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Model of Pressure Distribution in Vortex Flow Controls

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Abstract

Vortex valves belong to the category of hydrodynamic flow controls. They are important and theoretically interesting devices, so complex from hydraulic point of view, that probably for this reason none rational concept of their operation has been proposed so far. In consequence, functioning of vortex valves is described by CFD-methods (computer-aided simulation of technical objects) or by means of simple empirical relations (using discharge coefficient or hydraulic loss coefficient). Such rational model of the considered device is proposed in the paper. It has a simple algebraic form, but is well grounded physically. The basic quantitative relationship, which describes the valve operation, i.e. dependence between the flow discharge and the circumferential pressure head, caused by the rotation, has been verified empirically. Conformity between calculated and measured parameters of the device allows for acceptation of the proposed concept.

Key words: rotational flow, flow controls, vortex valve, storm waste water

Notation

The following symbols are used in this paper:
Latin letters:

A_{in}	–	inflow cross section,
B	–	coefficient,
d_{in}	–	inflow diameter,
e	–	auxiliary parameter,
$e_r,\ e_t$	–	versors, radial and tangential respectively,
g	–	gravity acceleration,
h	–	chamber depth,
H	–	total overpressure head,
H_M	–	local energy loss (“local uplift”),
H_V	–	rotational overpressure head (“rotational uplift”),

L_c	–	geometrical scale of flow,
$M_{in},\ M_{out}$	–	flowing fluid power, introduced and dissipated respectively,
p_a	–	atmospheric pressure,
p_v	–	pressure induced by fluid rotation,
Q	–	discharge of flow,
r	–	radial coordinate,
r_M	–	radius of measuring point,
r_w	–	outflow radius,
R	–	chamber radius,
$\mathbf{u}$	–	velocity vector,
$u_r,\ u_t$	–	velocity components, radial and tangential respectively,
v_c	–	characteristic velocity,
v_{in}	–	inlet velocity,
v^*	–	shear velocity,
V	–	volume.

Greek letters:

α	–	coefficient of discharge,
λ	–	Nikuradse hydraulic loss coefficient,
μ_τ	–	dynamic coefficient of turbulent viscosity,
ξ	–	total coefficient of local hydraulic loss,
ξ_M	–	coefficient of local hydraulic loss (“local uplift”),
ξ_v	–	coefficient of rotational hydraulic loss (“rotational uplift”),
ρ	–	fluid density,
ω	–	angular velocity.

1. General Characteristics of the Problem

Rotational flow of liquids is a very significant and interesting kind of motion. Among its specific features, one should mention the increase of pressure towards the outer part of the vortex (for pressure chambers), which is marked by the free-surface rise along the outer wall in open chambers.

Both these effects are utilized in practice. As an important and curious example of such an application one can quote vortex flow controls. This category of devices can be divided into two groups:

– vortex valves (when the increase of pressure reduces the discharge of the supply conduit), called also vortex diode (Yoder et al 2011);
– vortex dividers (when the rise of liquid free-surface along the vortex perimeter provides facilities for dividing of the influent into two parts – main stream, flowing through the drain hole of the device and side stream, directed to the upper edge of the chamber, which works as a storm overflow).

A relatively full description of this kind of flow one can obtain making use of computer aided methods (e.g. Frith and Duggins 1986). Without any doubts, Computational Fluid Dynamics (CFD) offers very powerful possibilities, but from the other hand it also shows some important disadvantages (Sawicki 2014). So, simplified methods are welcome, especially by practically oriented specialists.

