Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 193 (2017) 136 – 143

Procedia Engineering

www.elsevier.com/locate/procedia

International Conference on Analytical Models and New Concepts in Concrete and Masonry Structures AMCM’2017

Verification of selected calculation methods regarding shear strength in reinforced and prestressed concrete beams

Krystyna Nagrodzka-Godycka[a] , Marta Wiśniowska[a,]*

[a]*Gdańsk University of Technology, Gabriela Narutowicza 11/12, 80-233 Gdańsk, Poland*

Abstract

The purpose of this article was an attempt to compare selected calculation methods regarding shear strength in reinforced and prestressed concrete beams. Several calculation methods were tested. This included codes: PN-EN 1992-1-1:2008 [1], ACI 318-14 [2] and *fib* Model Code for Concrete Structures 2010 [3]. The analysis also consists of methods published in technical literature. Calculations of shear strengths were made based on experimental works found in literature. The shear strength ratios V_{test}/V_{calc} were chosen to be the yardstick of comparison, where V_{test} is the experimental shear strength and V_{calc} is the calculated shear strength. A wide range of variables including shear span/depth ratio, compressive strength of concrete, longitudinal steel percentage helped to verify the applicability of calculation methods. Although most of authorial techniques proved to be unstable, they succeeded to show that codes' formulas for calculating shear strength could still be improved.

Peer-review under responsibility of the scientific committee of the International Conference on Analytical Models and New Concepts in Concrete and Masonry Structures

Keywords: shear strength; shear design; building codes; reinforced concrete; prestressed beams; stirrups

* Corresponding author. Tel.: +48 507-297-979.
E-mail address: wisniowska.marta1@gmail.com

Peer-review under responsibility of the scientific committee of the International Conference on Analytical Models and New Concepts in Concrete and Masonry Structures
doi:10.1016/j.proeng.2017.06.196

1. Introduction

Over the years much research and many debates have taken place all around the world to explore the shear mechanism in beams [4,5,6,7]. Although many experiments and analysis have been carried out, the provisions regarding shear strength provide results that often differ from experimental data [8,9,10]. Therefore it seemed reasonable to assess if the observed growth in shear design equations [11] resulted in noticeable improvement.

Nomenclature

a	shear span
b_w	web width
d	effective beam depth
f_c	compressive stress in concrete
f_y	yield stress of steel
s	spacing of shear reinforcement
ρ_w	longitudinal reinforcement ratio

1.1. Primary assumptions

To ensure the coherence of the analysis, initial restrictions were taken:

- Beams were either reinforced concrete or prestressed concrete;
- All cross sections were either rectangular or T-beams;
- All beams had stirrups as transverse reinforcement;
- All beams failed in shear;
- All beams were single – span;
- No limit on material properties was imposed;
- In order to compare analytical results with experimental data, all units were taken without reduction factors.

2. Selection of calculation methods

The fundamental approach to the shear problem is the truss analogy model presented by Mörsch [12]. Being modified many times it became a leading model in most European Codes. In the following analysis PN-EN 1992-1-1:2008 [1] (eq. (1)) was chosen to be its representative. Eurocode 2 (with Polish Annexes) is based on evaluation of truss model with variable angle of inclination of the struts and without concrete contribution. Experiments held by members of the American Concrete Institute proved that the truss analogy model does not cover the concrete contribution that was observed during laboratory tests. Therefore a semi - empirical formula was developed and adopted in ACI 318 - 14 [2] (eq. (2, 3a, 3b, 4)). The last standard that was taken into consideration in the following analysis is Model Code 2010 [3] with its levels of approximation that each base on different shear model. The second (eq. (5)) and the third (eq. (6)) level of approximation were analyzed.

