

Dual-wavelength laser polarimeter and its performance capabilities

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abstract

A dual-wavelength optical polarimetric approach has been proposed as a means of elimination of the systematic errors and estimation of the optical anisotropy parameters for a single DKDP crystal. Our HAUP-related polarimeter uses two semiconductor lasers with the neighbouring wavelengths of 635 nm and 650 nm. Based on the temperature dependence analysis of small characteristic azimuths of light polarization with respect to the axis of the sample, we found the parameters of imperfections of polarization system. We acquired eigen waves ellipticities in a DKDP crystal and found perpendicular to the optic axis value of the optical rotatory power. Our results correlate positively with previously measured data for KDP crystals.

Keywords: Polarimetry, Crystal, Dual-wavelength, Birefringence, Optical activity

1. Introduction

High accuracy polarimetric methods are the most precise in crystal optics research. These methods are effectively applied to obtain information about temperature or wavelength dependencies of main optical anisotropy parameters of crystals.

Physical properties of crystals, such as linear birefringence, circular birefringence (also known as optical rotation or optical activity), linear and circular dichroism can be simultaneously studied by high-accuracy universal polarimeter (HAUP) [1–4]. Several modifications of HAUP exist, most widely used is the one invented by Kobayashi et al. and which have undergone numerous improvements over time [5–10]. An alternative to solve crystal optics problems is the so-called tilter polarimeter, presented by Glazer and Kaminsky [11,12]. Another option is a spectropolarimeter built by Moxon and Renshaw [13,14]. Main feature of these polarimeters is the full computerization of the measurements process. However, there is a constant need of improvement of the sensitivity, generality and speed of polarimeters by utilizing the up-to-date advances in laser optics.

Main principles of high-accuracy polarimetry can be stated as follows: as little number of elements in the polarization scheme as possible, use of monochromatic light sources (lasers), small input azimuths of light polarization before the sample, precise measuring

of light intensities for different polarizer and analyser positions, elimination of systematic errors (parasitic ellipticity and angular error), control of intermediate measuring results for verification of their correctness, full results processing only after completion of the measuring cycle, universality of methods, and possibility of the simultaneous measuring of crystal optical anisotropy parameters.

Universal laser polarimeter, which is similar to HAUP, but uses different principles for data gathering and processing, was designed with participation of the authors of the current work and is used for studying numerous important crystals used in application [15–20]. Polarimeter is based on the basic optical scheme PSA (polarizer-sample-analyser) and allows for a full computer control over the measurements process.

2. Principles of measurements and experimental technique

2.1. Transmission function and characteristic azimuths

For a PSA-system, transmission J of monochromatic light is determined as a function of small changes in the polarizer azimuth θ and the analyser azimuth χ angles (less than 0.01 rad for both) which are measured from the principal crystal axes [16]

$$J(\theta, \chi) = (\theta - \chi)^2 + 4\theta\chi \sin^2(\Gamma/2) + 2\theta[(k + q)\sin \Gamma - \delta\chi \cos \Gamma] - 2\chi[(k - p) - \delta\chi] + \text{const}, \quad (1)$$

where $\Gamma = 2\pi \Delta n d / \lambda$ is the phase difference, d is the thickness of specimen, Δn is the linear birefringence, k is the ellipticity of eigen waves in a studied crystal predefined by optical activity, and λ

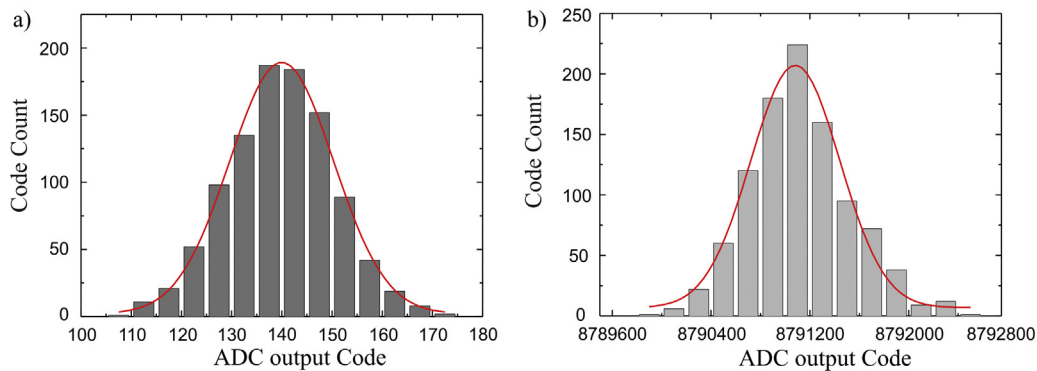


Fig. 1. Random noise histogram for 1000 total number of codes with grounded input (a) and for a half-scale input from photo detection unit (b).

