This is a post-peer-review, pre-copyedit version of an article published in IEEE Antennas and Wireless Propagation Letters . The final authenticated version is available online at: http://dx.doi.org/10.1109/LAWP.2017.2778712

Uniform Sampling in Constrained Domains for Low-Cost Surrogate Modeling of Antenna Input Characteristics

Slawomir Koziel, *Senior Member, IEEE*, Ari T. Sigurðsson, and Stanislaw Szczepanski

Abstract**—In this letter, a design of experiments technique that permits uniform sampling in constrained domains is proposed. The discussed method is applied to generate training data for construction of fast replacement models (surrogates) of antenna input characteristics. The modeling process is design-oriented with the surrogate domain spanned by a set of reference designs optimized with respect to the performance figures and/or operating conditions that are of interest. The reference designs are triangulated and the resulting simplexes are extended in orthogonal directions. Our methodology is demonstrated using two examples: a dual-band dipole and an ultra-wideband monopole. The results indicate that the proposed sampling technique leads to considerable improvement of the surrogate model predictive power as compared to random sampling. Numerical results are supported by application studies (antenna optimization) and experimental validation.**

Index Terms**—Antenna design, surrogate modeling, performance-driven modeling, approximation models, uniform sampling, simulation-driven design, constrained domain.**

I. Introduction

HIGH fidelity computational models are necessary tools of contemporary antenna design. Their advantages include versatility and reliability. A principal disadvantage of full-wave EM simulation tools is high evaluation cost. Although this is not a problem for design verification, it may become a bottleneck in case of design tasks that require numerous analyses of the structure at hand, such as parametric optimization (both local [1] and global [2]), statistical analysis [3], yield-driven design [4], or tolerance-aware design [5]. In these situations, fast replacement models (or surrogates) are indispensable.

Surrogate models can be categorized into two major classes: data-driven models, where the surrogate is constructed by approximating sampled simulation data [6], and physics-based ones with the model obtained by appropriate correction of an underlying low-fidelity representation (e.g., an equivalent network [7]). The most popular data-driven techniques include polynomial regression [8], kriging [9], radial-basis function [10], neural networks [11], Gaussian process regression [12], and support-vector regression [13]. The most popular method of the other class is space mapping [7]; other techniques are primarily various kinds of response correction methods (e.g., [14], [15]).

In case of antennas, physics-based surrogates are of limited use due to unavailability of fast low-fidelity models. On the other hand, construction of data-driven models is difficult because of highly-nonlinear responses of antennas and typically large number of geometry/material parameters of modern structures. Due to curse of dimensionality, conventional modeling techniques are limited to low-dimensional spaces (up to 4-6 parameters), and, more importantly, relatively narrow parameter ranges. The latter is a particularly limiting factor: in order to be practically useful, the model needs to cover sufficiently wide range of operating conditions and/or material parameters, which normally implies wide parameter ranges.

In [16], a constrained modeling technique has been introduced with the aim of reducing the surrogate model training data set size. This was achieved by restricting the model domain to a region that contains potentially useful designs, and realized by defining the domain as a vicinity of a manifold spanned by several reference designs optimized with respect to a selected figure of interest (e.g., operating frequency). Extension of this technique presented in [17] permits handling two independent figures of interest. A drawback of [17] is that a particular number and allocation of the reference design is necessary. Furthermore, both [16] and [17], although superior over conventional modeling methods, use a simple design of experiments approach, which does not allow for uniform sample allocation.

In this work, a procedure for uniform data allocation in constrained domain is proposed, which allows for improving the predictive power of the surrogate over constrained domain without increasing the number of data samples. Furthermore, a generalization of the surrogate model definition of [17] is introduced that can handle any number of figures of interest and allows arbitrary allocation of the reference designs. This is especially important if certain number of (optimized) designs are already available and can be re-used. Two antenna examples are provided. It is demonstrated that our technique is superior over both conventional modeling methods and constrained approach with rudimentary sampling.

Manuscript submitted on October 19, 2017. This work was supported in part by the Icelandic Centre for Research (RANNIS) Grant 174114051, and by National Science Centre of Poland Grant 2015/17/B/ST6/01857.

