditional statistics, we investigate the comovements between self-employment and GDP. To this aim adopting the methodological framework originally proposed by Burns & Mitchell (1946), we calculate cross-correlation coefficients (δ) between self-employment and GDP cyclical components ($Self_{cycle}$ versus GDP_{cycle}). Following King & Rebelo (1993) oraz Carmona et al. (2010), we define *comovements* of given pair of variable – x and y, as when: (1) – both variables x and y stand cyclical components and they

commove in the same direction over the business cycle so that the correlation coefficient between these two results to be positive; or – (2) – reversely, when variables x and y standing for cyclical components commove in opposite direction over the business cycle so that the correlation coefficient between these two results to be negative. We also may argue that if the correlation coefficient between variables x and y stand cyclical components is close to zero, then x and y do not commove. Regarding the business cycle analysis and comovements of different variables over it, we may say that if the calculated correlation coefficient between the variable explaining the business cycle and another defined variable y is positive, it raises arguments that variable y behaves pro-cyclically; while when the correlation coefficients is negative – variable y behaves counter-cyclically. We may also state that if the correlation coefficients are either negative or positive, by close to zero – the variable y demonstrates rather a-cyclical behavior. In other words, we treat calculated cross-correlation coefficients (δ) as a statistical measure of comovements between each pair of self-employment and GDP cyclical components series. Moreover, as argued by Prescott (1986), Kydland & Prescott (1990) or Harvey & Jaeger (1993), the calculated cross-correlation coefficient (δ) allows concluding on the phase shift of one series of data compared to another series of data. Put differently, the consecutive correlation coefficients are calculated between the detrended time series which are shifted backward or shifter toward by n periods; henceforth we may conclude whether one time series lags or – conversely, leads the other time series (in our case – self-employment and GDP).
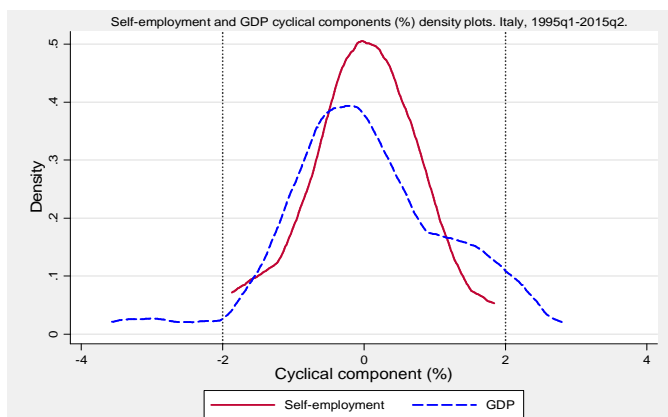
Figure 10. Self-employment and GDP cyclical components
(%) density plots. Italy, 1995q1-2015q2
Source: authors' elaboration.

Table 9. Self-employment and GDP cyclical components – summary statistics. Italy, 1995q1-2015q2

Variable	No. of obs.	Mean	Std. Dev.	Min. value	Max. value
Self-employment cyclical component	82	-0.0012758	0.806851	-1.865586 (2005q4)	1.840425 (2004q4)
GDP cyclical component	82	-0.0031055	1.190239	-3.563782 (2009q2)	2.854013 (2008q1)

Source: Authors` calculations.

Fig.11 and Table 10 comprehensively summarize the results of the analysis of $c\_Self_{It,1995-2015}$; $c\_GDP_{It,1995-2015}$ comovements over analyzed period in Italy. To be more specific, Table 5 reports cross-correlations between $c\_Self_{It,1995-2015}$ and $c\_GDP_{It,1995-2015}$ at different lags and leads; henceforth, the numbers defined as $(t-n)$ or $(t+n)$, show whether detrended time series on self-employment (entrepreneurship cycle) *lead or lag* business cycle (expressed as GDP deviations from long-run trend) by $n-periods$ (in here $-n-quarters$). If cross-correlation coefficient results to be highest at $t=0$, then we it is argued that the self-employment and business cycle tend to move contemporaneously.
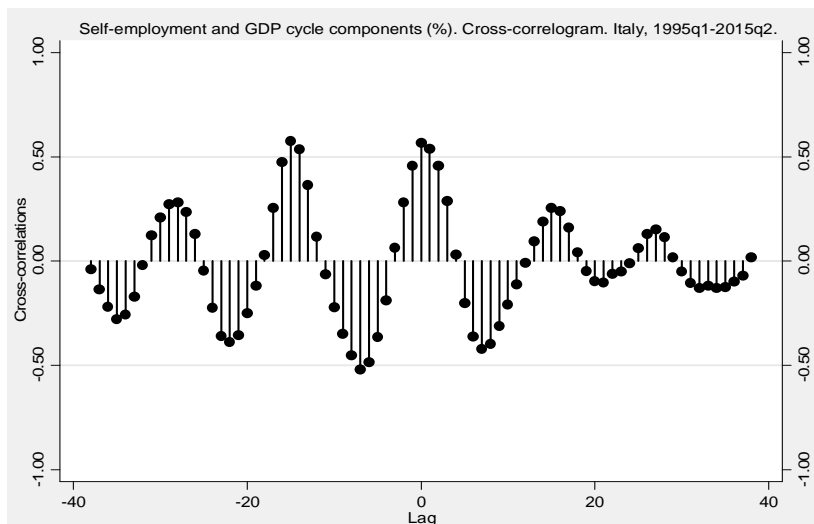

Figure 11. Self-employment (entrepreneurship cycle)
and GDP (business cycle). Cross-correlogram. Italy, 1995q1-2015q2
Source: Authors` elaboration.