In addition to the standards, three authorial methods published in technical literature were taken into consideration. All three pose an attempt to improve ACI 318-11 [2] provisions. Frosch Method [9] (eq. (7)) modified Standard's provisions in two ways. First it replaced the effective beam depth (d) with a cracked transformed section neutral axis depth (c). Secondly it eliminated longitudinal reinforcement ratio (ρ_w) in the expression for V_c as it is already taken into account in calculating c. Another attempt to modify ACI 318-11 provisions was carried at the University of Houston [13] (eq.(8)). The UH Method, dedicated to prestressed concrete beams, introduced a function of a shear span to depth radio ($(a/d)^{-0,7}$) that reflects the arch action in the beam. The last method is a compound of the two former. Kuo, Hsu and Hwang [13], later called KHH method, ((eq. (9)) used shear span to depth radio ($(a/d)^{-0,7}$) according to UH Method and the depth of uncracked compression zone.

$$V_{PN-EN} = \min\begin{Bmatrix} V_{Rd,s} = \dfrac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot ctg\Theta \\ V_{\max} = \dfrac{\alpha_{cw} \cdot b_w \cdot z \cdot \nu_1 \cdot f_{cd}}{ctg\Theta + tg\Theta} \end{Bmatrix} \tag{1}$$

Due to Author's prior analysis [14], based on formula presented by Wesołowski [15], the angle of concrete compression strut in eq. (1) was taken as $ctg\Theta = 2.5$.

$$V_{ACI} = V_c + V_s \tag{2}$$

$$V_c = 0.17 \cdot \sqrt{f_c^{'}} \cdot b_w \cdot d \tag{3a}$$

$$V_c = \min\begin{Bmatrix} (0.05 \cdot \lambda \cdot \sqrt{f_c^{'}} + 4.8 \cdot \dfrac{V_u \cdot d_p}{M_u}) \cdot b_w \cdot d \\ (0.05 \cdot \lambda \cdot \sqrt{f_c^{'}} + 4.8) \cdot b_w \cdot d \\ 0.42 \cdot \lambda \cdot \sqrt{f_c^{'}} \cdot b_w \cdot d \end{Bmatrix} \tag{3b}$$

Equation (3a) refer to RC beams and equation (3b) refer to prestressed beams.

$$V_s = A_w \cdot f_y \cdot \frac{d}{s} \tag{4}$$

$$V_{MC,II} = \min\begin{Bmatrix} V_s = \dfrac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot ctg\Theta \\ V_{\max} = k_c \cdot f_c \cdot b_w \cdot z \cdot \sin\Theta \cdot \cos\Theta \end{Bmatrix} \tag{5}$$

$$V_{MC,III} = \min\begin{Bmatrix} V_{Rd,s} + V_{Rd,c} = \dfrac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot ctg\Theta + k_v \cdot \sqrt{f_c} \cdot b_w \cdot z \\ V_{\max} = k_c \cdot f_c \cdot b_w \cdot z \cdot \sin\Theta \cdot \cos\Theta 1 \end{Bmatrix} \tag{6}$$

$$V_{Frosch} = V_s + V_c = 0.42 \cdot \sqrt{f_c^{'}} \cdot b_w \cdot c + A_w \cdot f_y \cdot N_v \tag{7}$$

$$V_{UH} = \min\begin{Bmatrix} V_s + V_c = A_w \cdot f_y \cdot (\tfrac{d}{s} - 1) + \min\left\{ 1.17 \cdot \left(\dfrac{a}{d}\right)^{-0,7} \cdot \sqrt{f_c^{'}} \cdot b_w \cdot d ; 0.83 \cdot \sqrt{f_c^{'}} \cdot b_w \cdot d \right\} \\ V_{\max} = 1.33 \cdot \sqrt{f_c^{'}} \cdot b_w \cdot d \end{Bmatrix} \tag{8}$$

$$V_{K,H,H} = \min\left\{\begin{matrix} V_s + V_c = A_w \cdot f_y \cdot (\frac{d}{s} - 1) + \min\left\{1.17 \cdot \left(\frac{a}{d}\right)^{-0,7} \cdot \sqrt{f_c^{'}} \cdot b_w \cdot c; 0.83 \cdot \sqrt{f_c^{'}} \cdot b_w \cdot c\right\} \\ V_{\max} = 1.33 \cdot \sqrt{f_c^{'}} \cdot b_w \cdot d \end{matrix}\right\} \quad (9)$$

3. Beams database

The assessment of the effectiveness of the selected calculation methods was based on test results of beams published in technical literature and given to the author by Professors of Concrete Structures in Gdańsk University of Technology. 53 reinforced concrete and 42 prestressed concrete beams were chosen to the analysis. Table 1. presents properties of the analysed beams that are known to affect shear strength.

