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Reliable low-cost surrogate modeling and design optimisation of antennas using implicit space mapping with substrate segmentation

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Abstract: In this work, a reliable methodology for fast simulation-driven design optimisation of antenna structures is proposed. The authors' approach exploits implicit space mapping (ISM) technology. To adopt it for handling antenna structures, they introduce substrate segmentation with separate dielectric permittivity value assigned for each segment as ISM preassigned parameters. At the same time, the coarse model for space mapping is established by reducing discretisation level of the EM analysis of the antenna structure at hand. Further, the proposed approach is generalised to permit execution of ISM in case of dielectric resonator antennas (DRAs), which is realised by segmentation of the DR and appropriate assignment of permittivity values. The discussed methodology is demonstrated through optimisation of a bandwidth-enhanced patch antenna and a triangular DRA. Comparison with state-of-the-art surrogate-assisted design methods, here, frequency and output space mapping, as well as conventional optimisation using pattern search, is also provided.

1 Introduction

Design of contemporary antenna structures is heavily based on computational tools, in particular, full-wave electromagnetic (EM) simulation models. EM analysis permits accurate evaluation of antenna performance; however, it might be expensive for various reasons such as complexity of the antenna topology as well as necessity of including additional components into the analysis that may affect the antenna operation (e.g. connectors [1] etc.). High cost of computational models is a major problem from the point of view of simulation-driven design of antenna structures, specifically adjustment of geometry parameter values. Both hands-on methods (such as parameter sweeps) and automated numerical optimisation procedures (e.g. [2]) require large number of EM analysis to yield a satisfactory design. The cost of these procedures may be prohibitive. This is particular pertinent to population-based metaheuristics, where computational cost may be as high as thousands or tens of thousands of antenna evaluations [3]. At the same time, parameter sweeping is unable to handle a large number of parameters (typical for modern antennas) not to mention multiple design goals or constraints.

Because EM-driven design closure is nowadays mandatory in vast majority of cases, development of methods that may lead to reduction of its high cost became a practical necessity. Such methods either aim at improving efficiency of conventional numerical optimisation techniques or shift the optimisation burden into a cheaper representation of the expensive computational model at hand. A notable example in the first group is utilisation of adjoint sensitivity techniques [4] that permits considerable speedup of gradient-based algorithms [5]. Another option is explicit parallelisation (applicable, e.g. in case of pattern search algorithms [6]). The second group involved surrogate-based optimisation (SBO) techniques [7–9]. Here, one needs to distinguish SBO with approximation surrogates as well as physics-based ones. Approximation models are fast and generic but expensive to set up, therefore, only applicable for lower-dimensional or narrowed down search spaces. Physics-based methods utilise a suitably corrected underlying low-fidelity model. Perhaps the most popular approach of this kind in microwave and antenna engineering is space mapping (SM) [7]. Other methods include various response correction techniques [8], manifold mapping [8], or feature-based optimisation [10].

Application of physics-based methods in case of antenna design is challenging due to the fact that low-fidelity model is normally obtained through coarse-discretisation EM simulations, which makes it relatively expensive. In this work, we focus on implicit SM (ISM) [8], which is probably the most interesting version of SM. It is easy to implement and it does not affect the underlying coarse model domain, which is important in case of constrained optimisation.

ISM utilises – as degrees of freedom for coarse model correction – a special set of so-called preassigned parameters. So far, ISM was mostly utilised for microwave circuit optimisation [8, 11] where the most suitable preassigned parameters are dielectric permittivity (or substrate height) of the transmission line components in an equivalent network representation of the structure under design [11]. As mentioned before, in case of antennas, the low-fidelity models do not offer this kind of flexibility. Consequently, utilisation of ISM is limited.

In this work, we propose a novel implementation of ISM for antenna design with preassigned parameters being dielectric permittivity values of the substrate partitions in the coarse-mesh EM model. Such a setup allows for significant flexibility in terms of the number preassigned parameters but also their location (e.g. with respect to antenna metallisation). Furthermore, it is easy to implement as it only requires modification of the antenna substrate and its parameterisation. Our approach is further generalised to handle DRAs. The proposed methodology is demonstrated through the design of a bandwidth-enhanced patch antenna and a triangular DRA. Numerical results are validated experimentally. Benchmarking using state-of-the-art conventional and surrogate-assisted methods is also provided.