A very important role in technical applications plays the classical attitude, according to which the vortex control unit is treated as if it was a special kind of the orifice. Its hydraulic characteristics is described by the well-known algebraic formula (Kotowski and Wójtowicz 2010, Lecornu et al 2008), which combines discharge Q and overpressure head H by the empirical coefficient of discharge α:

$$Q = \alpha A_{in} \sqrt{2gH}, \tag{1}$$

or in the converse form, making use of the local hydraulic loss coefficient ξ:

$$H = \xi \frac{V_{in}^2}{2g} = \xi \frac{8Q^2}{\pi^2 g d_{in}^4}. \tag{2}$$

However this concept, quite useful because of its formal simplicity, doesn't explain physical aspects of the considered device. This situation generates a feeling of lack of a rational model of vortex controls operation. Such model should be mathematically simple (at the best – algebraic) and as precise physically, as possible. It could be applied as an independent technical method during the vortex controls design, construction and exploitation, but also useful as an auxiliary tool in CFD procedures (especially when the inverse problem has to be solved). The key meaning for the analyzed question has determination of the pressure inside the object. The paper is devoted to the presentation of some model of this variable.

2. The Idea of Pressure Determination

Velocity and pressure, basic variables which characterize the fluid motion, for turbulent flow conditions are described by the equation of continuity and Reynolds equation of momentum conservation (Landau and Lifshitz 1987). However, as it was underlined above, in more general cases, these relations can be solved by means of computer methods only.

In order to fulfil requirements, formulated hereinbefore for the relations in demand (mathematically simple and physically rational), a kinematic approach was used. This means that:

- the velocity field was described by means of an algebraic relation, selected and derived on the ground of available empirical data (kinematic evaluation of the velocity field) and appropriate physical statements;
- the pressure field was calculated from the Reynolds equation.

In a general attitude this method is well known and quite often applied in hydromechanics and hydraulics. As an classical example one can quote here the plane potential flow model (when the kinematic condition for the velocity field determination has the form rot $u = 0$) and the screw motion (when the velocity and velocity rotation vectors are parallel).

On the ground of the liquid motion through chamber of the vortex flow control, taking into account its geometrical features (Fig. 1) it was stated, that it is purposeful to assume a model of plane and axisymmetric motion for this caste:

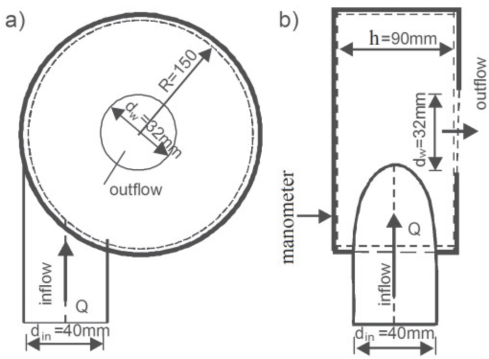

Fig. 1. Diagram of vortex valve (a – plan view, b – side view)

$$u(r) = u_r(r)e_r + u_t(r)e_t. \tag{3}$$

For the radial velocity component (from the outer wall to the centrally oriented outflow) the typical for cylindrical fluid-flow reactors (e.g. for radial settling tanks) relation was accepted:

$$u_r(r) = -\frac{Q}{2\pi rh}. \tag{4}$$

For the tangential velocity component in turn, taking into account classical measurements (Stairmand 1951), a general expression proposed in (Rhodes 2008) was assumed:

$$(r) = \frac{B}{u^{0.5}}. \tag{5}$$

The method of the multiplier B determination will be presented in the next point of this paper. For a such kinematic picture of the considered flow, the Reynolds equation (its $0r$ projection) takes the form:

$$\frac{\delta p_v}{\delta r} = \rho u_r \frac{\delta u_r}{\delta_r} - \frac{\rho u_t^2}{r}. \tag{6}$$

Substituting two already defined components of velocity (Eqs. 4, 5) and integrating this relation with the initial condition:

$$r = r_w, \quad p_v = p_a, \tag{7}$$

which corresponds to the free liquid outflow through the bottom orifice, one obtains:

$$p_v(r) = p_a + \rho B^2 \left(\frac{1}{r_w} - \frac{1}{r} \right) + \frac{\rho Q^2}{8\pi^2 h} \left(\frac{1}{r_w^2} - \frac{1}{r^2} \right). \tag{8}$$

3. Tangential Velocity Profile in Pressure Chamber

The unknown so far multiplier B, which appears in Eq. 5, can be determined on the ground of balance of two mechanical energy fluxes:

– introduced into the rotating liquid (power M_{in});
– dissipated by the rotating liquid (power M_{dis}).