Table 10. Cyclical time paths of gross domestic output – correlation of self-employment and gross domestic output (GDP) BW-filtered cyclical components (%) at different leads (t-n) and lags (t+n). Italy, 1995q1-2015q2

	$cross-correlations\ (Self_{i,t}\ ;\ GDP_{i,t})$										
	$t-5$	$t-4$	$t-3$	$t-2$	$t-1$	$t=0$	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$
Self-employment (quarterly levels)	-0,36	-0,18	0,06	0,28	0,45	**0,56**	0,53	0,45	0,28	0,03	-0,20
	$Self_{i,t}$ leads $GDP_{i,t}$ (business cycle)					$Self_{i,t}$ and $GDP_{i,t}$ move contemporaneously	$Self_{i,t}$ lags $GDP_{i,t}$ (business cycle)				

Source: authors` calculations. Note: complete list of correlation coefficients for all 38 leads and lags – see Appendix X.

The results of cross-correlation analysis comprehensively suggest that, over analyzed period, the correlation coefficients are typically positive, which again speaks in support of the hypothesis on pro-cyclicality of entrepreneurship. Moreover, the highest cross-correlation coefficient is at t=0, hence entrepreneurship (self-employment) may be de-

fined as moving contemporaneously with the business cycle. Regarding the periods (t-1), (t+1) and (t+2), the cross-correlations only slightly differ from the result at t=0, but most importantly they are still positive. Finally, we observe that the cross-correlations for the remaining periods are substantially smaller, switching from being negative to positive.

Entrepreneurship Versus Business Cycle – Pro-Cyclical, Counter-Cyclical or Random Walk? Exemplification of New Methodology

This section targets to exemplify adoption of new methodology designed to verify whether entrepreneurship (measured as total self-employment) behaves pro-cyclically or counter-cyclically with the business cycle. In here, we deploy analogous economic time series as in Sect. 5.2, hence seasonally adjusted quarterly data on total self-employment and total gross domestic product (corrected for inflation) in Italy over the period 1995-2015. In what follows we present the results of our analysis, which are confronted with those formerly discussed in Sect. 5.2.

Similarly to the analysis in Sect. 5.2, original time series on self-employment and gross domestic product, using Butterworth filter, have been decomposed in long-term trends ($y\_Self_{It,1995-2015}$; $y\_GDP_{It,1995-2015}$) and cyclical components ($c\_Self_{It,1995-2015}$; $c\_GDP_{It,1995-2015}$) measuring from trends deviations. Figs. 12 and 13 plot $c\_Self_{It,1995-2015}$ versus $c\_GDP_{It,1995-2015}$, which exhibit statistical relationship between examined variables. In Figs. 12 and 13, two-dimensional reference grid has been plotted – red vertical line represent GDP trend and blue horizontal line represent for self-employment trend, which allows discriminating between pro-cyclical entrepreneurship behavior during expansion phase ($\rightarrow$ observations falling into first quarter – Q1), pro-cyclical entrepreneurship behavior during recession phase ($\rightarrow$ observations falling into third quarter – Q3), counter-cyclical entrepreneurship behavior during expansion phase ($\rightarrow$ observations falling into second quarter – Q2) and counter-cyclical entrepreneurship behavior during recession phase ($\rightarrow$ observations falling into fourth quarter – Q4).