S. Koziel and A.T. Sigurðsson are with Engineering Optimization and Modeling Center of Reykjavik University, Reykjavik, Iceland (e-mails: koziel@ru.is, aris14@ru.is); S. Koziel and S. Szczepanski are also with the Faculty of Electronics, Telecommunications and Informatics, Gdansk University of Technology, 80-233 Gdansk, Poland (e-mail: stanisla@eti.pg.gda.pl).

II. DESIGN-ORIENTED CONSTRAINED SURROGATE MODELING

In this section, we briefly describe the concept of generalized design-oriented constrained modeling. The proposed uniform sampling technique is described in Section III.

A. *Performance Figures and Reference Designs*

Conventional data-driven modeling construct the surrogate in a hypercube defined by lower and upper bounds of the antenna parameters. Clearly, vast majority of the model domain defined this way is of no interest because it does not contain designs that are "good" from the point of view of the considered figures of interest, operating conditions or material parameters (e.g., operating frequency, bandwidth, substrate parameters, etc.). Consequently, constructing the model in the entire space is a waste of resources or even infeasible because of excessive computational expense.

We denote by F_k, $k = 1, \ldots, N$, the performance figures of interest considered for a given structure (e.g., operating frequencies). Constrained modeling attempts to restrict the model domain so as to limit it only to the most promising regions [16]. Necessary information is obtained from a set of reference designs $\boldsymbol{x}^{(j)}$, $j = 1, \ldots, p$, obtained by optimizing the antenna for selected values $\boldsymbol{F}^{(j)} = [F_1^{(j)} \ \ldots \ F_N^{(j)}]$. Here, a generalized version of constrained modeling is presented which, as opposed to [17], allows any number and allocation of the reference designs, as well as permits handling any number of figures of interest.

Given $\boldsymbol{x}^{(j)}$, Delaunay triangulation [18] is used to create simplexes $S^{(k)} = \{\boldsymbol{x}^{(k.1)},\ldots, \boldsymbol{x}^{(k.N+1)}\}$, $k = 1, \ldots, N_S$, where $\boldsymbol{x}^{(k.j)} \in \{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(N)}\}$, $j = 1, \ldots, N + 1$, are vertices. Example two-dimensional objective space along with triangulated reference designs has been shown in Fig. 1.

B. *Surrogate Model Definition*

We define a manifold M as a union of the convex hulls $h(S^{(k)})$ of the simplexes $S^{(k)}$

$$M = \bigcup_k \{ \boldsymbol{y} = \sum\nolimits_{j=1}^{N+1} \alpha_j \boldsymbol{x}^{(k.j)} : 0 \le \alpha_j \le 1, \sum\nolimits_{j=1}^{N+1} \alpha_j = 1 \} \tag{1}$$

Given an arbitrary point $\boldsymbol{z}$, we consider its projection $P_k(\boldsymbol{z})$ onto the hyper-plane H_k containing the convex hull $h(S^{(k)})$. Let $\boldsymbol{x}^{(0)} = \boldsymbol{x}^{(k.1)}$ (simplex "anchor") and $\boldsymbol{v}^{(j)} = \boldsymbol{x}^{(k.j+1)} - \boldsymbol{x}^{(0)}$, $j = 1, \ldots, N$ (simplex spanning vectors). The projection $P_k(\boldsymbol{z})$ corresponds to expansion coefficients $\alpha^{(j)}$ obtained by solving

$$\arg \min_{[\alpha^{(1)},\ldots,\alpha^{(N)}]} \left\| \boldsymbol{z} - \left[\boldsymbol{x}^{(0)} + \sum\nolimits_{j=1}^{N} \alpha^{(j)} \boldsymbol{v}^{(j)} \right] \right\|^2 \tag{2}$$

Furthermore, we have

$$\left[\alpha^{(1)} \ldots \alpha^{(N)} \right]^T = (\boldsymbol{V}^T \boldsymbol{V})^{-1} \boldsymbol{V}^T (\boldsymbol{z} - \boldsymbol{x}^{(0)}) \tag{3}$$

The location of $P_k(\boldsymbol{z})$ can be identified based on the expansion coefficients (5). In particular, $P_k(\boldsymbol{z}) \in h(S^{(k)})$ if and only if $\alpha^{(j)} \ge 0$ for $j = 1, \ldots, N$, and $\alpha^{(1)} + \ldots + \alpha^{(N)} \le 1$. We define $\boldsymbol{x}_{\max} = \max\{\boldsymbol{x}^{(k)}, k = 1, \ldots, p\}$ and $\boldsymbol{x}_{\min} = \min\{\boldsymbol{x}^{(k)}, k = 1, \ldots, p\}$. The vector $\boldsymbol{dx} = \boldsymbol{x}_{\max} - \boldsymbol{x}_{\min}$ determines the range of variation of the antenna parameters within the manifold M.