Table 1. Database of beams

Author	No.	f_c[MPa]	f_y[MPa]	ρ_w [%]	ρ_{sw}[%]	b_w [cm]	a/d	k
Sarsam, Al-Musawi [16]	14	39.0 - 80.1	477 - 506	2.23 - 3.51	0.19 - 0.37	18	2.5 - 4.0	0
Yoon [17]	9	36.0 - 87.0	400	2.28	0.08 - 0.21	37.5	3.3	0
Lee, Kim [18]	10	19.7 - 40.8	525 - 550	0.93 - 2.79	0.10 - 0.18	35	3.0 - 5.0	0
Tompos, Frosch [9]	2	36.4 - 72.3	483 - 552		0.05 - 0.12	22.9 - 45.7	3	0
Moody [19]	2	21.9	302	4.25	0.27 - 0.47	22.9	1.5	0
Sokołowski [20, 21]	3	43.4 - 55.0	572	4.49	0.57	10	1.45 - 3.38	0
Diab, Godycki-Ćwirko [22, 23, 24]	2	41 - 43.2	254	4.29 - 4.36	0.48	9.92 - 10.2	2.41 - 3.21	0
Hanoon, Jaafar, Abed [25]	11	19.9 - 60.1	420	1.05 - 3.50	0.29 - 1.00	18	1.89 - 2.09	0
Elzanaty, Nilson, Slate [26]	16	40 - 73	434	0.26 - 0.8	5.1 - 7.6	3.8 - 5.8	1	1
Sokołowski [22, 23]	18	44.8 - 57.4	572	0.23	8.3 - 10.2	1.4 - 3.3	0.3 - 1	0.3-1
Diab, Godycki-Ćwirko [22, 23, 24]	6	37.4 - 42	254	0.48 - 0.51	20 - 20.2	2.5 - 3.5	0.3 - 1	0.3-1
Silva et al. [27]	3	41.8 - 49.3	683	0.18 - 0.35	15	3	1	1

4. Shear strengths analysis of RC beams

Fig. 1 presents distributions of shear strength ratios V_{test}/V_{calc} for RC beams where V_{test} is the experimental shear strength and V_{calc} is the calculated shear strength by each shear design procedure. Table 2 presents statistic date of the results. The University of Houston method was excluded from the analysis as it mostly demonstrated V_{test}/V_{calc} ratios significantly below the safety value of V_{test}/V_{calc}=1.

The only method that never estimated shear strength to be greater than it appeared during tests was Frosch method [9]. However table 2 shows that it had the greatest variation coefficient of all the methods. The method to have the greatest number of exact shear strength ratios (V_{test}/V_{calc}=1) was the method of Korean scientists [13]. However it can be observed on fig.1 that it nearly had the same number of underestimations as the number of acceptable values, what makes it unlikely to rely on.

Fig. 1 and table 2 also demonstrates the differences between the results using second (MC2010*) or third (MC 2010**) level of approximation of Model Code 2010 [3]. The accuracy and variety of the shear strength ratios using more detailed approach proved to be significantly higher than while using the simpler approach.

Both PN-EN 1992-1-1:2008 [1] and ACI 318-14 [2] provided acceptable results, with low number of underestimations and good statistics shown in table 2.