Figure 12. Self-employment and GDP cyclical components (%). Italy, 1995q1-2015q2
Source: authors` elaboration. Note: Red line $\rightarrow$ GDP trend; blue line $\rightarrow$ self-employment trend.
Vertical and horizontal dash lines are drawn to capture most densely 'populated' areas.
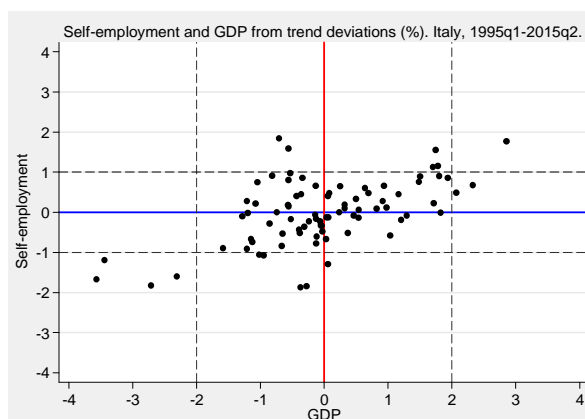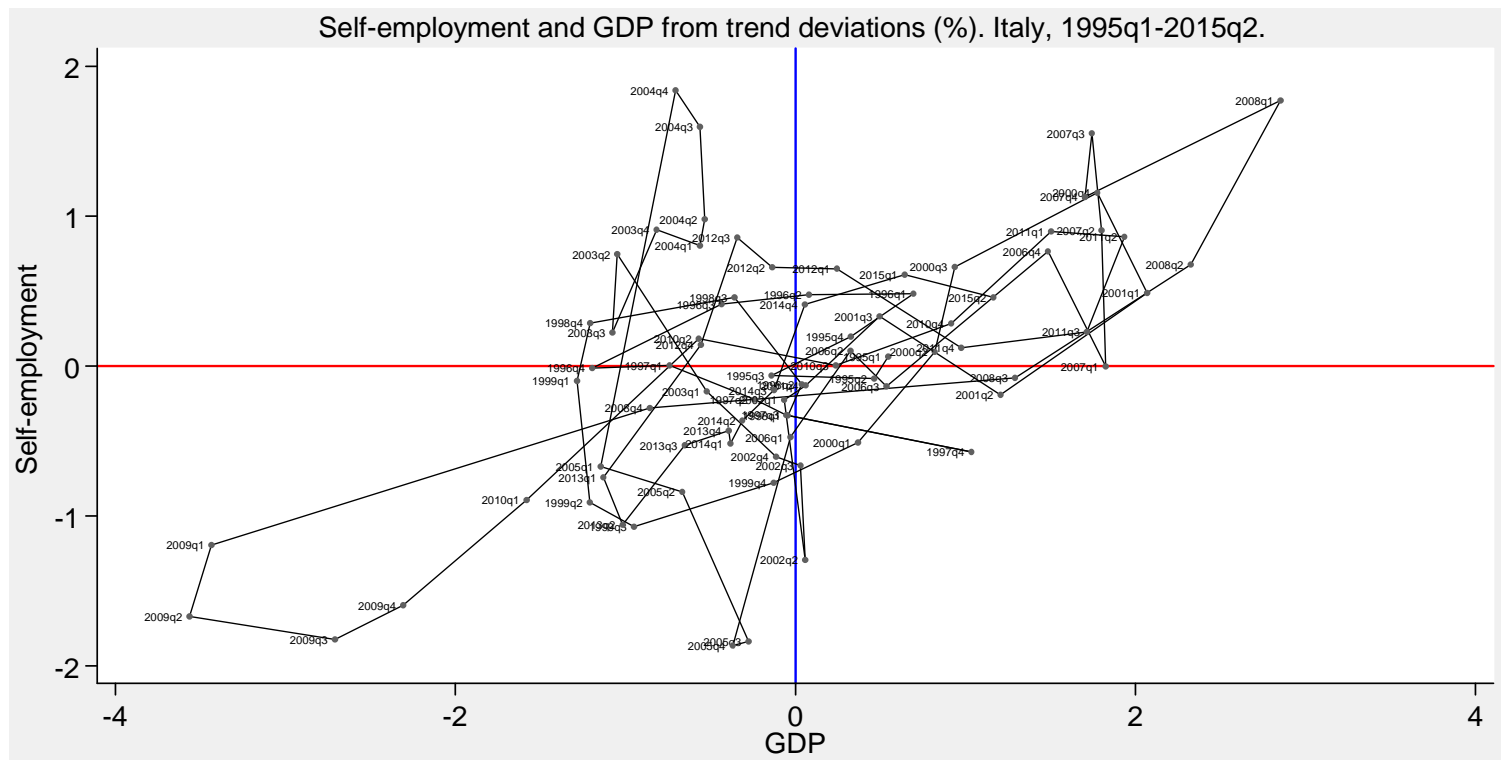
Figure 13. Self-employment and GDP cyclical components (%). Italy, 1995q1-2015q2
Source: authors` elaboration. Note: Red line → GDP trend; blue line → self-employment trend

Casual observation of Fig. 12 demonstrates that, in Italy over analyzed period, regardless the business cycle phase, self-employment – with only few exceptions, deviates from long-term trend at about (+/- 1%), which proofs that patterns of entrepreneurial activity do not demonstrate abrupt ups and downs. Over the same time period in Italy, GDP fluctuations around long-term trend are found to be far more intensive – at about (+/-2%), compared to self-employment fluctuations. The latter may suggest that, over analyzed period in Italy, regardless entrepreneurial patters are found to be pro-cyclical or counter-cyclical during expansion and/or recession phase of business cycle, the reaction of entrepreneurial activity to changes in GDP is relatively weak.

However confirming or rejecting this supposition yields more detailed analysis, which results are discussed in the reminder of this section.

Discriminating between defined four distinct quarters – Q1, Q2, Q3 and Q4, implies splitting the original empirical sample, into four sub-samples. Hence our initial sample covering 82 quarterly observations has been divided into four sub-samples, while each one identifies different relationship emerging between $c\_Self_{It,1995-2015}$ and $c\_GDP_{It,1995-2015}$, in Italy between 1995q1 and 2015q2.

Table 11 summarizes all observations regarding $c\_Self_{It,1995-2015}$ and $c\_GDP_{It,1995-2015}$ falling into Q1, Q2, Q3 and Q4, along with respectively calculated Entrepreneurship Vulnerability Coefficients. Additionally, to shed more light on the nature of examined relationships, Figs. 14-17 graphically display identified relationships between $c\_Self_{It,1995-2015}$ and $c\_GDP_{It,1995-2015}$ and respective Entrepreneurship Vulnerability Coefficients in Q1, Q2, Q3 and Q4.