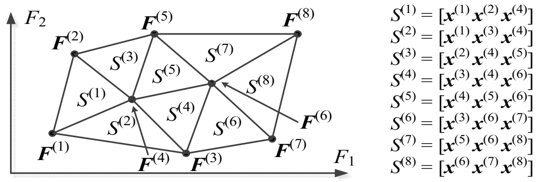


Fig. 1. Conceptual illustration of reference designs and their triangulation in two-dimensional space of figures of interest.

The surrogate model domain X_S is defined as orthogonal extension of M, the „thickness" of which is determined by a user-selected parameter $d_{\max}$. A point $\boldsymbol{y} \in X_S$ if and only if

1. Set $K(\boldsymbol{y}) = \{k \in \{1, \ldots, N_S\} : P_k(\boldsymbol{y}) \in S^{(k)}\} \neq \varnothing$;
2. $\min\{\|\boldsymbol{y} - P_k(\boldsymbol{y})\|/\|\boldsymbol{dx}\| : k \in K(\boldsymbol{y})\} \le d_{\max}$.

The first condition states that the point $\boldsymbol{y}$ has to be "within" the manifold M in the "tangential" sense; the second condition requires that $\boldsymbol{y}$ is sufficiently close to M in the "orthogonal" sense (measured in relation to $\boldsymbol{dx}$).

Clearly, the fundamental benefit of setting up the surrogate model in X_S rather than in the interval $[\boldsymbol{x}_{\min}, \boldsymbol{x}_{\max}]$ is that the volume of X_S is significantly smaller than the volume of $[\boldsymbol{x}_{\min}, \boldsymbol{x}_{\max}]$. At the same time, the designs of interest (i.e., optimal w.r.t. the selected figures of interest) will be either in X_S or close to it. The surrogate itself is constructed here using kriging interpolation [10].

III. UNIFORM SAMPLING IN CONSTRAINED DOMAIN

Design of experiments, i.e., training data allocation in the domain X_S is challenging due to complex geometry of this set. Conceptually, the simplest sampling method (utilized in [17]) is pure random sampling in the interval $[\boldsymbol{x}_{\min}, \boldsymbol{x}_{\max}]$, and accepting those samples that are within X_S. The process is continued until the required number of samples, say, K, has been found. A principal drawback of this method is poor uniformity of the obtained data set, which is particularly due to the fact that the domain X_S is "thin" (i.e., with large dimensions along the manifold M and small in orthogonal directions).

In this work, an improved design of experiments technique is proposed that aims at obtaining a sample set of improved uniformity, and, consequently, improved predictive power of the surrogate. The procedure works as follows:

1. Calculate the volumes V_k of simplexes $S^{(k)}$;
2. Set $K_k = \lceil KV_k/\sum_j V_j \rceil$ (here, $\lceil\ \rceil$ is a ceiling function);
3. For each $k = 1, \ldots, N_S$:
   - Allocate mK_k LHS [10] samples in a unit hypercube $[0,1]^N$ and choose samples lying in the unit N-simplex;
   - Map the samples selected in the previous step onto convex hull of simplex $S^{(k)}$ as follows: $\boldsymbol{x} \to \boldsymbol{x}^{(0)} + \sum_j x_j \boldsymbol{v}^{(j)}$, where $\boldsymbol{x}^{(0)}$ and $\boldsymbol{v}^{(j)}$ are defined under (1), and $\boldsymbol{x} = [x_1 \ \ldots \ x_n]^T$;
   - Perturb the mapped samples by adding vectors $\boldsymbol{x}_d = \boldsymbol{r} - P_k(\boldsymbol{r})$, where $\boldsymbol{r}$ is a random vector in the interval $[-\boldsymbol{d}, \boldsymbol{d}]$, $\boldsymbol{d} = \boldsymbol{dx} \cdot d_{\max}$, and $P_k(\boldsymbol{r})$ is a projection of $\boldsymbol{r}$ onto the convex hull of $S^{(k)}$ (cf. Section II).