For the steady state, during the normal object exploitation, one can write:

$$M_{in} = M_{dis}. \tag{9}$$

This method was successfully applied for aerated grit chambers (Sawicki 2004) and rotational separators functioning description (Gronowska-Szneler and Sawicki 2014).

The power of the influent can be expressed by the obvious technical relation:

$$M_{in} = \rho Q \frac{v_{in}^2}{2} = \frac{8\rho Q^3}{\pi^2 d_{in}^4}. \tag{10}$$

Next value, the power of the rotational motion energy consumption, can be described by the intensity of the energy dissipation. It is very convenient to express this power by the flow rotation (Serrin 1959, Slattery 1999). For the assumed velocity model, in turbulent motion, one can write (as the angular velocity equals half of velocity rotation):

$$M_{dis} = \int_v 4\mu_\tau \omega^2 dV = \int_v 4\mu_\tau \frac{B^2}{r^3} dV. \tag{11}$$

The coefficient of turbulent viscosity can be conveniently described by the algebraic model (Launder and Spalding 1972)

$$\mu_\tau = 0.00113 \rho v_c L_c. \tag{12}$$

.

This expression differs from the respective formula used in the paper (Sawicki 2012), devoted to the functioning of circulative separators, what results from the distinct geometry of the considered objects. Namely, the circulative separators are symptomatic of the free-surface, whereas the flow controls, analyzed hereby, work under the pressure. For this reason, description of characteristic velocity and mixing length in formulae for the turbulent viscosity are different for both these flow systems.

The characteristic flow velocity in the considered case (pressure flow) can be expressed by the shear velocity, making use of the Nikuradse resistance coefficient:

$$v_c = v^* = \sqrt{\frac{\lambda}{8}} u_t, \tag{13}$$

where as the characteristic scale of motion L_c – by some fraction of the distance from the outer wall of the chamber:

$$L_C = R - r. \tag{14}$$

Substituting Eqs. 13, 14 into Eq. 12, one obtains:

$$\mu_T(r) = 0.0004\rho\sqrt{\lambda}u_t(r)(R - r). \tag{15}$$

Taking into account cylindrical shape of the device (Fig. 1a, b), the elementary volume is given by the following relation:

$$dV = 2\pi r dr. \tag{16}$$

Formal calculation of the integral Eq. 11 yields:

$$M_{dis} = 0.002\sqrt{\lambda}\rho B^2 \pi R \left(r_w^{-1.5} - R^{-1.5}\right). \tag{17}$$

On account of technical proportions ($r_w << R$) the subtrahend in the above brackets can be rejected. Comparing so simplified Eq. 17 with Eq. 10, according to Eq. 9, one obtains the formula, describing the multiplier B in Eq. 5:

$$B = \frac{5.08 r_w^{1/2} Q}{\lambda^{1/6} d_{in}^{4/3} R^{1/3}}. \tag{18}$$

Making use of Eq. 5, the following function for the tangential velocity profile can be written:

$$u_t(r) = \frac{5.08Q}{\lambda^{1/6} d_{in}^{4/3} R^{1/3} h^{1/3}} \left(\frac{v_r}{r}\right)^{1/2}. \tag{19}$$

4. Calculation of Pressure Distribution in Vortex Valve

The expression describing the pressure distribution (Eq. 8) contains three right-hand terms. The first one ($TF = p_a$) is a constant value, the second one (TS) relates to the tangential velocity, whereas the third one (TT) is connected with radial velocity component. In order to estimate the influence of two variable terms, their mutual proportion was analyzed:

$$e = \frac{TT}{TS} = \frac{0.0005\lambda^{1/3} d_{in}^{8/3} R^{2/3}}{(h^{4/3} r_w)} \left(\frac{1}{r_w} - \frac{1}{r}\right). \tag{20}$$