Let us consider the first quarter (Q1) (see also Fig.14), which encompasses 26 (32% out of total 82) observations demonstrating **pro-cyclical behavior of self-employment during expansion phase** of business cycle. That is to say that self-employment and GDP from trends deviations are positive ($c\_Self_{It,Q1}$>0 and $c\_GDP_{It,Q1}$>0). In Q1, calculated Entrepreneurship Vulnerability Coefficients for consecutive time periods are predominantly below 1, which suggest pro-cyclical behavior of self-employment during expansion phase of business cycle unveils generally weak vulnerability to gross domestic product fluctuations. If 26 observations are included, the average $Q1\_EVC_{i,t,q}^{self\leftrightarrow gdp}$ is at 1.02 (see also Table 12) that indicates close to neutral entrepreneurship vulnerability to business cycle. However if 3 outlying observations – 1996q2, 2012q1 and 2014q4 (see Table 11) – are excluded, the average $Q1\_EVC_{i,t,q}^{self\leftrightarrow gdp}$ significantly decreases, and results to be at about 0.46, which may be interpreted that pro-cyclical fluctuations of entrepreneurship during expansion phase of business cycle are at about twice weaker if compared to fluctuations of GDP over analogous time periods, and thus entrepreneurial activity maybe claimed as of weak vulnerability to business cycle.

Observations reported in the third quarter (Q3) (see also Fig. 15), encompassing 30 observations (36% out of total 82) also demonstrate **pro-cyclical behavior of self-employment however during the recession phase of business cycle**. This means that both self-employment and GDP from trends deviations are negative ($c\_Self_{It,Q3}$<0 and $c\_GDP_{It,Q3}$<0); hence decreases in GDP are accompanied by falling number of self-employed persons over analogous time periods. In Q3, calculated Entrepreneurship Vulnerability Coefficients in 15 time periods (quarters) exceed 1 ($Q3\_EVC_{i,t,q}^{self\leftrightarrow gdp} > 1$),

which exhibits that in 50% of examined cases (time periods) entrepreneurial activity unveils strong vulnerability to business cycle. Put differently, during recession phase of business cycle, GDP negative deviations from trend are accompanied by relatively higher self-employment negative deviations from long-term trend. Alternatively we may state that self-employment demonstrates relatively strong reaction to business cycle. If calculate the average calculated Entrepreneurship Vulnerability Coefficient, but exclusively for those observations where $Q3_{EVC_{i,t,q}^{self\leftrightarrow gdp}} > 1$, it results to be at about 4.32. However, importantly to note, also in Q3 another 15 observations are reported where the Entrepreneurship Vulnerability Coefficients are below 1 ($Q3_{EVC_{i,t,q}^{self\leftrightarrow gdp}} < 1$), which suggests that self-employment vulnerability to the business cycle is weak. in this case, the average calculated Entrepreneurship Vulnerability Coefficient, but only for these observations where ($Q3_{EVC_{i,t,q}^{self\leftrightarrow gdp}} < 1$), is at barely 0.52. All these results allow drawing more general conclusion that during recession phase of business cycles when self-employment behaves pro-cyclicaly, strong entrepreneurship vulnerability is evidently dominant, which

Table 11. Self-employment and GDP from trends deviations (%), and Entrepreneurship Vulnerability Coefficients. Italy, 1995q1-2015q2

Q1 (expansion phase) (pro-cyclical behaviour)				Q2 (recession phase) (counter-cyclical behaviour)				Q3 (recession phase) (pro-cyclical behaviour)				Q4 (expansion phase) (counter-cyclical behaviour)			
Time	Self	GDP	V_1	Time	Self	GDP	V_2	Time	Self	GDP	V_3	Time	Self	GDP	V_4
1995q1	0,06	0,55	**0,12**	1996q3	0,41	-0,43	**-0,95**	1995q3	-0,06	-0,14	**0,45**	1995q2	-0,08	0,46	**-0,18**
1995q4	0,20	0,33	**0,60**	1997q1	0,00	-0,74	**0,00**	1996q4	-0,01	-1,20	**0,01**	1997q4	-0,57	1,04	**-0,55**
1996q1	0,48	0,69	**0,69**	1998q3	0,46	-0,36	**-1,28**	1997q2	-0,22	-0,24	**0,94**	1998q2	-0,12	0,04	**-3,29**
1996q2	0,48	0,08	**5,98**	1998q4	0,29	-1,21	**-0,24**	1997q3	-0,33	-0,05	**6,01**	2000q1	-0,51	0,37	**-1,39**
2000q2	0,09	0,82	**0,11**	2003q2	0,75	-1,05	**-0,71**	1998q1	-0,33	-0,04	**7,71**	2001q2	-0,19	1,21	**-0,16**
2000q3	0,66	0,94	**0,70**	2003q3	0,22	-1,08	**-0,21**	1999q1	-0,10	-1,29	**0,08**	2001q4	-0,13	0,06	**-2,13**
2000q4	1,16	1,78	**0,65**	2003q4	0,91	-0,82	**-1,11**	1999q2	-0,91	-1,21	**0,75**	2002q2	-1,29	0,06	**-22,93**
2001q1	0,49	2,07	**0,24**	2004q1	0,80	-0,56	**-1,43**	1999q3	-1,07	-0,95	**1,13**	2002q3	-0,66	0,03	**-21,97**
2001q3	0,33	0,50	**0,67**	2004q2	0,98	-0,53	**-1,83**	1999q4	-0,78	-0,13	**6,03**	2006q3	-0,14	0,53	**-0,26**
2006q2	0,10	0,32	**0,31**	2004q3	1,60	-0,56	**-2,84**	2002q1	-0,23	-0,07	**3,44**	2007q1	0,00	1,82	**0,00**
2006q4	0,76	1,49	**0,51**	2004q4	1,84	-0,71	**-2,61**	2002q4	-0,61	-0,11	**5,34**	2008q3	-0,08	1,29	**-0,06**
2007q2	0,90	1,80	**0,50**	2010q2	0,18	-0,57	**-0,32**	2003q1	-0,17	-0,52	**0,32**				
2007q3	1,55	1,74	**0,89**	2012q2	0,66	-0,14	**-4,84**	2005q1	-0,67	-1,15	**0,58**				
2007q4	1,13	1,70	**0,66**	2012q3	0,86	-0,34	**-2,50**	2005q2	-0,84	-0,67	**1,26**				
2008q1	1,77	2,85	**0,62**	2012q4	0,14	-0,56	**-0,26**	2005q3	-1,84	-0,28	**6,68**				
2008q2	0,68	2,32	**0,29**					2005q4	-1,87	-0,37	**5,07**				
2010q3	0,00	0,24	**0,02**					2006q1	-0,47	-0,03	**16,29**				
2010q4	0,28	0,92	**0,31**					2008q4	-0,28	-0,86	**0,33**				
2011q1	0,90	1,50	**0,60**					2009q1	-1,19	-3,43	**0,35**				
2011q2	0,86	1,93	**0,45**					2009q2	-1,67	-3,56	**0,47**				
2011q3	0,23	1,71	**0,13**					2009q3	-1,82	-2,71	**0,67**				
2011q4	0,12	0,98	**0,12**					2009q4	-1,60	-2,31	**0,69**				
2012q1	0,65	0,25	**2,64**					2010q1	-0,89	-1,58	**0,57**				
2014q4	0,41	0,06	**7,38**					2013q1	-0,74	-1,13	**0,66**				
2015q1	0,61	0,64	**0,95**					2013q2	-1,06	-1,02	**1,04**				
2015q2	0,46	1,16	**0,39**					2013q3	-0,53	-0,65	**0,81**				
								2013q4	-0,43	-0,39	**1,10**				
								2014q1	-0,52	-0,38	**1,36**				
								2014q2	-0,36	-0,31	**1,17**				
								2014q3	-0,16	-0,13	**1,28**				