2 Fundamentals of SM technology

In this section, we recall the basics of SM technology. Remarks on ISM and the proposed implementation are provided in Section 3.

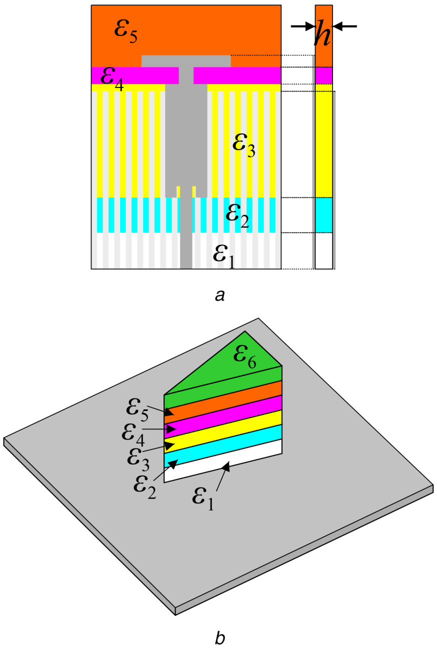

Fig. 1 *ISM for antenna optimisation. Illustration of preassigned parameters incorporation*
(a) Substrate of the planar antenna with six parameters, ***(b)*** DRA structure with six parameters

2.1 Problem formulation

We will denote by $\boldsymbol{R}_f(\boldsymbol{x})$ the computational model of the antenna structure under design. Normally, $\boldsymbol{R}_f$ is evaluated using high-fidelity full-wave EM analysis. $\boldsymbol{R}_f(\boldsymbol{x})$ stands for all relevant antenna responses (e.g. reflection versus frequency, gain, radiation pattern, etc.); $\boldsymbol{x}$ is a vector of designable (geometry parameters). The design optimisation problem can be stated as a non-linear minimisation problem

$$\boldsymbol{x}^* = \arg \min_{\boldsymbol{x}} U(\boldsymbol{R}_f(\boldsymbol{x})) \tag{1}$$

where U is an objective function encoding given design specifications. Typically, the problem (1) is constrained by imposing lower/upper bounds on design parameters as well as other constraints (e.g. to ensure physical consistency of the structure).

2.2 SM algorithm

SM is an iterative procedure that yields a series $\boldsymbol{x}^{(i)}$, $i = 0, 1, \ldots,$ of approximations to $\boldsymbol{x}^*$ as follows (referred to as surrogate model optimisation, SO)

$$\boldsymbol{x}^{(i+1)} = \arg \min_{x} U(\boldsymbol{R}_s^{(i)}\boldsymbol{x})) \tag{2}$$

In (2), $R_s^{(i)}$ is a surrogate model constructed by correcting the underlying (low-fidelity) coarse model $\boldsymbol{R}_c$. The generic SM surrogate model can be written as $R_s^{(i)}(x) = R_s^{\#}(x, p^{(i)})$, where the parameters $\boldsymbol{p}^{(i)}$ are obtained through parameter extraction (PE) process [11]

$$\boldsymbol{p}^{(i)} = \arg \min \{\boldsymbol{p}\colon \| \boldsymbol{R}_f(\boldsymbol{x}^{(i)}) - \boldsymbol{R}_s^{\#}(\boldsymbol{x}^{(i)}, \boldsymbol{p} \| \} \tag{3}$$

The PE process aims at reducing misalignment between the coarse and the fine model, which can be realised at the most recent design as in (3) or at several designs [11].

A popular type of SM is input SM with the surrogate model being

$$\boldsymbol{R}_s^{\#}(\boldsymbol{x}, \boldsymbol{p}) = \boldsymbol{R}_s^{\#}(\boldsymbol{x}; \boldsymbol{B}, \boldsymbol{c}) = \boldsymbol{R}_c(\boldsymbol{B}\boldsymbol{x} + \boldsymbol{c}) \tag{4}$$

with $\boldsymbol{B}$ and $\boldsymbol{c}$ being a square matrix and a column vector, respectively. The entries of $\boldsymbol{B}$ and $\boldsymbol{c}$ are the surrogate model parameters in this case.