is the wavelength of monochromatic light. In general, one should also define the following values: parasitic ellipticity of the polarizer and analyser p , q , and the angular systematic error $\delta\chi$. In (θ, χ) coordinate system, the transmission function describes the second order surface and projections of the cuts of this surface by the equi-intensity planes have a shape of ellipses. A full set of these projections forms the so-called HAUP maps [6,17].

The main feature of our polarimetric method is the search of the positions of three characteristic azimuth angles $\theta_0, \theta_1, \theta_2$ of the incident light in the PSA system, with the preceding coordination of the polarizers and analysers scales in PA system. The relations for these azimuths can be expressed as [15,16]

$$\theta_0 = (k - p) \cot(\Gamma/2) - \frac{\delta\chi}{(1 - \cos \Gamma)}, \quad (2)$$

$$\theta_1 = (k - p) \cot \Gamma - \frac{(k + q)}{\sin \Gamma}, \quad (3)$$

$$\theta_2 = -(1/2)(p + q) \cot(\Gamma/2) - \frac{\delta\chi}{2}. \quad (4)$$

In fact, it is impossible to measure the absolute values of these angles, because the start point for polarizer azimuth is unknown, that is why experimentally only their differences $\Delta\theta_{01} = \theta_0 - \theta_1$, $\Delta\theta_{02} = \theta_0 - \theta_2$, $\Delta\theta_{12} = \theta_1 - \theta_2$ are analyzed. These differences are related to each other through equations derived from Eqs. (2)–(4)

$$\Delta\theta_{01} \sin \Gamma = 2k - p + q - \delta\chi \cot(\Gamma/2). \quad (5)$$

It is easy to see that $\Delta\theta_{01}(1 + \cos \Gamma) = 2\Delta\theta_{02}$ and $\Delta\theta_{01}(1 - \cos \Gamma) = -2\Delta\theta_{12}$. Therefore, it is enough to determine just two of the characteristic azimuth angles θ_0, θ_1 and θ_2 , or one of the differences $\Delta\theta_{01}, \Delta\theta_{02}, \Delta\theta_{12}$ of characteristic angles and get $\cos \Gamma$. But experimental verification of Eq. (5) is an additional criterion of correct measurements procedures. One should consider the fact that precision of the measurement of the characteristic angles depends greatly on the phase difference Γ .

Characteristic difference (5) alone is not enough to find k and systematic errors $p, q, \delta\chi$. Therefore, we rotate the crystal for 90° around the light beam axis and consider the sign change of k and Γ in Eqs. (2)–(5). By changing the specimen temperature we can alter birefringence and get characteristic azimuths differences that depend on Γ .

Using two sources of light with the values of the wavelengths λ_1 and λ_2 which almost coincide, we can neglect the effects of k value dispersion with some approximation and assume values $p, q, \delta\chi$ to be constant. In this dual-wavelength polarimetric system systematic errors can be differently eliminated. In particular, we will have a set of data for characteristic azimuths which were measured with different laser sources of light. From this point of view the differences $\Delta\theta_{01}, \Delta\theta_{02}, \Delta\theta_{12}$ for separately λ_1 and λ_2 , but also

the differences $\Delta\theta_{i\lambda} = \theta_i(\lambda_1) - \theta_i(\lambda_2)$, ($i = 0, 1, 2$) can be successfully analyzed.

It is enough to use the relation (5) for the elimination of the parasitic errors p, q and $\delta\chi$. In fact, using the relations (2) and (3) for θ_0 and θ_1 azimuth angles the differences $\Delta\theta_{0\lambda}$ and $\Delta\theta_{1\lambda}$ can be expressed as

$$\Delta\theta_{0\lambda} = A_1(k - p) - B_1\delta\chi, \quad (6)$$

$$\Delta\theta_{1\lambda} = A_2(k - p) - B_2(k + q). \quad (7)$$

Here $A_1 = \cot(\Gamma_1/2) - \cot(\Gamma_2/2)$, $B_1 = (1 - \cos \Gamma_1)^{-1} - (1 - \cos \Gamma_2)^{-1}$, $A_2 = \cot \Gamma_1 - \cot \Gamma_2$, $B_2 = 1/\sin \Gamma_1 - 1/\sin \Gamma_2$, $\Gamma_1 = \Gamma(\lambda_1)$ and $\Gamma_2 = \Gamma(\lambda_2)$.