Source: authors` calculations.

additionally may be supported by the fact that average $Q3\_EVC_{i,t,q}^{self\leftrightarrow gdp}$, if calculated for all 30 observation, is at 2.4 (see Table 7).
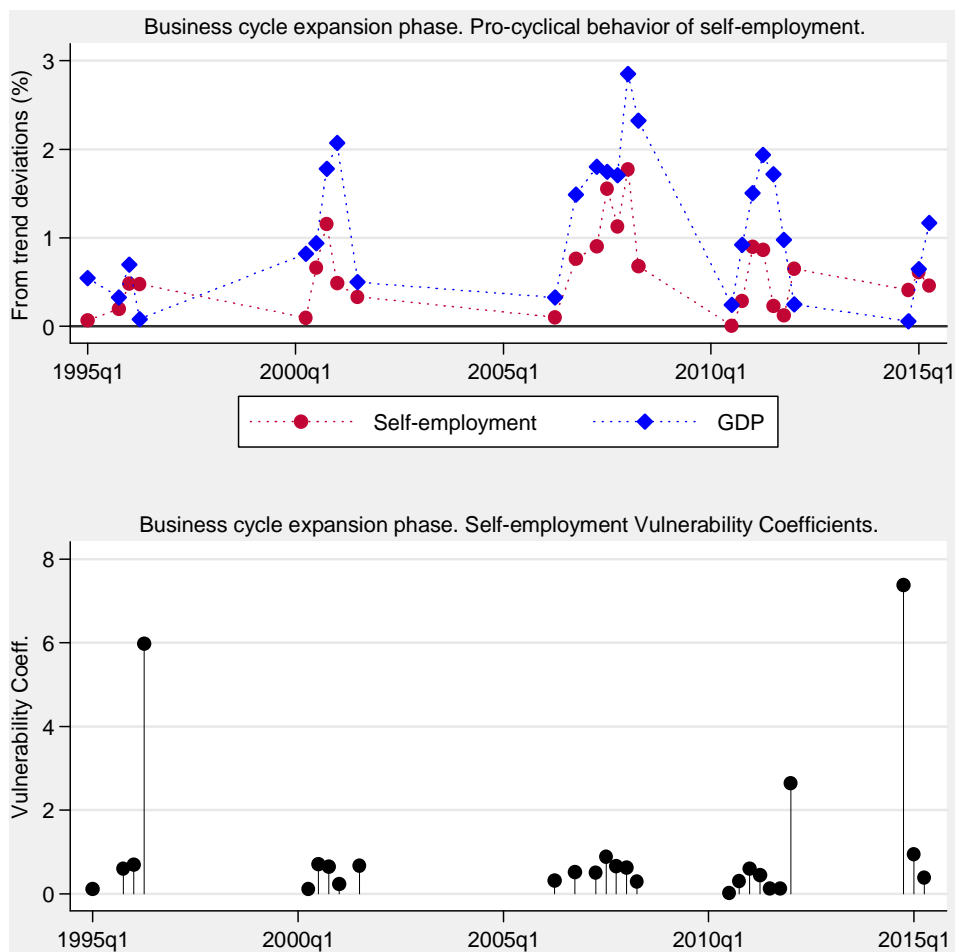