It should be emphasised that the critical assumption behind SM is that the coarse model is much faster than the fine model so that neither PE nor SO incur significant computational cost (despite the fact that both are normally solved using conventional algorithms). In case of antennas, this assumption may not be satisfied because the coarse model requires EM simulation. Therefore, application of SM for antenna design requires that both the number of coarse and fine model evaluations is carefully controlled in the optimisation process.

3 Antenna optimisation using variable-fidelity EM models

In this work, we are interested in ISM for the reasons explained in the introduction: easily controlled surrogate model flexibility as well as no effect on coarse model domain (as opposed to, e.g. input SM). Here, we explain the proposed concept of ISM implementation through substrate segmentation as well as describe our ISM algorithmic framework.

3.1 Implicit space mapping (ISM) with substrate segmentation

In ISM [11], the surrogate model (preassigned) parameters $\boldsymbol{p}$ are normally unrelated to design variables. They are normally fixed in the fine model but can be freely adjusted in the coarse model in order to facilitate the model alignment. As mentioned before, they are typically substrate parameters (e.g. permittivity) of transmission line components in the equivalent network model.

Here, in order to apply ISM for antenna design, we utilise coarse-mesh EM simulation model as a coarse model, whereas preassigned parameters are introduced as dielectric permittivity values of the substrate segments as indicated in Fig. 1. In case of planar antennas, the segments may be allocated in correlation with particular functional units of the antenna (e.g. a radiator, feeding line etc.). At the same time, the concept can be extended to other cases, here, represented by DRAs, cf. Fig. 1*b*. In this case, it is a bulk of the DR that is subjected to the segmentation process, along with other pieces of the dielectric, if applicable.

3.2 Algorithm framework

In our implementation, ISM is combined with frequency SM (FSM) [7] as well as output SM (OSM) [7]. Both FSM and OSM can be realised with negligible computational cost and allow for correcting residual model misalignment both in terms of frequency shifts (FSM) and level discrepancy (OSM).

The surrogate model is defined as

$$\boldsymbol{R}_s^{(i)}(\boldsymbol{x}) = \boldsymbol{R}_c(\boldsymbol{x}, \boldsymbol{p}^{(i)}, F^{(i)}(\Omega)) + \boldsymbol{d}^{(i)} \tag{5}$$

The ISM parameters $\boldsymbol{p} = [\varepsilon_1 \ \ldots \ \varepsilon_N \ h]^{\mathrm{T}}$, where N is the number of substrate segments (cf. Fig. 1); the mapping $F^{(i)}$ represents scaling of the frequency sweep $\Omega = [\omega_1 \ \ldots \ \omega_m]$ (i.e. the coarse model is evaluated at the frequencies $F^{(i)}(\Omega)$ instead of Ω).

When applying SM for antenna design it is imperative to reduce the number of coarse model evaluations in the optimisation process. To achieve this goal, the ISM PE process is realised iteratively as

$$\boldsymbol{p}^{(i.k+1)} = \arg \min_{\| \boldsymbol{p} - \boldsymbol{p}^{(i.k)} \| \le \delta_p^{(k)}} \| \boldsymbol{R}_f(\boldsymbol{x}^{(i)}) - \boldsymbol{L}_p^{(k)}(\boldsymbol{x}^{(i)}, \boldsymbol{p}, \Omega) \| \tag{6}$$

where $\boldsymbol{p}^{(i.k)}$, $k = 0, 1, \ldots,$ approximates $\boldsymbol{p}^{(i)}$ (we set $\boldsymbol{p}^{(i.0)} = \boldsymbol{p}^{(i-1)}$). The surrogate $\boldsymbol{R}_s^{(i)}$ is replaced in (6) by its linear expansion model