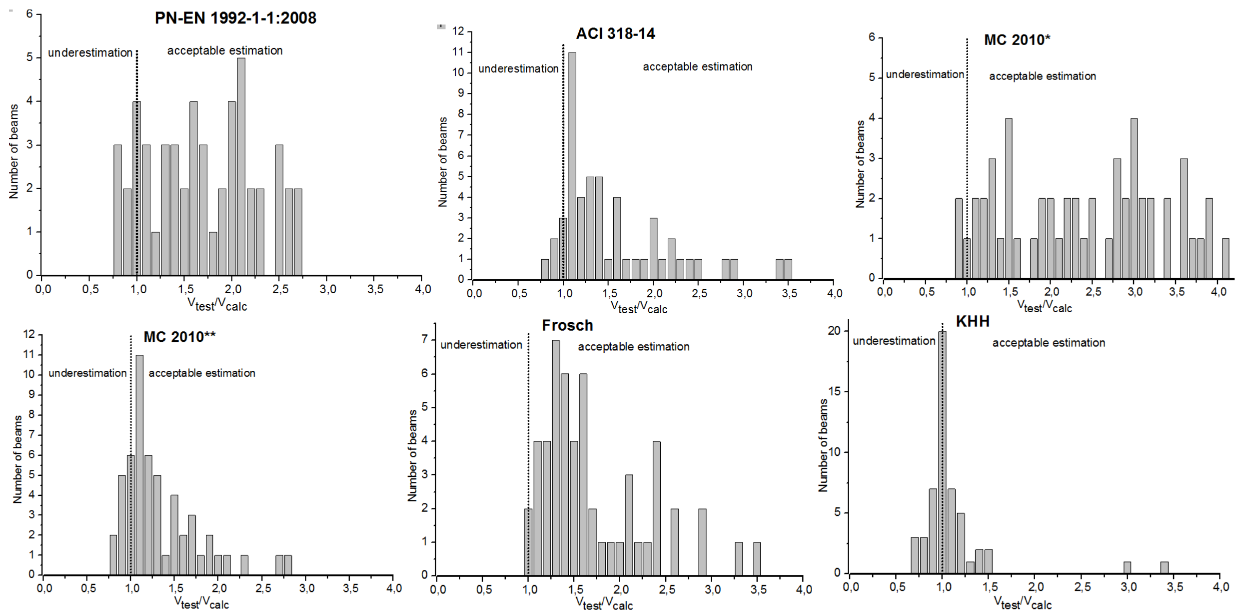

Fig. 1. Distributions of shear strength ratios for RC beams

Table 2. Statistic data for RC beams

	$V_{test}/V_{PN\text{-}EN}$	V_{test}/V_{ACI}	$V_{test}/V_{MC,II}$	$V_{test}/V_{MC,III}$	V_{test}/V_{Frosch}	V_{test}/V_{KHH}
Number of beams	53	53	53	53	53	53
Average	1.62	1.54	2.39	1.87	1.34	1.11
Standard deviation	0.59	0.63	0.92	0.44	0.60	0.46
Variation coefficient	36.5%	40.9%	38.5%	23.8%	44.5%	41.7%

5. Shear strengths analysis of prestressed beams

Fig. 2 presents distributions of shear strength ratios V_{test}/V_{calc} for prestressed beams where V_{test} is the experimental shear strength and V_{calc} is the calculated shear strength by each shear design procedure. Table 3 presents statistic date of the results.

The method to have the greatest number of exact shear strength ratios (V_{test}/V_{calc}=1) in case of RC beams, the KHH method [13], proved no to have much use in case of prestressed beams with the average ratio of 3.99 and variation coefficient of 60%. The other authorial method, the UH method [13] also didn't show to be a significant improvement of code's requirements.

Although the second level of approximation was the only method that never estimated shear strength to be greater than it appeared during tests, it had much higher variety of the results than the third level of approximation. It can be also seen in table 3 that the statistics for second level of approximation are worse than for the third level of approximation. Both PN-EN 1992-1-1:2008 and ACI 318-14 provided acceptable results, with low number of underestimations, however it was the third level of approximation of Model Code 2010 that had the most accurate results.