Figure 14. Pro-cyclical behavior of entrepreneurship; Expansion phase of business cycle
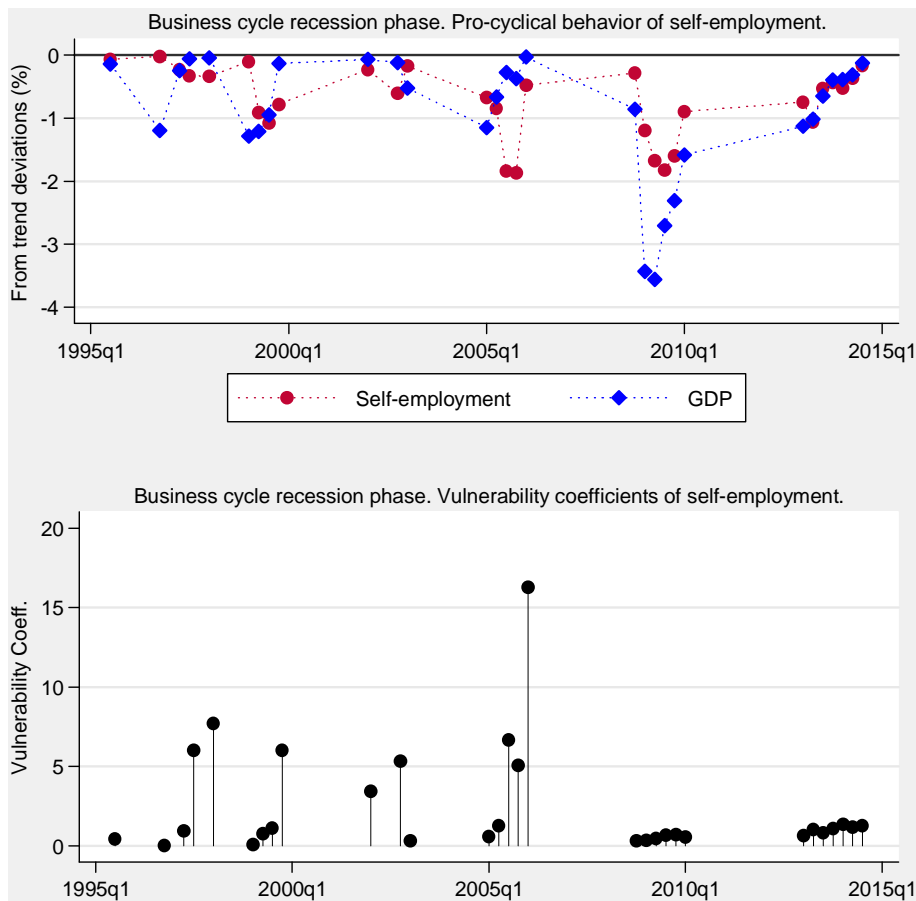Source: authors` elaboration.

Figure 15. Pro-cyclical behavior of self-employment; Recession phase of business cycle
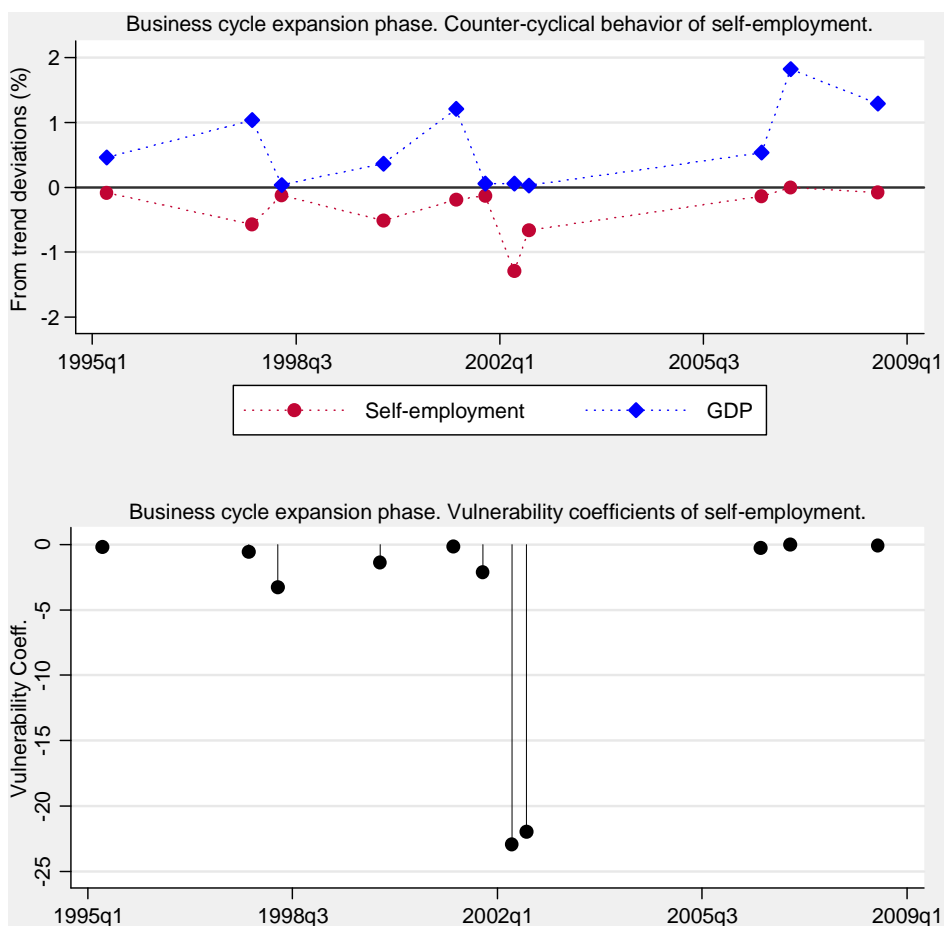Source: authors` elaboration.