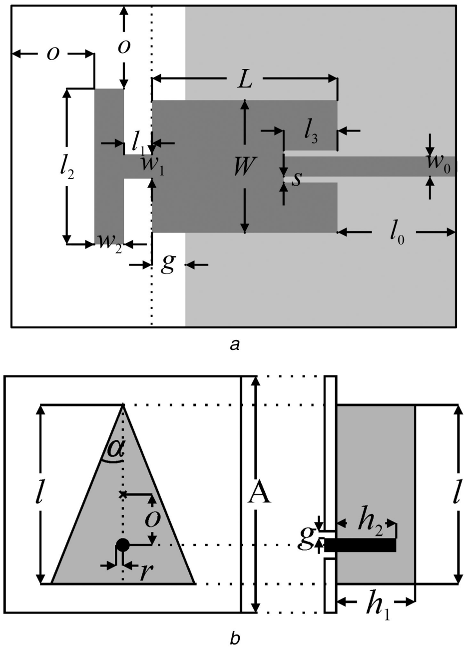

Fig. 2 *Geometries of the considered antenna structures*
(a) Bandwidth-enhanced planar antenna (note that g is defined w.r.t. dotted line: its negative/positive values mean trimming ground plane below patch/monopole) [13], ***(b)*** Triangular DRA [15]

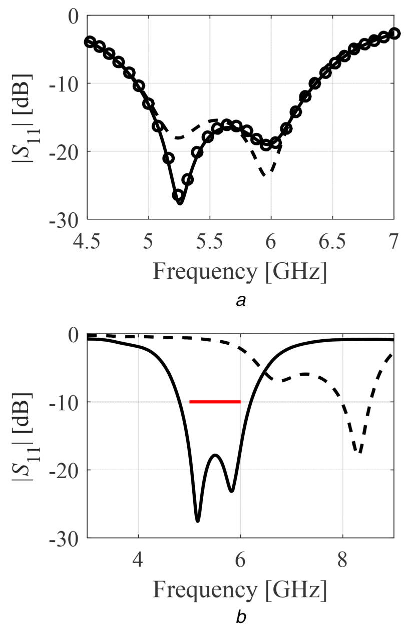

Fig. 3 *Corrected antenna structure*
(a) The effect of ISM/FSM: coarse (- - -), fine (—), and ISM/FSM corrected coarse model (o) responses at the selected design (excellent alignment between the SM surrogate and the fine model), ***(b)*** Comparison of the initial (– –) and optimised antenna response

$$L_p^{(k)} = R_c(\boldsymbol{x}^{(i)}, \boldsymbol{p}^{(i.k)}, \Omega) + J_p(\boldsymbol{x}^{(i)}, \boldsymbol{p}^{(i-1)}, \Omega) \cdot (\boldsymbol{p} - \boldsymbol{p}^{(i.k)}) \tag{7}$$

in which $\boldsymbol{J}_p(\boldsymbol{x}^{(i)},\boldsymbol{p}^{(i-1)},\Omega)$ is an estimated Jacobian of $\boldsymbol{R}_c$ w.r.t. $\boldsymbol{p}$ evaluated by finite differentiation (FD). Step size for FD is determined based on visual inspection of responses.

The PE process (6) is embedded in a trust region (TR) framework to ensure convergence. The TR radius $\delta_p^{(k)}$ is updated using the standard rules [12], i.e. new design is accepted only if gain ratio $\rho > 0$. The initial radius value $\delta_p^{(0)}$ is set to 1. It is increased (doubled) if $\rho > 0.75$ and decreased (divided by three) if $\rho < 0.25$. It should be emphasised that for the sake of cost saving, the Jacobian $\boldsymbol{J}_p$ is not updated throughout the iterations of (6).

In order to provide better flexibility, frequency SM is implemented as a polynomial scaling $F^{(i)}(\omega) = f_{0.i} + f_{0.i}\omega + f_{0.i}\omega^2$ (as opposed to the standard FSM, typically implemented as an affine transformation [7]). Coefficients f_k are found by minimising the expression $||\boldsymbol{R}_f(\boldsymbol{x}^{(i)} - \boldsymbol{R}_c(\boldsymbol{x}^{(i)},\boldsymbol{p}^{(i)},F^{(i)}(\Omega))||$. The computational cost of FSM is negligible because the scaled response is obtained by re-interpolating the response at the original sweep. The OSM term $\boldsymbol{d}^{(i)}$ in (5) is simply calculated as $\boldsymbol{d}^{(i)} = \boldsymbol{R}_f(\boldsymbol{x}^{(i)} - \boldsymbol{R}_c(\boldsymbol{x}^{(i)},\boldsymbol{p}^{(i)},F^{(i)}(\Omega))$ [7]. It ensures perfect alignment of the surrogate and the fine model at the current design $\boldsymbol{x}^{(i)}$.