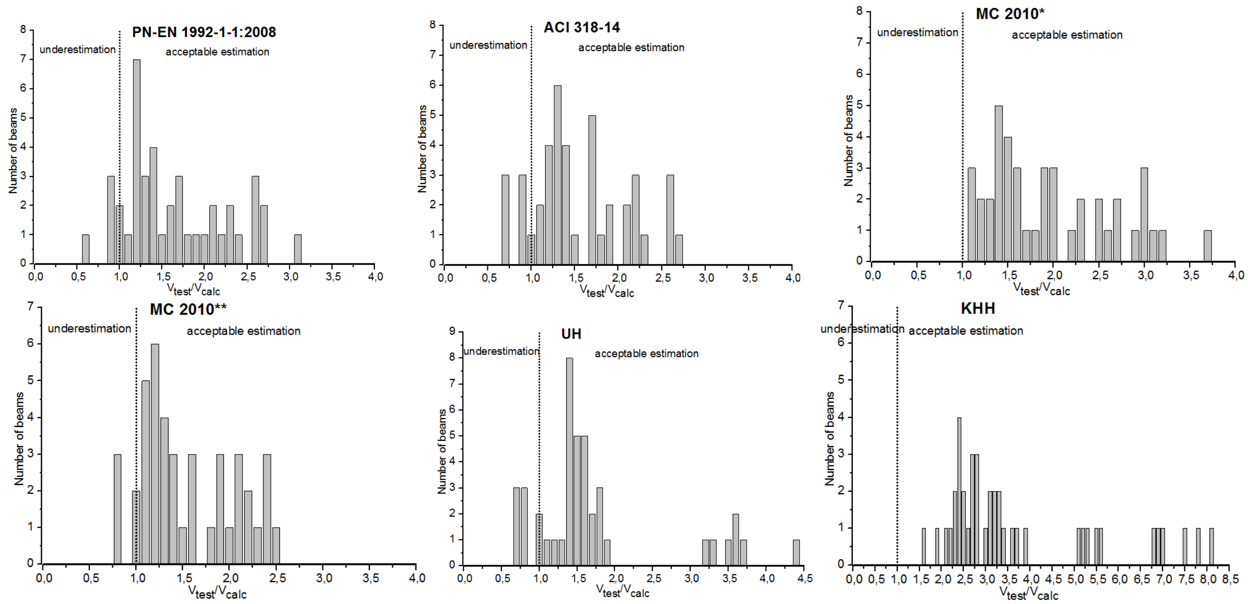

Fig. 2. Distributions of shear strength ratios for prestressed beams

Table 3. Statistic data for prestressed beams

	$V_{test}/V_{PN\text{-}EN}$	V_{test}/V_{ACI}	$V_{test}/V_{MC,II}$	$V_{test}/V_{MC,III}$	V_{test}/V_{UH}	V_{test}/V_{KHH}
Number of beams	42	42	42	42	42	42
Average	1.66	1.56	1.98	1.55	1.73	3.99
Standard deviation	0.61	0.56	0.69	0.51	0.92	2.40
Variation coefficient	36.7 %	35.6 %	35.0 %	33.1 %	53.2 %	60.0 %

6. Partial factor covering uncertainty in the resistance model according to European standards

The ratio of experimental to calculated load bearing capacity $\bar{\eta} = V_{test}/V_{calc}$ for RC beams ranged from 1.1 to 2.4 depending on the standard. As for prestressed beam it was from 1.55 to 3.99.According to EN 1990 [28] the design resistance R_d is expressed as

$$R_d(f_m) = \frac{1}{\gamma_{Rd}} R_{test}(f_m) \tag{10}$$

where

γ_{Rd} is a partial factor covering uncertainty in resistance model,

The verification result may be considered – following formula (10) – as positive, if the ratio $R_{test,mean}$ to $R_d(f_m)$ is greater than a defined γ_{Rd} value.

Lewicki suggests [29] to define γ_{Rd} as ratio of 5% fractile of statistical distribution of material strength $f_{m,exp}$ and of test results R_{exp}. For practical application this ratio may be reduced to

$$\gamma_{Rd} = \frac{1-1.64\nu_m}{1-1.64\nu_R} = \frac{\eta_m}{\eta_R} \tag{11}$$

where

ν_m and ν_R – respectively – coefficient of variation of material strength and of test results.

If derived γ_{Rd} value is lower than the ratio $\bar{\eta}$ it can be assumed that the calculation formulas are correct and are fully applicable for structural design.