Figure 16. Counter-cyclical behavior of self-employment; Expansion phase of business cycle
Source: authors` elaboration.

Business cycle recession phase. Counter-cyclical behavior of self-employment.

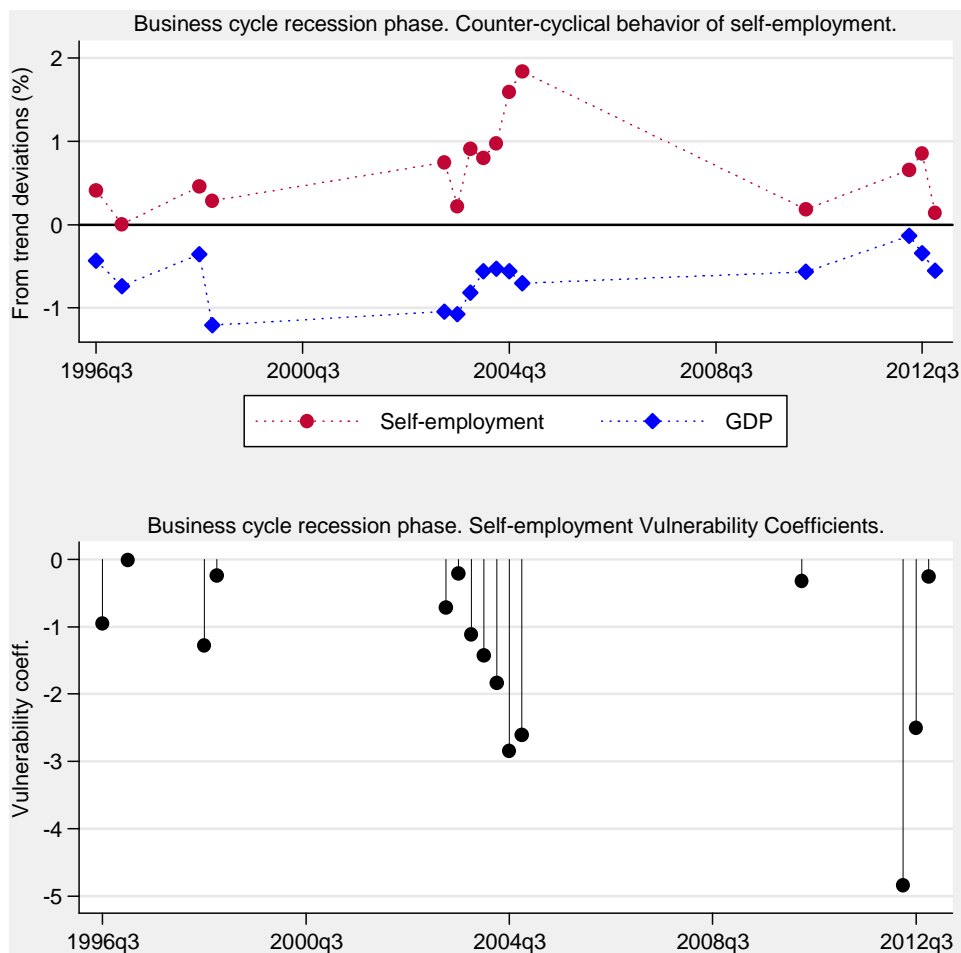Business cycle recession phase. Self-employment Vulnerability Coefficients.

Figure 17. Counter-cyclical behavior of entrepreneurship; Recession phase of business cycle
Source: authors` elaboration.

Now, turning to the analysis of counter-cyclical behavior of entrepreneurship during both expansion and recession phase of business cycle, we discuss the results of Q2 and Q4 respectively. All 15 observations (19% out of total 82) falling into Q2, are classified are those representing counter-cyclical patterns of entrepreneurship during recession phase of business cycle (see Fig. 17). That is to say that decreases in GDP are accompanied by rises in self-employment ($c\_Self_{It,Q2}$>0 and $c\_GDP_{It,Q2}$<0). In this case calculated average $Q2\_EVC_{i,t,q}^{self \leftrightarrow gdp}$, for all 15 observations, is at about $|-1.4|$, which may suggest strong entrepreneurship vulnerability to business cycle. Importantly, in 7 cases (time periods), the $Q2\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ is higher than 1, while in another 8 is lower than 1; however average values of $Q2\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ in these sub-samples are $|-2.31|$ and $|-0.38|$

respectively. Basing on these calculations, we may raise arguments speaking in support of hypothesis that during recession phase of business cycle pro-cyclical behavior of self-employment is characterized by strong vulnerability to GDP changes.