Finally, the surrogate model optimisation process (1) is arranged similarly as in (6). We generate a sequence $\boldsymbol{x}^{(i+1.k)}$, $k = 0, 1,\ldots,$ of approximations to $\boldsymbol{x}^{(i+1)}$ (here, $\boldsymbol{x}^{(i+1.0)} = \boldsymbol{x}^{(i)}$) by solving

$$\boldsymbol{x}^{(i+1.k+1)} = \arg \min_{\| \boldsymbol{x} - \boldsymbol{x}^{(i+1.k)} \| \le \delta_{SO}^{(k)}} U(\boldsymbol{G}_s^{(i,k)}(\boldsymbol{x})) \tag{8}$$

The initial value of the threshold $\delta_{SO}^{(0)}$ is set to 1. In (8), the surrogate $\boldsymbol{R}_s^{(i)}$ is replaced by its linear expansion model

$$\boldsymbol{G}_s^{(i.k)}(\boldsymbol{x}) = \boldsymbol{R}_s^{(i)}(\boldsymbol{x}^{(i+1.k)} + \boldsymbol{J}_{\boldsymbol{R}c}(x^{(i+1.k)}, \boldsymbol{p}^{(i)}, \boldsymbol{F}^i\Omega)) \cdot (\boldsymbol{x} - \boldsymbol{x}^{(i+1.k)}) \tag{9}$$

where the Jacobian $\boldsymbol{J}_{\boldsymbol{R}c}$ with respect to $\boldsymbol{x}$ is obtained using FD. Similarly as in (7), the step size for FD is obtained based on visual inspection of response characteristics.

4 Verification examples

In this section, we present two verification examples: a planar enhanced-bandwidth patch antenna, and a triangular DRA. For the first example, numerical results are validated experimentally. In both cases, benchmarking using state-of-the-art optimisation techniques is provided.

4.1 Enhanced-bandwidth planar antenna

As a first example, consider a planar antenna with enhanced bandwidth shown in Fig. 2*a* [13]. The substrate is a Taconic RF-35 ($\varepsilon_r = 3.5$, tan $\delta = 0.0018$, $h = 0.762$ mm). Design variables are $\boldsymbol{x} = [L\ l_1\ l_2\ l_3\ W\ w_1\ w_2\ g]^T$. Parameters: $o = 7$, $w_0 = 1.7$, $l_0 = 10$ and $s = 0.5$ (all in mm). The design consists of a patch with inset feed excited through a 50 Ω microstrip line. Bandwidth enhancement is achieved by using a parasitic monopole radiator introducing additional resonance. Undercut of the ground plane below the patch (controlled by parameter g, cf. Fig. 2*a*) enables further enhancement of the bandwidth. It should be noted that negative/positive values of g correspond to ground plane trimming below patch/monopole radiator. The fine and coarse models are both implemented in CST Microwave Studio ($\boldsymbol{R}_f$: ∼550,000 mesh cells, simulation time 3 min; $\boldsymbol{R}_c$: ∼30,000 cells, 15 s) [14]. The design objective is to minimise $|S_{11}|$ in 5 to 6 GHz range. The initial design is $\boldsymbol{x}^{(0)} = [13\ 2.5\ 6.0\ 0.25\ 6.0\ 1.7\ 1.7 - 2.6]^T$. The lower bounds $\boldsymbol{l} = [10\ 1\ 5\ 0.01\ 2\ 0.2\ 0.2 - 3]^T$ and upper bounds $\boldsymbol{u} = [20\ 4\ 15\ 0.45\ 10\ 3.2\ 3.2\ 3]^T$ ensure consistency of the optimised design.

For the sake of optimisation, five substrate segments are utilised with ISM parameters $\boldsymbol{p} = [h\ \varepsilon_1\ \varepsilon_2\ \varepsilon_3\ \varepsilon_4\ \varepsilon_5]^T$ as indicated in Fig. 1*a*. Fig. 3*a* shows the effect of ISM combined with FSM at a selected antenna design. The correction allows for obtaining perfect alignment of the surrogate and the fine model responses.