The first two steps aim at assigning, to each simplex, the numbers of samples that are proportional to the simplex volumes. In the third step, the first operation is uniform allocation of the prescribed number of samples within unit simplexes (m is the volume ratio of the unit hypercube and the simplex of appropriate dimension). Subsequently, an affine transformation is applied to obtain uniform distributions within the simplexes. Finally, the samples are perturbed in orthogonal directions (with respect to the simplex spanning hyperplanes H_k) in order to fill in the domain X_S. Note that due to using the ceiling function in Step 2 as well as quasi-uniform distribution obtained by LHS, the actual (total) number of samples will be slightly different from the required number K but this can be easily corrected by adding/removing individual samples where necessary. Figure 2 shows an example distribution obtained using the reference and the proposed sampling techniques.

IV. Verification Examples

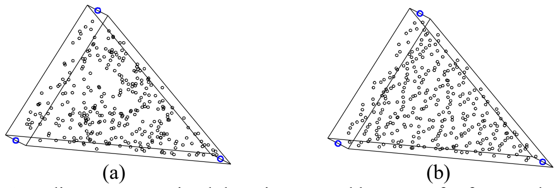
A. Dual-Band Uniplanar Dipole Antenna

As a first example, consider a dual-band uniplanar dipole antenna shown in Fig. 3 [19]. The structure is implemented on Taconic RF-35 ($\varepsilon_r = 3.5$, $h = 0.762$ mm). The structure consists of two ground plane slits interconnected through a thick slot. It is fed by a 50 Ohm CPW. The variables are: $\boldsymbol{x} = [l_1\ l_2\ l_3\ w_1\ w_2\ w_3]^T$, whereas $l_0 = 30$, $w_0 = 3$, $s_0 = 0.15$ and $o = 5$ are fixed (all dimensions in mm). The EM antenna model is simulated in CST.

The objective is to construct the surrogate model of the antenna which covers the following ranges of operating frequencies: 2.0 GHz $\leq f_1 \leq$ 4.0 GHz (lower band), and 4.5 GHz $\leq f_2 \leq$ 6.5 GHz (upper band). There are twelve reference designs selected and optimized for the pair of operating frequencies $\{f_1,f_2\}$ [GHz]: {2.0,4.6}, {2.1,5.9}, {2.2,6.2}, {2.4,5.0}, {2.8,4.6}, {2.8,5.4}, {3.0,6.0}, {3.1,6.5}, {3.4,4.8}, {3.8,5.4}, {3.9,5.8}, {4.0,6.4}. The lower and upper bounds for design variables were set using the reference designs as $\boldsymbol{l} = [25.0\ 6.0\ 14.0\ 0.2\ 1.6\ 0.5]^T$, and $\boldsymbol{u} = [35.0\ 15.0\ 21.0\ 0.55\ 4.0\ 2.0]^T$.

The surrogate model has been constructed using 100, 200, and 500 training samples, using constrained surrogate of Section II with random sampling and uniform sampling of Section III. Unconstrained sampling in the interval $[\boldsymbol{x}_{\min}, \boldsymbol{x}_{\max}]$ has also been carried out for comparison. Table I gathers the modeling errors (average RMS error estimated using 30-fold cross-validation), see also Fig. 4. Note that utilization of the proposed sampling results in significant accuracy improvement.

The surrogate model has been used to design the antenna for two pairs of operating frequencies, f_1 = 2.5 GHz and f_2 = 4.7 GHz, as well as f_1 = 3.6 GHz and f_2 = 6.4 GHz. Figure 5 shows the responses of the optimized constrained surrogate model with uniform sampling as well as the responses of the EM simulation model. The first of the two designs has been fabricated and measured (cf. Fig. 6). The agreement between simulated and measured responses is very good (slight frequency shifts are due to the fact that the SMA connector is not incorporated in the EM antenna model).



(a) (b)

Fig. 2. Sampling on constrained domain spanned by a set of reference designs: (a) random sampling, (b) proposed uniform sampling. For clarity, only one simplex is shown.