Finally, we have solely 11 observations classified for Q4, which exhibit counter-cyclical behavior of entrepreneurship during expansion phase of business cycle (see Fig. 16). That is to say that increases in GDP are accompanied by drops in self-employment ($c\_Self_{It,Q4}$ <0 and $c\_GDP_{It,Q4}$>0). If all 11 observations are considered, the average $Q4\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ is at about $|-4.8|$; however it shall be borne in mind, that for two periods – 2002q2 and 2002q3, the values of entrepreneurship vulnerability coefficient were -22.9 and -21.9 respectively, which heavily affects the calculated average. Hence, to obtain more reliable result we exclude these two observations, and the corrected average $Q4\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ results at around -0.9. Considering the, corrected for outlying observation, average $Q4\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ suggests relatively weak entrepreneurship vulnerability to the business cycle. However, one should be rather caution when drawing conclusions on the features of counter-cyclical entrepreneurial behavior during expansion phase of business cycles, as these results are violated by two outliers, are drawn basing on very limited number of observations and hence may lack representativeness and robustness.

Table 12. Self-employment and GDP from trends deviations (%), and Entrepreneurship Vulnerability Coefficients – summary statistics. Italy, 1995q1-2015q2

	Variable	Obs.	Mean	Std. dev.	Min. Value	Max. Value	Absolute Difference between Max and Min Values
Q1 (expansion phase; **pro-cyclical** behavior of self-employment)	$c\_Self_{It,Q1}$	26	0,6	0,4	0,004	1,7	1,7
	$c\_GDP_{It,Q1}$	26	1,3	0,76	0,05	2,8	2,8
	$Q1\_EVC_{i,t,q}^{self \leftrightarrow gdp}$	26	**1,02**	1,7	0,016	7,3	7,3
	$Q1\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ (outliers excluded)	23	**0,46**	-	-	-	-
Q2 (recession phase; counter-cyclical behavior of self-employment)	$c\_Self_{It,Q2}$	15	0,6	0,5	0,003	1,8	1,8
	$c\_GDP_{It,Q2}$	15	-0,6	0,3	-1,2	-0,1	1,1
	$Q2\_EVC_{i,t,q}^{self \leftrightarrow gdp}$	15	**-1,4**	1,3	-4,8	-0,005	4,8
	$Q2\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ (outliers excluded)						
Q3 (recession phase; **pro-cyclical** behavior of self-employment)	$c\_Self_{It,Q3}$	30	-0,7	0,5	-1,8	-0,02	1,8
	$c\_GDP_{It,Q3}$	30	-0,9	0,9	-3,5	-0,03	3,5
	$Q3\_EVC_{i,t,q}^{self \leftrightarrow gdp}$	30	**2,4**	3,5	0,01	16,3	16,3
	$Q3\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ (outliers excluded)						
Q4 (expansion phase; counter-cyclical behavior of self-employment)	$c\_Self_{It,Q4}$	11	-0,3	0,4	-1,3	-0,004	1,3
	$c\_GDP_{It,Q4}$	11	0,6	0,6	0,03	1,8	1,8
	$Q4\_EVC_{i,t,q}^{self \leftrightarrow gdp}$	11	**-4,8**	8,7	-22,9	-0,002	22,9
	$Q4\_EVC_{i,t,q}^{self \leftrightarrow gdp}$ (outliers excluded)	9	-0.9				

Source: authors` calculations.

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Appendix:

Appendix X. Cyclical time paths of gross domestic output – correlation of self-employment and gross domestic output (GDP) BW-filtered cyclical components (%) at different leads ($t-n$) and lags ($t+n$). Italy, 1995q1-2015q2.

Period	$cross-correlations(Self_{i,t}; GDP_{t,t})$	Period	$cross-correlations(Self_{i,t}; GDP_{t,t})$	Period	$cross-correlations(Self_{i,t}; GDP_{t,t})$	Period	$cross-correlations(Self_{i,t}; GDP_{t,t})$
t-38	-0.0391	t-18	0.0284	t+1	0.5388	t+21	-0.1014
t-37	-0.1363	t-17	0.2548	t+2	0.4566	t+22	-0.0603
t-36	-0.2194	t-16	0.4752	t+3	0.2888	t+23	-0.0489
t-35	-0.2779	t-15	0.5758	t+4	0.0305	t+24	-0.0093
t-34	-0.2560	t-14	0.5377	t+5	-0.2018	t+25	0.0613
t-33	-0.1704	t-13	0.3652	t+6	-0.3623	t+26	0.1312
t-32	-0.0200	t-12	0.1177	t+7	-0.4215	t+27	0.1515
t-31	0.1238	t-11	-0.0631	t+8	-0.3958	t+28	0.1139
t-30	0.2102	t-10	-0.2212	t+9	-0.3109	t+29	0.0179
t-29	0.2722	t-9	-0.3485	t+10	-0.2073	t+30	-0.0502
t-28	0.2809	t-8	-0.4508	t+11	-0.1108	t+31	-0.1058
t-27	0.2366	t-7	-0.5194	t+12	-0.0074	t+32	-0.1290
t-26	0.1303	t-6	-0.4842	t+13	0.0960	t+33	-0.1186
t-25	-0.0462	t-5	-0.3648	t+14	0.1891	t+34	-0.1290
t-24	-0.2241	t-4	-0.1884	t+15	0.2550	t+35	-0.1241
t-23	-0.3594	t-3	0.0635	t+16	0.2409	t+36	-0.0975
t-22	-0.3889	t-2	0.2823	t+17	0.1612	t+37	-0.0692
t-21	-0.3544	t-1	0.4585	t+18	0.0422	t+39	0.0177
t-20	-0.2507	t=0	0.5666	t+19	-0.0486		
t-19	-0.1174			t+20	-0.0954		

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