The final design $\boldsymbol{x}^* = [15.85\ 3.98\ 10.09\ 0.02\ 5.53\ 3.17\ 2.98 - 1.22]^T$ has been obtained in four iterations of the SM algorithm (cf. Section 2). The maximum in-band reflection level is −16.6 dB. The structure response is shown in Fig. 3*b*.

The computational cost of the optimisation process corresponds to 23 evaluations of the fine model (∼70 min of the CPU-time) and includes: 194 evaluations of the coarse model (42 for PE and 152 for surrogate model optimisation) as well as seven evaluations of $\boldsymbol{R}_f$. Fast convergence of the algorithm should be emphasised.

Table 1 Bandwidth-enhanced antenna: optimisation results

Optimisation algorithm	Optimisation cost				Performance
	Model evaluations		Total cost		Maximum in-band $\|S_{11}\|$, dB
	$\boldsymbol{R}_c$	$\boldsymbol{R}_f$	Relative to $\boldsymbol{R}_f$	Absolute, min	
OSM only	251	10	30.9	92.8	−16.3
Pattern search	—	304	304	912	−15.9
ISM (this work)	194	7	23	70	−16.7

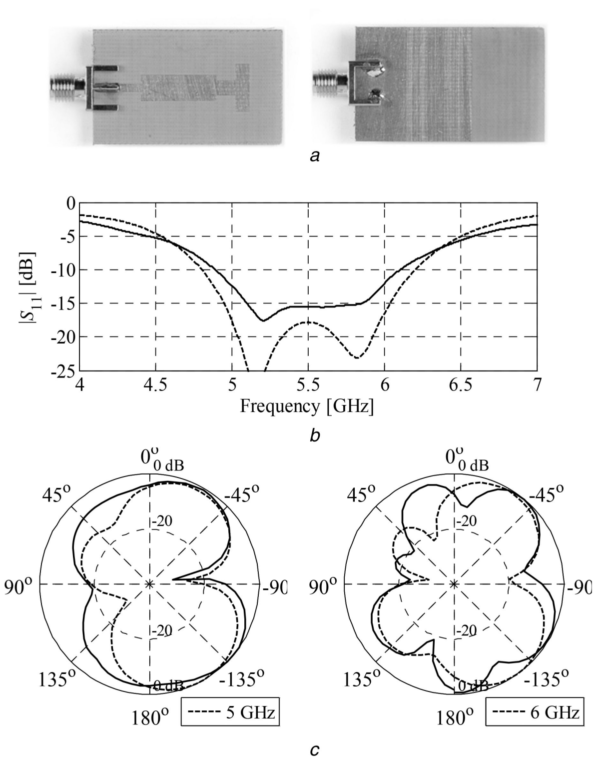

Fig. 4 *Bandwidth-enhanced planar antenna*

(a) Photographs of the structure prototype, as well as comparison of simulated (– –) and measured (—), ***(b)*** Reflection characteristics, ***(c)*** *E*-plane radiation patterns

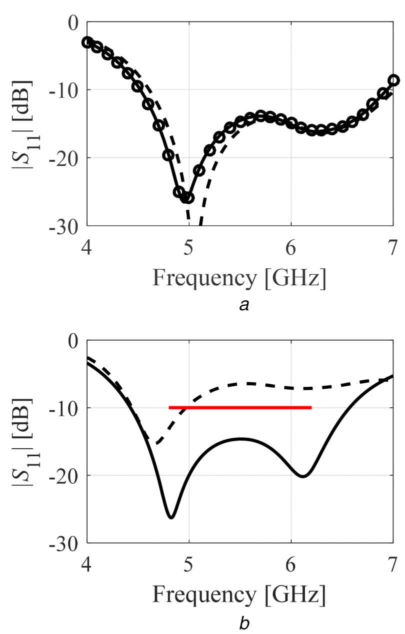

Fig. 5 *Triangular DRA*

(a) The effect of ISM/FSM: coarse (- - -), fine (—), and ISM/FSM corrected coarse model (o) responses at the selected design, ***(b)*** Comparison of the antenna responses at the initial (– –) and final (—) designs

Moreover, the design with maximum in-band $|S_{11}|$ of −16.0 dB has been obtained after just two iterations.