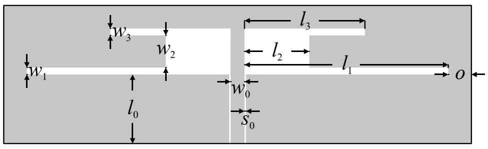


Fig. 3. Geometry of a dual-band uniplanar dipole antenna [7].

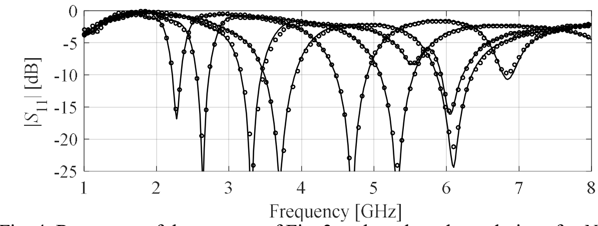


Fig. 4. Responses of the antenna of Fig. 3 at the selected test designs for N = 500: high-fidelity EM model (—), constrained surrogate with uniform sampling (o).

TABLE I Surrogate Model Accuracy

Number of training samples	Average RMS Error		
	Unconstrained Surrogate	Constrained Surrogate (random sampling)	Constrained Surrogate (Uniform sampling)
100	15.6 %	7.7 %	4.8 %
200	11.7 %	4.6 %	3.7 %
500	7.8 %	3.2 %	2.5 %

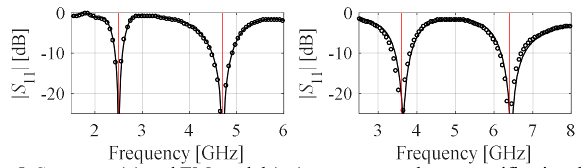


Fig. 5. Surrogate (o) and EM model (—) responses at the two verification designs corresponding to (a) f_1 = 2.5 GHz and f_2 = 4.7 GHz, and (b) f_1 = 3.6 GHz and f_2 = 6.4 GHz. Vertical lines indicate the required operating frequencies.

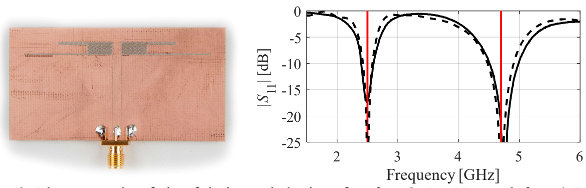


Fig. 6. Photograph of the fabricated design for f_1 = 2.5 GHz and f_2 = 4.7 GHz as well as simulated (- - -) and measured (—) reflection characteristics.

B. UWB Monopole

The second example is a UWB monopole shown in Fig. 7(a). The structure consists of a rectangular radiator with elliptical corner cuts as well as stepped-impedance feed line. The antenna is implemented on a 0.76-mm-thick substrate. The design parameters are $\boldsymbol{x} = [L_g\ L_1\ L_2\ W_1\ L_p\ W_p\ a\ b]^T$ (dimensions in mm). The feeding line width W_0 is adjusted for a given substrate permittivity to ensure 50 ohm input impedance. The EM antenna model $\boldsymbol{R}$ is implemented in CST (~900,000 mesh cells, simulation time 2 minutes). The model includes the SMA connector.

We aim at constructing a surrogate model of the antenna input characteristic assuming various dielectric permittivities ε_r of the substrate. The reference designs are optimized for minimum in-band reflection at ε_r = 1.8, 3.0, 4.5, and 6.0. We have $\boldsymbol{x}^{(1)}$ = [9.86 4.17 6.46 2.08 21.1 29.7 0.52 0.44]^T, $\boldsymbol{x}^{(2)}$ = [9.45 4.02 6.33 1.41 19.9 26.7 0.57 0.41]^T, $\boldsymbol{x}^{(3)}$ = [9.17 3.54 6.55 1.26 19.6 26.8 0.58 0.39]^T, and $\boldsymbol{x}^{(4)}$ = [9.91 5.04 5.91 1.05 18.5 24.9 0.62 0.38]^T. Based on these designs, the lower and upper bounds for design variables are established as $\boldsymbol{l}$ = [8.5 3.0 5.5 1.0 18.0 24.0 0.5 0.35]^T, and $\boldsymbol{u}$ = [10.0 5.5 7.0 2.5 22.0 30.0 0.65 0.45]^T. The triangulation of the reference design yields three simplexes (intervals): $\{\boldsymbol{x}^{(1)},\boldsymbol{x}^{(2)}\}$, $\{\boldsymbol{x}^{(2)},\boldsymbol{x}^{(3)}\}$, and $\{\boldsymbol{x}^{(3)},\boldsymbol{x}^{(4)}\}$.