Introducing technical evaluation of typical relations among the basic dimensions of the device:

$$R \sim h, \quad r_w \sim 0.1h, \quad d_{in} \sim 0.5h, \tag{21}$$

the maximal value of the quotient in Eq. 20 is close to (for $r = R$ and $\lambda = 0.02$):

$$e = \frac{TT}{TS} \sim 0.0005. \tag{22}$$

This result means, that the third term (TT) in Eq. (8) reaches the value negligibly smaller than the second one (TS). In consequence this term can be rejected. Finally, substituting the multiplier B according to Eq. (18) into Eq. (8) one obtains the following function, describing the overpressure head in the vortex valve:

$$H_v(r) = \frac{p(r) - p_a}{p\rho} = \frac{25.8 r_w Q^2}{\left(\lambda^{1/3} d_{in}^{8/3} R^{2/3} h^{2/3} \rho\right)} \left(\frac{1}{r_w} - \frac{1}{r}\right). \tag{23}$$

This expression has been practically checked during the laboratory measurements.

5. Empirical Verification of Pressure Distribution in Vortex Valve

The laboratory test stand was made of Bakelite, according to the schematic diagram shown in Fig. 1, taking the following dimensions: $R = 150$ mm, $h = 90$ mm, $d_{in} =$ 40 mm, for two different outlet radii – $r_w = 16$ mm and $r_w = 24$ mm. The inlet pipe was supplied by the water conduit, fitted with a water meter and a flow stabilizer. The pressure inside the device was measured, for different flow intensities, by a spring-type manometer, placed at a distance $r_M = 115$ mm from the valve axis.

Results of measurements are shown as the set of experimental point, presenting relation between the overpressure head and water discharge:

$$H\left(Q, r = r_M\right) = \frac{p\left(Q, r = r_M\right) - p_a}{p\rho}. \tag{24}$$

These points are juxtaposed with broken lines H_V (Q), which illustrate the theoretical Eq. 23 (for r_w = 16 mm in Fig. 2 and for r_w = 24 mm in Fig. 3).

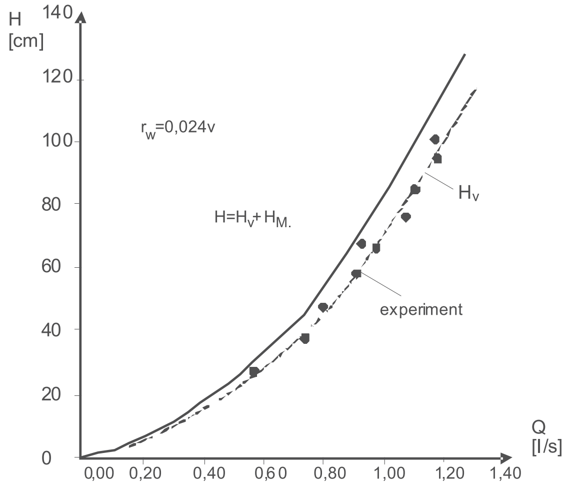

Fig. 2. Laboratory stand verification (r_w = 16 mm)

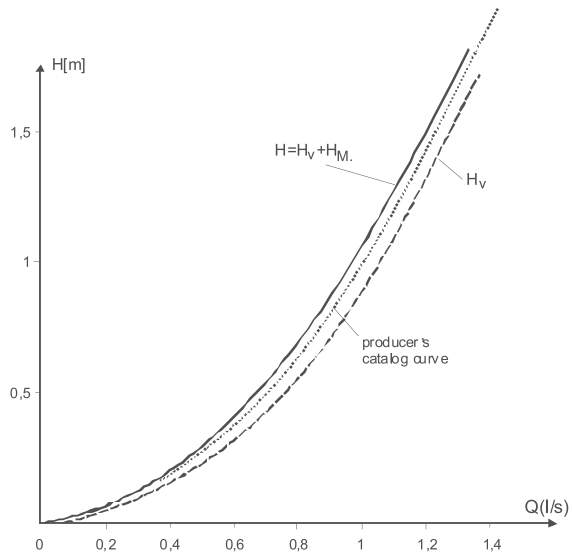

Fig. 3. Laboratory stand verification (r_w = 24 mm)