Since the relations, which are expressed by Eqs. (6) and (7), are linear, the magnitude of systematic errors p and $\delta\chi$ can be defined using procedure of the straight line approximation. As a result, the number of the equations which can be used to eliminate the systematic errors is increased.

2.2. Experimental setup

The measuring procedure is fully automated with independent rotations of both polarizer and analyser, controlled by the stepper motors. In order to measure small intensity changes, the so-called delta-sigma analogue-to-digital converter (ADC) with high resolution is used. Both the polarizer and the analyser are Glan type calcite prisms with a clear aperture of $10 \text{ mm} \times 10 \text{ mm}$. Angle resolution of the setup of stepper motors is approximately of 1.3×10^{-3} deg over the 10° range.

The light transmitted through the analyser is detected with a photodiode FD-288A followed by high impedance operational amplifier AD8646 and 24-bit ADC, which was connected to PC. In photovoltaic mode photodiode generates a small current which is proportional to the level of light intensity I over 6 to 9 decades [21].

Full noise of the registering system in polarimeter depends on the amplifier and ADC parameters and laser stability. Fig. 1 is a noise histogram of the 24-bit ADC with a grounded analogue input (a), and about 2.5 V input range (b). The normal or Gaussian distribution indicates that the noise is random. According to Fig. 1(b) standard derivation $\sigma_I = 360 \pm 45$ LSB (less than 9 bits), so the signal-to-noise ratio is equal to 87 dB. This data confirms that the chosen setup of precise intensity measurements by linearity and signal/noise ratio in a HAUP-type polarimeter is comparable to the well-known photon counting technique [22,23].

For measurements we chose deuterated potassium dihydrogen phosphate $\text{K}(\text{H}_{1-x}\text{D}_x)_2\text{PO}_4$ or DKDP single crystal with a level of deuteration $x = 0.93$. Grown by a water-solution method these crystals have a high optical quality and are currently used for electro-optical modulation and frequency conversion [24]. The plate with $d = 0.63 \text{ mm}$ was cut from a crystal perpendicularly to

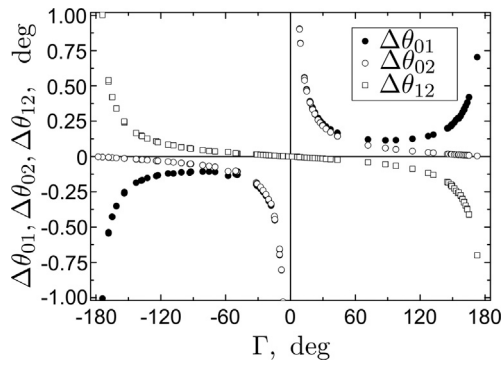


Fig. 2. Dependencies of characteristic angles differences $\Delta\theta_{01}$, $\Delta\theta_{02}$, $\Delta\theta_{12}$ on the phase change value Γ for DKDP crystal and wavelength of 635 nm, Γ changes from $\Gamma = -\pi$ to $\Gamma = \pi$.

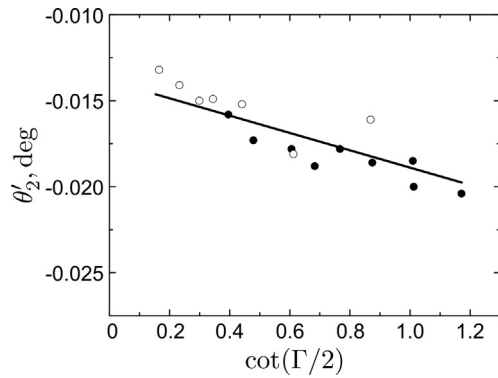


Fig. 3. Dependencies of characteristic angles θ_2 on $\cot(\Gamma/2)$ in DKDP crystal for two wavelengths 635 nm (○) and 650 nm (●).

its optical axis and placed in thermostat without optical windows. Heating the sample from room temperature to 30–35 °C allowed to modify the phase difference in the range of approximately 1.0 rad, which is enough to verify formulas (6) and (7), find the systematic errors and ellipticity of eigen waves k .