Assuming in lack of precise data (when test results are coming from different researchers) for concrete strength ν_m =0.12 - what leads to η_m = 0.80 - one gets for $\nu=\nu_R$ = 0.385 for results acc. to MC II, as shown in Fig. 1 and Tab. 2.

$$\gamma_{Rd} = \frac{0.80}{1-1.64\cdot 0.385} = 2.17 < \bar{\eta} = 2.39$$

For the III[rd] level of approximation according to MC with respect to the reinforced concrete beams the value of γ_{Rd} coefficient is 1.31 and the relationship $\gamma_{Rd} < \bar{\eta}$ is also fulfilled. The values of γ_{Rd} obtained from other procedures are higher than the adequate values $\bar{\eta}$. As for prestressed beams only MC II[nd] level of approximation procedure fulfils this condition $\gamma_{Rd} < \bar{\eta}$.

$$\gamma_{Rd} = \frac{0.80}{1-1.64\cdot 0.35} = 1.88 < \bar{\eta} = 1.98$$

The calculations for all procedures are presented in Table 4.

Table 4. Values of a partial factor covering uncertainty in resistance model, ratio of experimental to calculated load bearing capacity and variation coefficient

RC beams							
Calc. method	$V_{test}/V_{PN\text{-}EN}$	V_{test}/V_{ACI}	$V_{test}/V_{MC,II}$	$V_{test}/V_{MC,III}$	V_{test}/V_{Frosch}	V_{test}/V_{UH}	V_{test}/V_{KHH}
$\bar{\eta}$	1.62	1.54	2.39	1.87	1.34	0.53	1.11
ν	36.5 %	40.9 %	38.5 %	23.8 %	44.5 %	52.6 %	41.7 %
γ_{Rd}	1.99	2.43	2.17	1.31	2.96	5.82	2.53
Prestressed beams							
Calc. method	$V_{test}/V_{PN\text{-}EN}$	V_{test}/V_{ACI}	$V_{test}/V_{MC,II}$	$V_{test}/V_{MC,III}$	V_{test}/V_{UH}	V_{test}/V_{KHH}	
$\bar{\eta}$	1.66	1.56	1.98	1.55	1.73	3.99	
ν	36.7 %	35.6 %	35.0 %	33.1 %	53.2 %	60.0 %	
γ_{Rd}	2	1.92	1.88	1.75	6.3	52	

7. Conclusions

The purpose of this article was an attempt to compare selected calculation methods regarding shear strength in reinforced and prestressed concrete beams. The analysis was limited due to the number of beams and variety of properties affecting shear strength. Nevertheless it succeeded to prove that the provisions regarding shear strength could still be improved to give more accurate and reliable results. It should be stressed that the aim of the article was not to decide which method is the most accurate, but rather if the available methods provide us with reliable results.

