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An optimised placement of the hard quality sensors for a robust monitoring of the chlorine concentration in drinking water distribution systems

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Abstract

The problem of an optimised placement of the hard quality sensors in drinking water distribution systems under several water demand scenarios for a robust monitoring of the chlorine concentration is formulated in this paper. The optimality is understood as achieving a desired trade off between the sensors and their maintenance costs and the accuracy of estimation of the chlorine concentration. The contribution of this work is a comprehensive approach to optimised sensor placement by addressing a single, bi and multi-objective problem formulations including a comparison of the proposed methods in terms of the number of hard sensors placed and the performance of the monitoring system. During the design of optimised sensors placement algorithms, the interval observer, recently developed by the authors is applied as the soft sensors. Finally, for the purpose of validating the performance of the algorithms, they are applied to the model of a real drinking water distribution system.

Keywords: algorithms, monitoring, sensors placement, optimization problems, water quality

Principal symbols and abbreviations

DWDS	drinking water distribution system
NSGA-II	Non-dominated Sorting Genetic Algorithm II
$\pm$	mark of upper and lower bounds, respectively
$\lvert(\cdot)\rvert$	number of elements in a set $(\cdot)$
$\lVert\cdot\rVert$	Euclidean norm
$\leq$	mark of element-wise compare

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$\delta_l(\upsilon_\gamma)$	value of the lth objective function at the γth Pareto solution
$\delta_{\mathrm{rv},l}$	relative value of the lth objective function and $l \in L$
$\boldsymbol{\varepsilon}_{c_{\mathrm{out}}}(t)$	measurement error
$\lvert\boldsymbol{\varepsilon}_{c_{\mathrm{out}}}(t)\rvert$	absolute value of measurement error
$\boldsymbol{\varepsilon}_{c_{\mathrm{out}}}^{\max}$	bound of measurement error
η	real, arbitrary, positive and constant parameter
Γ	set of Pareto solutions
Ω_1	set of all nodes in a DWDS
Ω_2	set of all tanks in a DWDS
$\Omega_{\mathrm{E}} \subset \Omega_1$	set of monitored nodes in a DWDS
υ_γ	the γth Pareto solution and $\gamma \in \Gamma$
υ^*	the best solution among Pareto solutions
$\boldsymbol{A}(t) \in \mathbb{R}^{n\times n}$	state matrix
ASR	number of available sensors
$\boldsymbol{b}(t) \in \mathbb{R}^n$	vector of inputs
$c_{\mathrm{f},h}^+(k)$	upper envelope bounding unknown chlorine concentration in the hth tank
$c_{\mathrm{f},h}^-(k)$	lower envelope bounding unknown chlorine concentration in the hth tank
$c_{\mathrm{out},r}^+(k)$	upper envelope bounding unknown chlorine concentration at the rth node
$c_{\mathrm{out},r}^-(k)$	lower envelope bounding unknown chlorine concentration at the rth node
$c_{\mathrm{f},h,ob}^+(k)$	upper envelope bounding unknown chlorine concentration in the hth tank for the obth water demand scenario
$c_{\mathrm{f},h,ob}^-(k)$	lower envelope bounding unknown chlorine concentration in the hth tank for the obth water demand scenario
$c_{\mathrm{out},r,ob}^+(k)$	upper envelope bounding unknown chlorine concentration at the rth node for the obth water demand scenario
$c_{\mathrm{out},r,ob}^-(k)$	lower envelope bounding unknown chlorine concentration at the rth node for the obth water demand scenario
$dis_{\mathrm{rv},\upsilon_\gamma}$	relative distance of the γth Pareto solution from a coordinate system origin
d_t	derivative with respect to t
$f_{\mathrm{p},(\cdot)}$	penalty function identified by $(\cdot)$
$g_{sfr} \in SFR$	decision variable allocating hard sensor to sensor feasible nodes
$h \in \Omega_2$	individual tank
$\boldsymbol{I}$	identity matrix
k	discrete time instant and $k = 1, 2, ..., K$ and $K = \frac{T}{T_{\mathrm{QP}}}$
L	set of objective functions
m	number of measured quality state variables
n	number of quality state variables

$\boldsymbol{N}_1 \in \mathbb{R}^{s \times s}$	invertible matrix proportional to the identity matrix
ob	individual water demand scenario
$P_{(\cdot)}$	positive, real number identified by $(\cdot)$
$P_{r,3}$	positive, real number for the rth node
$P_{h,4}$	positive, real number for the hth tank
$r \in \Omega_{\mathrm{E}}$	individual monitored node
$\mathbb{R}$	set of real numbers
s	number of unmeasured quality state variables
$\boldsymbol{S}^{\pm}$	interval observer
SC	set of all considered water demand scenarios
$SFR \subset \Omega_1$	set of nodes where sensors can be located (sensor feasible nodes)
t	time instant
T	considered time horizon
T_{QP}	quality sampling interval
$\boldsymbol{w}(t)$	auxiliary variable
$\boldsymbol{x}(t) \in \mathbb{R}^n$	vector of quality state variables
$\boldsymbol{x}_1(t) \in \mathbb{R}^s$	vector of unmeasured quality state variables
$\hat{\boldsymbol{x}}_1^{\pm}(t)$	upper and lower bounds on estimated state variables, respectively
$\boldsymbol{x}_2(t) \in \mathbb{R}^m$	vector of measured quality state variables
$\boldsymbol{x}_2^{\pm}(t)$	upper and lower bounds on measured state variables, respectively
$\widetilde{\boldsymbol{x}}_2(t)$	vector of indirectly measured state variables (pseudo measurements)
$X_{1,\max,r}$	upper limit on estimation accuracy at the rth node
$X_{2,\max,h}$	upper limit on estimation accuracy in the hth tank
$\boldsymbol{y}_{\mathrm{cout}}(t)$	vector of measurements