3. Results and discussion

3.1. Systematic errors

Typical experimental dependencies of characteristic angles on Γ for DKDP crystals are shown in Fig. 2. As one can see, close to $\Gamma = \pm m\pi$ ($m = 0, 1, 2, \dots$) differences of the characteristic azimuths become too big compared to the typical value of $k \approx 0.02^\circ$. With the considerable change of characteristic azimuths it is difficult to find the systematic errors p , q , $\delta\chi$. At the same time, optimal values of Γ can be achieved by modifying the crystal thickness and temperature. In a dual-wavelength laser polarimeter we use two semiconductor lasers with the neighbouring wavelengths of $\lambda_1 = 635$ and $\lambda_2 = 650$ nm. We consider that in Eqs. (2)–(5) all quantities but Γ do not depend on the small wavelength change $\delta\lambda = \lambda_2 - \lambda_1 = 15$ nm.

Consecutive measurements of characteristic angles for two wavelengths in stable experiment conditions allow to find the desired quantities. Fig. 3 shows that angle θ_2 and superposition of systematic errors do not depend on the wavelength. Using Eq. (4) we could find the sum of parasitic ellipticities of polarizer and analyser $p + q = (0.14 \pm 0.03) \times 10^{-4}$. Reference value for azimuths $\lambda_0, \theta_1, \theta_2$ is unknown and we can find them with precision to the nearest constant, that is why θ_2 is implemented in Fig. 3.

Then, by plotting the dependencies $\Delta\theta_{1\lambda}/A_2 = k - p - B_2(k + q)/A_2$ for alternative crystal orientations (Fig. 4) we found the four quantities in radians: $k - p = (3.35 \pm 0.94) \times 10^{-4}$, $-k - q = -(3.72 \pm 0.35) \times 10^{-4}$, $-k - p = -(3.86 \pm 0.92) \times 10^{-4}$ and $k - q = (3.80 \pm 0.23) \times 10^{-4}$, which are averaged over the wavelengths range from 635 to 650 nm. Rotation of the specimen by 90° around the beam propagation direction ensures a sign reversal for the Γ and k parameters [1–3]. We consider also, that influence of the crystal temperature change (from 18 to 35 °C) on the eigen waves ellipticity k is insignificant.

With these data we can have all the necessary information about the polarimeter systematic errors p and q , and using Eq. (6) we can find the angular error mean value $\delta\chi = (3.0 \pm 1.0) \times 10^{-4}$. We should note, that precision of acquiring the angle of the slope on Fig. 4 is higher than that of getting the point of intercept with y axis.

Average polarizer ellipticity in PA system (polarizer-analyser) can be found using the generalized Malus law (see [25–27])

$$I(\alpha) = I_0(\sin^2 \alpha + \varepsilon^2), \quad (8)$$

where I_0 implies the maximum transmitted intensity, ε is the light ellipticity that depends on the quality of PA system employed. For $\alpha \ll 1$ we have $\sin^2 \alpha \approx \alpha^2$, so that Eq. (8) may be written in a parabolic form. After least-square fitting of the experimental data $I(\alpha)$ one can obtain three parabolic parameters and then determine ε^2 . We acquired for two wavelengths of 635 and 650 nm the values $\varepsilon_1^2 = (6.98 \pm 1.56) \times 10^{-6}$ and $\varepsilon_2^2 = (5.40 \pm 1.01) \times 10^{-6}$ correspondingly. Considering the parasitic ellipticities of polarizer and analyser to be identical for PA system, we could find that

$$p_{PA} = q_{PA} = \left[\frac{(\varepsilon_1^2 + \varepsilon_2^2)}{2} \right]^{1/2} = (0.18 \pm 0.04) \times 10^{-4}.$$

In PSA system ellipticities values of polarization devices $p = 0.25 \times 10^{-4}$ and $q = 0.04 \times 10^{-4}$. Precision of calculation of these values is not very high. At the same time they are way smaller than achieved with the same polarimeter while measuring OA of $\text{Ca}_3\text{Ga}_2\text{Ge}_4\text{O}_{14}$ crystals at 633 nm wavelength [19]. Dependencies of the systematic errors of the optical elements in the measuring system on specimen quality do not allow to use standard HAUP procedure to find the value of p . In HAUP setup for additional measurements one use inactive ($k = 0$) reference lithium niobate (LiNbO_3) and calcite (CaCO_3) crystal [3,9,10,15]. In theory, it gives chance to find parasitic ellipticity of the polarizer p , which is the first element of PSA system, and light propagation through it should not depend on the crystal quality.