For the sake of comparison, the antenna was also optimised using just output SM [7], as well as a pattern search algorithm [2] (see Table 1). It can be observed that the proposed ISM algorithm ensures as good quality of the design as direct fine model optimisation at a fraction of the computational cost of the latter. At the same time, OSM-only optimisation features comparable cost but the design quality is lower.

For additional validation, the ISM-optimised antenna prototype has been fabricated and measured. Fig. 4 shows the comparison of the simulated and measured reflection responses, as well as *E*-plane radiation patterns at selected frequencies. Agreement between the results is acceptable.

4.2 Triangular DRA

Our second example is a DRA shown in Fig. 2*b* [15]. The design consists of a triangular DRA fed through a circular probe. The DR material is Eccostock HiK ($\varepsilon_r = 10$, $\tan\delta = 0.002$). The vector of design variables is $\boldsymbol{x} = [\alpha\ l\ h_1\ h_{2r}\ o_r]^T$. Parameters $h_2 = h_1 \cdot h_{2r}$, $o = 0.5l + r - 2o_r(0.5l_2 + r)$ are relative, whereas $r = 0.63$, $g = 0.82$ remain fixed. All dimensions except h_{2r} and o_r are in mm. Both the fine and coarse models are implemented in CST Microwave Studio ($\boldsymbol{R}_f$: ∼2,100,000 cells, simulation time 11 min; $\boldsymbol{R}_c$: ∼45,000 cells, 50 s) [14]. The design objective is to minimise antenna reflection in 4.8 to 6.2 GHz range. The initial design is $\boldsymbol{x}^{(0)} = [14.0\ 20\ 15\ 0.7\ 0]^T$. The bounds $\boldsymbol{l} = [10\ 10\ 10\ 0\ 0]^T$ and $\boldsymbol{u} = [30\ 30\ 30\ 1\ 1]^T$ ensure consistency of the structure.

Six DRA segments with ISM parameters $\boldsymbol{p} = [\varepsilon_1\ \varepsilon_2\ \varepsilon_3\ \varepsilon_4\ \varepsilon_5\ \varepsilon_6]^T$ are utilised for optimisation (see Fig. 1*b*). The effect of ISM combined with FSM at a selected antenna design is shown in Fig. 5*a*. An excellent alignment of the surrogate and the fine model can be observed upon applying ISM/FSM.

The final design $\boldsymbol{x}^* = [12.1\ 18.1\ 22.3\ 0.50\ 0.008]^T$ has been obtained in four iterations of the SM algorithm of Section 2. Antenna reflection at the final design is shown in Fig. 5*b*.

The optimisation cost corresponds to 20 evaluations of the fine model (∼220 min of the CPU time). Cost breakdown is as follows: 30 $\boldsymbol{R}_c$ and 141 $\boldsymbol{R}_c$ simulations for PE and surrogate model optimisation as well as seven evaluations of $\boldsymbol{R}_f$.

The antenna was also optimised using just output SM, as well as a pattern search algorithm. Comparison of the results has been provided in Table 2. Note that the proposed ISM algorithm ensures as good quality of the design as direct fine model optimisation at a fraction of the computational cost of the latter.

5 Conclusion

ISM for antenna design has been proposed involving partitioning of the structure dielectric and exploiting permittivity parameters of the segments as degrees of freedom for model alignment. The number of ISM parameter can be readily adjusted by changing segmentation arrangement. This is the first implementation of ISM with substrate segmentation with extends application of ISM for antenna design where equivalent network low-fidelity models are not available. Our results indicate that segmentation-based ISM allows for reducing cost of the optimisation process compared with OSM as well as conventional methods (here, pattern search).

Table 2 Triangular DRA: optimisation results

optimisation algorithm	Optimisation cost				Performance
	Model evaluations		Total cost		Maximum in-band $\|S_{11}\|$, dB
	$\boldsymbol{R_c}$	$\boldsymbol{R_f}$	Relative to $\boldsymbol{R_f}$	Absolute, min	
OSM only	210	10	25.9	285	−15.3
Pattern search	—	201	201	2211	−15.4
ISM (this work)	171	7	20.0	220	−15.2

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