Similarly, as for the first example, the surrogate model has been constructed using 100, 200, and 500 training samples. The modeling errors are provided in Table II (see also Fig. 7(b)). It can be noted that uniform sampling results in reduction of the modeling error although not as significant as for the previous case which is because the simplexes $S^{(k)}$ in this case are just one dimensional. For the sake of verification, the antenna has been optimized for ε_r = 2.2 and 3.5 (cf. Fig. 8). The latter design has been fabricated and measured as shown in Fig. 9.

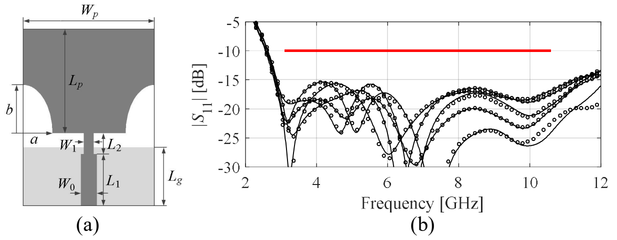


Fig. 7. Planar UWB antenna: (a) geometry (ground plane marked with ligh-gray shade), (b) responses at the selected test designs for N = 500: EM model (—), constrained surrogate model with uniform sampling (o).

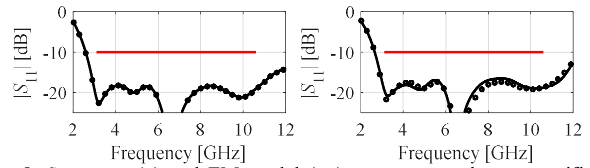


Fig. 8. Surrogate (o) and EM model (—) responses at the two verification designs corresponding to (a) ε_r = 2.2, and (b) ε_r = 3.5. Horizontal line indicates matching requirements (–10 dB for 3.1 GHz to 10.6 GHz).

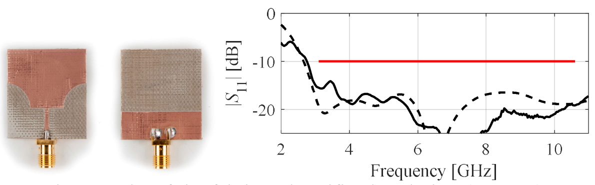


Fig. 9. Photographs of the fabricated verification design (ε_r = 3.5) as well as simulated (- - -) and measured (—) reflection characteristics. Horizontal line indicates matching requirements (–10 dB for 3.1 GHz to 10.6 GHz).

TABLE II UWB MONOPOLE: SURROGATE MODEL ACCURACY

Number of training samples	Average RMS Error		
	Unconstrained Surrogate	Constrained Surrogate (random sampling)	Constrained Surrogate (uniform sampling)
100	68.3 %	15.3 %	14.4 %
200	68.9 %	11.9 %	10.3 %
500	68.4 %	9.6 %	8.8 %

V. CONCLUSION

In the letter, a novel technique for uniform sampling in constrained domains has been proposed for reliable surrogate modeling of antenna input characteristics. Our methodology has been demonstrated using two antenna structures, an ultra-wideband monopole and a dual-band dipole. The results indicate superiority over rudimentary random sampling. Application case studies and experimental validation confirm usefulness of the approach for antenna design purposes.

REFERENCES

[1] J. Nocedal and S. Wright, *Numerical Optimization*, 2nd edition, Springer, New York, 2006.

[2] M. Fernandez Pantoja, A. Rubio Bretones, and R. Gomez Martin, "Benchmark antenna problems for evolutionary optimization algorithms," *IEEE Trans. Ant. Prop.*, vol. 55, no. 4, pp. 1111-1121, 2007.