A producer's catalogue characteristics of a technical version of the vortex valve (offered by the Polish firm ECOL-UNICON) was taken as an additional source of information. This object has the same form as the one shown in Fig. 1, but slightly different size (R = 123 mm, h = 40 mm, r_w = 22.5 mm, the inlet has a rectangular shape 40 × 42 mm, so the substitutive diameter was determined, equal to d_{in} = 46.3 mm).

Calculated relation $H_V(Q, r = R)$ for this technical valve is shown in Fig. 4 (broken line), together with the catalogue curve (dotted line, drowned for completely filled valve chamber).

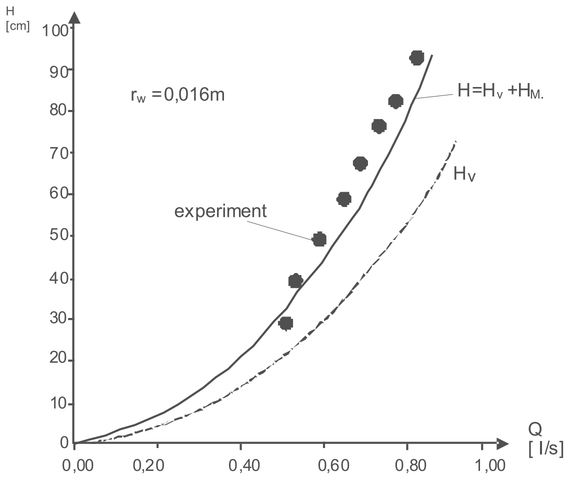

Fig. 4. Technical valve verification

The analysis of these results leads to the statement, that the simplified theory of the vortex valve operation, proposed in this paper, is well confirmed by measurements. This statement is particularly justified for two cases shown in Figs. 3, 4, where the calculated pressure head differs not more than 6% from the measured heads. More serious divergence appears for the case presented in Fig. 2, where the difference between measured and calculated heads reaches 35%.

6. Influence of Local Energy Loss

As it proceeds from the concept proposed in this paper, the circumferential pressure head, determined by Eq. 23, is caused by the rotation of the valve content. The difference pointed out above (measured values are higher than calculated ones) apparently means, that the observed rise of the overpressure head $H(Q)$ above its calculated value H_V (Q) must be induced by some additional energy loss.

In the hydraulic practice this factor is usually treated as the local energy loss and taken into account by an empirical coefficient ξ_M, as in Eq. 2. So, one can expect that the enlargement of the "rotational uplift" H_V, calculated from Eq. 23, by the "local uplift" H_M:

$$H_M = \xi_m \frac{v_{out}^2}{2g}, \tag{25}$$

should improve the coincidence between theory and empirical results in Figs. 2, 3, 4.

There is a question, how should be determined the coefficient ξ_m – as exactly as possible, but without any individual investigations, using the existing tools of hydraulics. It seems that the main element of the considered structure, inducing the drag force, is a bottom orifice (outflow). Basic value of the discharge coefficient equals about 0.6 in this case (Sawicki 2009). However the experimental value of this parameter for the vortex valve, working without the swirl (i.e. for low discharge), is approximately equal to 0.2 (Kotowski 2011). In this situation a mean value of this coefficient was taken till further calculations (see also Eq. 28):

$$\alpha_M = 0.4, \quad \xi_M = 6.25. \tag{26}$$

This evaluation made possible to calculate the local energy loss and in consequence – the overall value of the pressure head:

$$H(Q) = H_V(\text{Eq. 23}) + H_M(\text{Eq. 25}). \tag{27}$$

As it is seen in Figs. 2, 3, 4, this additional factor didn't change too much for the second and third case (where the "local uplift" is considerably less than the "rotational uplift"), but has evidently improved the situation for the smallest bottom orifice (Fig. 2).