References

[1] PN-EN 1992-1-1 Eurokod 2: Projektowanie konstrukcji z betonu. Część 1-1: Reguły ogólne i reguły dla budynków. PKN, Warszawa 2008
[2] ACI Committee: Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary. American Concrete Institute, Farmington Hills, MI, 2014
[3] *fib* Model Code for Concrete Structures 2010, Ernst und Sohn, Berlin, 2013; polish version: Ajdukiewicz A.: Pre - Norma Konstrukcji Betonowych fib Model Code 2010, tom 2, Stowarzyszenie Producentów Cementu, Polska Grupa Narodowa fib, 2014
[4] T. Godycki – Ćwirko, Ścinanie w Żelbecie, Arkady, Warszawa, 1968
[5] G.N.J. Kani, The Riddle of Shear Failure and Its Solution, Journal of the American Concrete Institute, April (1964) 441-465.
[6] B. Bresler, A.C. Scordelis, Shear strength of Reinforced Concrete Beams, ACI Journal, Jan. (1963) 51-74.
[7] F. Leonhardt, R. Walther, Schubversuche an einfeldigen Stahlbetonbalken mit und ohne Schubbewehrung, D.A.f.Stb., Heft 151,1962
[8] E.L. Labib, Y.L. Mo, T.T.C. Hsu, Shear Cracking of Prestressed Girders with High Strength Concrete, International Journal of Concrete Structures and Materials, March (2013) 71-78.
[9] E.J. Tompos, R.J., Frosch, Influence of Beam Size, Longitudinal Reinforcement and Stirrup Effectiveness on Concrete Shear Strength, ACI Structural Journal. Sept./Oct. (2002) 559-567.
[10] F.J. Vecchino, W. Shim, Experimental and Analytical Reexamination of Classic Concrete Beam Tests, Journal of Structural Engineering, Mar. (2004) 460-469
[11] D. Collins, D. Mitchell, P. Adebar, F.J. Vecchio, A general shear design method, ACI Structural Journal, January-February (1996) 36-45.
[12] E. Mörsch, Der Eisenbetonbau, seine Theorie und Anwendung, Bd I, Verlag K. Wittwer, Stuttgart, 1929
[13] Wu Wei Kuo, Thomas T.C. Hsu, Shyh Jiann Hwang, Shear Strength of Reinforced Concrete Beams, ACI Structural Journal, July/August (2014) 809-818.
[14] M. Wiśniowska, Ścinanie w belkach żelbetowych i sprężonych - próba porównania różnych podejść obliczeniowych, master'sthesis, Gdańsk University of Technology, Gdańsk 2016
[15] M. Wesołowski, Wymiarowanie strefy przypodporowej elementów żelbetowych a właściwości modelu kratownicowego, Inżynieria i Budownictwo, 10 (2009) 578-580.
[16] K.F. Sarsam, J.M.S. Al-Musawi, Shear Design of High- and Normal Strength Concrete Beams with Web Reinforcement, ACI Structural Journal, Nov./Dec. (1992) 658-664.
[17] Y.S. Yoon, W.D. Cook, D. Mitchell, Minimum Shear Reinforcement in Normal, Medium and High-Strength Concrete Beams, ACI Structural Journal, Sept./Oct. (1996) 576-584.
[18] J.Y. Lee, U.Y. Kim, Effect of Longitudinal Tensile Reinforcement Ratio and Shear Span-Depth Ratio on Minimum Shear Reinforcement in Beams, ACI Structural Journal, Mar.-Apr. (2008) 134-144.
[19] K.G. Moody, I.M. Viest, R.C. Elstner, E. Hognestad, Shear Strength of Reinforced Concrete Beams - Parts 1 and 2, ACI Journal, (1955) 417-434.
[20] J. Sokołowski, Badania eksperymentalne strefy przypodporowej żelbetowych belek częściowo sprężonych, Praca doktorska, Politechnika Gdańska, Gdańsk 2010
[21] J. Sokołowski, M. Wesołowski, Praca strzemion w elementach sprężonych w świetle badań eksperymentalnych, Przegląd Budowlany. 12 (2010) 30-33
[22] A.H. Diab, Shear Strength of Partially Prestressed Reinforced Concrete Beams, Ph Thesis, Łódź 1986
[23] T. Godycki - Ćwirko, A.M. Diab, Contribution of Stirrups to Shear Cracks Formation of Partially Prestressed Beams, X International Conference of the FIP, New Dehli, India, 1986
[24]T. Godycki - Ćwirko, A.M. Diab, Stan Graniczny Nośności Ścinania Belek Żelbetowych i Częściowo Sprężonych Zbrojonych Strzemionami, XXXIII Konferencja Naukowa KILIW PAN i KN PZITB, Gliwice-Krynica 1987
[25] A. Hanoon, M.S. Jaafar, I.J. Abed, Experimental investigation into the shear behavior of self-compacting RC beams with and without shear reinforcement, Construct II, 2 (2014) 15- 23.
[26] A.H. Elzanaty, A.H. Nilson, F.O. Slate, Shear Capacity of Prestressed Concrete Beams Using High-Strength Concrete, ACI Structural Journal, May-June (1986) 359-368.
[27] De Silva, H. Mutsuyoshi, E. Witchukrengkrai: Evaluation of shear crack width in I-shaped prestressed concrete beams, Journal of advanced concrete technology, vol. 6, no. 3, October 2008, pp. 443-458.
[28] EN 1990:2002, Eurokode – Basic of structural design
[29] B. Lewicki, Konstrukcje betonowe, żelbetowe i sprężone. Komentarz Naukowy do PN-B-03264:2002, tom 1, rozdział 1: Zapewnienie bezpieczeństwa konstrukcji, ITB, Warszawa 2005, ISBN 83-7413-651-0