[3] J.S. Ochoa, and A.C. Cangellaris, "Random-space dimensionality reduction for expedient yield estimation of passive microwave structures," *IEEE Trans. Microwave Theory Techn.*, vol. 61, no. 12, pp. 4313-4321, Dec. 2013.

[4] S. Koziel and J.W. Bandler, "Rapid yield estimation and optimization of microwave structures exploiting feature-based statistical analysis," *IEEE Trans. Microwave Theory Tech.*, vol. 63, no., 1, pp. 107-114, 2015.

[5] H.L. Abdel-Malek, A.S.O. Hassan, E.A. Soliman, S.A. Dakroury, "The ellipsoidal technique for design centering of microwave circuits exploiting space-mapping interpolating surrogates," *IEEE Trans. Microwave Theory Techn.*, vol. 54, no. 10, pp. 3731-3738, Oct. 2006.

[6] T.W. Simpson, J.D. Pelplinski, P.N. Koch, and J.K. Allen, "Metamodels for computer-based engineering design: survey and recommendations", *Engineering with Computers*, vol. 17, pp. 129-150, 2001.

[7] J.W. Bandler, N. Georgieva, M.A. Ismail, J.E. Rayas-Sánchez, and Q. J. Zhang, "A generalized space mapping tableau approach to device modeling," *IEEE Trans. Microwave Theory Tech.*, vol. 49, no. 1, pp. 67-79, Jan. 2001.

[8] J.L. Chavez-Hurtado, and J.E. Rayas-Sanchez, "Polynomial-based surrogate modeling of RF and microwave circuits in frequency domain exploiting the multinomial theorem," *IEEE Trans. Microwave Theory Tech.*, vol. 64, no. 12, pp. 4371-4381, 2016.

[9] I. Couckuyt, "Forward and inverse surrogate modeling of computationally expensive problems," Ph.D. Thesis, Ghent University, 2013.

[10] N.V. Queipo, R.T. Haftka, W. Shyy, T. Goel, R. Vaidynathan, and P.K. Tucker, "Surrogate-based analysis and optimization," *Progress in Aerospace Sciences*, vol. 41, no. 1, pp. 1-28, Jan. 2005.

[11] H. Kabir, Y. Wang, M. Yu, and Q.J. Zhang, "Neural network inverse modeling and applications to microwave filter design," *IEEE Trans. Microwave Theory Tech.*, vol. 56, no. 4, , pp. 867-879, April 2008.

[12] J. P. Jacobs and J. P. De Villiers, "Gaussian-process-regression-based design of ultrawide-band and dual-band CPW-fed slot antennas," *Journal of Electromagnetic Waves and Appl.*, vol. 24, pp. 1763-1772, 2010.

[13] G. Angiulli, M. Cacciola and M. Versaci, "Microwave devices and antennas modelling by support vector regression machines," *IEEE Trans. Magnetics*, vol. 43, pp. 1589-1592, 2007.

[14] S. Koziel and A. Bekasiewicz, "Computationally feasible narrow-band antenna modeling using response features," *Int. J. RF & Microwave CAE*, vol. 27, no. 4, 2017.

[15] S. Koziel and L. Leifsson, "Generalized shape-preserving response prediction for accurate modeling of microwave structures," *IET Microwaves, Ant. Prop.*, vol. 6, No. 12, pp. 1332-1339, 2012.

[16] S. Koziel, "Low-cost data-driven surrogate modeling of antenna structures by constrained sampling," *IEEE Ant. Wireless Prop. Lett.*, vol. 16, pp. 461-464, 2017.

[17] S. Koziel, A. Bekasiewicz, "On reduced-cost design-oriented constrained surrogate modeling of antenna structures," *IEEE Ant. Wireless Prop. Lett.*, vol. 16, pp. 1618-1621, 2017.

[18] H. Borouchaki, P.L. George, and S.H. Lo, "Optimal Delaunay point insertion," *Int. J. Numerical Methods in Eng.*, vol. 39, no. 20, pp. 3407-3437, 1996.

[19] Y.-C. Chen, S.-Y. Chen, and P. Hsu, "Dual-band slot dipole antenna fed by a coplanar waveguide," *IEEE Int. Symp. Ant. Prop.*, pp. 3589-3592, 2006.