7. Discharge Coefficient for Vortex Valve

As already mentioned, the main goal of this paper is a rational model of the vortex valve operation – formally simple, but possibly precise physically. Such model has been described above.

However in the technical attitude engineers often use simplified relations, like Eqs. 1, 2, containing empirical coefficients of discharge α or local loss coefficient ξ. The relation between these two values one can find comparing Eqs. 1, 2:

$$\alpha = \xi^{-0.5}, \quad \xi = \alpha^{-2}. \tag{28}$$

A good deal of information about these coefficients can be found in (Kotowski and Wójtowicz 2010, Kotowski 2011). In order to make use of these findings, Eq. 23 must be rearranged to the form of Eq. 2. Some evident transformations yield a theoretical, i.e. resulting from the model proposed in this paper, equation describing the coefficient of "rotational uplift" for the vortex valve. It can be written in the following form:

$$\xi_v = \frac{31.7}{\lambda^{1/3}} \left(\frac{d_{in}}{R}\right)^{2/3} \left(\frac{d_{in}}{h}\right)^{2/3} \left(\frac{R}{R - r_w}\right). \tag{29}$$

One has to underline, that Eq. 23 is responsible for this part of the total overpressure, which is a consequence of the rotational motion. In order to obtain the total circumferential overpressure head in the valve chamber, the local energy loss must be added (Eq. 25, 27).

The Eq. 29 enables us to compare the proposed method with the empirical data, mentioned above (Kotowski 2011). On account of the range of investigated variants, it would be rather extensive study, planned for the nearest future. This discussion is confined to the preliminary confrontation of both methods. Such comparison can be easily done for the laboratory stand, described in this paper (Fig. 1) and the technical valve presented by (Kotowski 2011) in series No. 5 (R = 145 mm, h = 82 mm, d_{in} = 50 mm, r_w = 25 mm), for which experimentally determined coefficient of hydraulic loss and discharge coefficient are equal respectively:

$$\xi = 41.6, \quad \alpha = 0.155. \tag{30}$$

For the laboratory stand in turn, the coefficient of "rotational uplift", calculated from Eq. 28 is equal to $\xi_v = 33.5$. Taking into account the coefficient of local hydraulic loss, already calculated (Eq. 26), the total coefficient of hydraulic loss is equal to:

$$\xi = \xi_v + \xi_M = 39.75. \tag{31}$$

Consequently the value of total discharge coefficient (Eq. 28) equals:

$$\alpha = 0.159. \tag{32}$$

Comparison of these results (Eqs. 30, 31, 32) speaks well for the model proposed in this paper.

8. Conclusions

The paper contains a rational mathematical model of the vortex valve operation, where the term "rationality" means a combination of formal simplicity and physical correctness. The core of this model is set by:

– the radial velocity field, given by the condition of the flow continuity (Eq. 4);
– the tangential velocity field (Eq. 19), defined by the empirical data analysis and physical balance of energy fluxes (Eq. 9);
– the pressure head distribution (Eq. 23), calculated from the momentum balance (Eq. 6).

The Eq. 23 enables determination of the circumferential pressure inside the valve chamber. This value has a crucial meaning for the valve operation, as it is a factor which throttles the inflow conduit and decides about the technical function of the valve.

The theoretical relation was verified empirically, on the base of the laboratory stand (Fig. 2, 3) and producer's catalogue characteristics of a technical valve (Fig. 4). Moreover, the discharge coefficient for the valve was calculated theoretically and roughly juxtaposed with some empirical value, taken from the literature.

For each case the results of comparison of theoretical and empirical pieces of information were very positive.

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