3.2. Optical activity of DKDP crystal

KDP group crystals belong to point group symmetry $\bar{4}2m$ in their paraelectric phase, they are uniaxial, optically active along the x - and y -axes, but the direction of the rotation of light is opposite and corresponding gyration tensor component $g_{11} = -g_{22}$ [24,28]. By definition, the x -axis is laevorotatory, and the y -axis is dextrorotatory [29]. It is also interesting that correct identification of the x - and y -axes in KDP group crystals can only be achieved by measuring physical properties like OA.

The relation between eigen wave ellipticity k and gyration tensor component g_{11} for uniaxial crystals is $g_{11} = 2k\Delta n\bar{n}$, where \bar{n} is the mean refractive index [30]. For the mean wavelength of $\lambda = 642$ nm the parameters that we are interested in are equal to $n_e = 1.4645$, $\bar{n} = 1.4833$, and $\Delta n = 0.0375$ [31]. With all the values of systematic errors average value of the eigen wave ellipticity $k = 3.70 \times 10^{-4}$. As the result, we can determine the magnitude of the gyration tensor component $g_{11} = 4.12 \times 10^{-5}$ for DKDP crystals.

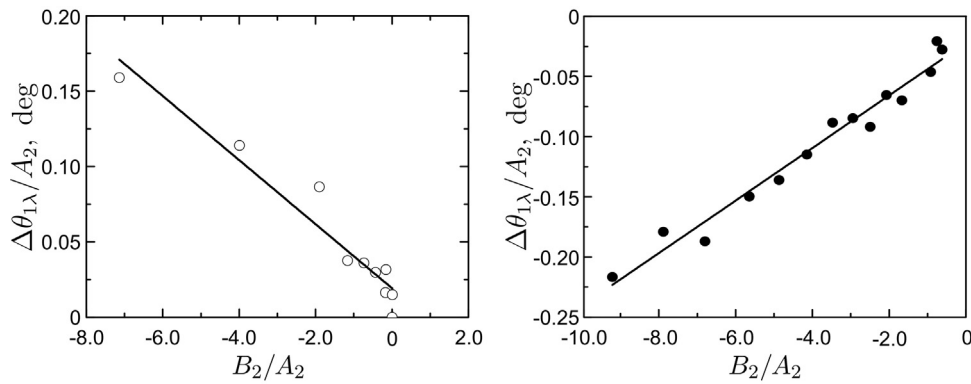


Fig. 4. The two plots of the parameters $\Delta\theta_{1\lambda}/A_2$ vs. B_2/A_2 correspond to alternative crystal orientations as obtained before (○) and after (●) rotating the specimen by 90° around the light beam direction. Solid lines represent the best linear fit.

Using the relation $\rho_{\perp} = \pi g_{11}/(\lambda n_e)$ for the optical rotatory power (ORP) perpendicular to the optic axis, we get $\rho_{\perp} = 7.89^{\circ}/\text{mm}$. This ORP value is very close to the one acquired earlier for KH_2PO_4 and $\lambda = 633 \text{ nm}$ in another experiment [9,18], but considerably smaller than in Ref. [32]. However, semiconductor lasers wavelength is strongly temperature dependent (typical dependence is $d\lambda/dT \approx 0.3 \text{ nm/K}$), thus, unstable radiation spectra of used lasers can influence our results, in particular the measurements errors.

4. Conclusions

We extend the HAUP-related polarimetric method by using two neighbouring laser wavelengths and applied new systematic errors elimination scheme for optical active birefringent DKDP crystal. In comparison to single laser polarimeter setup, our experiment allows us to measure simultaneously more parameters and simplify the experimental data processing. In particular, there is no need to use reference crystal, which usually has different optical anisotropy parameters compared to the studied crystal and often does not provide satisfactory results. Additionally, in flat parallel planes of lithium niobate crystal, multiple light reflections can occur, what changes significantly the effective phase difference Γ . For high measurements precision in our method it is advisable to use highly monochromatic gas or semiconductor lasers, because with small difference between $\Gamma(\lambda_1)$ and $\Gamma(\lambda_2)$ stability of radiation spectrum for neighbouring wavelengths is crucial. Obviously, while choosing the $\lambda_2 - \lambda_1$ interval, one should ensure that $\Gamma(\lambda_2) - \Gamma(\lambda_1)$ is significantly higher than the phase differences measurements error.

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