1. Introduction

A drinking water distribution system (DWDS) is rated as one of the Critical Infrastructure Systems that are essential for functioning of modern society and economy [1]. An operation of the DWDS aims at delivering to the users the required amount of water satisfying the quality requirements [2]. Achieving this goal is complicated, therefore, on-line suitable control and monitoring systems are needed. Moreover, two aspects must be taken into account during control and monitoring in the DWDS: quantity and quality of water [3]. They interact but the relationship is only one way, from the hydraulics to the water quality [4]. This was utilised in an integrated approach to control of water quantity and quality presented in [5]. In particular, two-level hierarchical control structure was proposed and investigated. Moreover, the details of designing the lower level controller in the mentioned hierarchical structure was shown in [6]. In turn, the main task of the monitoring system is to provide information on the state of the DWDS. Because, the two above mentioned issues must be considered, from the monitoring point of view, two cascaded systems can be distinguished: the water quantity and the water quality. The robust estimates of the water flow rates and hydraulic model parameters are produced by the quantity monitoring system [7]. Furthermore, these flows estimates are the input data to the water quality models, hence, to the water quality monitoring system [3]. It is worth to add that one of the important elements for designing a water quality monitoring system is the optimised placement of available hard quality sensors. This paper addressed this issue especially.

The water quality in the DWDS can be described by several factors. The most popular one is the disinfectant concentration. At present, the chlorine is commonly used as a disinfectant [3]. The water quality monitoring system exploits water quality measurements in order to gather knowledge on the state of water quality. In typical DWDSs the water quality measurements are made at network nodes and in tanks. Hence, these elements of the DWDS are called nodes or tanks with the hard sensors. The water quality may be measured in laboratories or by using on-line sensors. The bacteriology measurements (e.g. the number of coli bacteria) are the typical laboratory measurements in DWDSs. It is worth to add that modern sensors for on-line bacterial counts measuring will appear and they are tested in DWDSs [8]. However, currently they are not widespread and, therefore, primarily the free chlorine concentrations are measured on-line in DWDSs. Henceforth, this concentration will be considered as the water quality factor in this work. Moreover, without any loss on generality it is assumed that the hard sensors of the chlorine concentration can be placed only at the DWDS nodes. Hence, in this paper the water quality state is meant as a set of the chlorine concentrations in crucial elements (e.g. tanks) of the DWDS.

Unfortunately, placing the chlorine concentration sensors at all DWDS nodes is not possible. It is due to e.g. high costs of hard sensors as well as their maintenance and the access limitations for their installation. Therefore, in this paper the estimates of unmeasured chlorine concentrations called soft sensors are used to complete the measurement information delivered by the hard sensors. Typically, the DWDS is composed of: pumps, valves, pipes, nodes, tanks and reservoirs. The pumps and the valves are used to control the hydraulic quantities that are the water flow rates and pressures. Hence, the DWDS water quality model takes into account changes of the chlorine concentration at the nodes, in the tanks and along the pipes. A problem of changes in water quality in the DWDS was noticed e.g. in [9]. Because the chlorine reacts with organic and non-organic matter in water, the chlorine concentration decreases with time [10]. During formulation of models of the chlorine concentration decay, it is commonly assumed that the hydraulic solution of the DWDS (the values of the water flow rates within the pipes etc.) is known and it is constant over a specified time interval called the hydraulic step. The models of chlorine decay during water transfer through the DWDS can be found in many publications. For instance modelling of changes of the chlorine concentration in tanks was presented in [11]. In turn, a general description modelling and simulation of water quality can be found in [12]. While a comparison between the formulation and computational performance of four numerical methods for modelling chlorine concentration dynamics was shown in [13]. In turn, a continuous lumped model of the chlorine concentration can be found in [3]. Moreover, it is necessary to derive a model of the uncertainty. One of the practical approach is the set-membership [7]. The set bounded estimation with the interval observer of the chlorine concentration recently developed by the authors was presented in [3]. By integrating both models and measurements, the estimation algorithm is obtained. It provides the estimates of unmeasured chlorine concentrations. It is necessary to add that the preliminary results regarding the hard sensors placement clearly show that under the same number of sensors, the accuracy of resulting water quality estimates can vastly differ depending on the nodes of the DWDS where the sensors are located [14]. Thereupon, this work considers the optimised placement of the hard sensors of the chlorine concentration for robust monitoring of the water quality in the DWDS.

The problem of sensor placement is especially addressed for network systems such as systems of distribution of oil, electricity, water and gas [15]. According to the subject of the paper the problem of allocation of the water quality hard sensors has been investigated. In literature this issue has been presented for years. However, until today, there is no universal method of determining their location [16]. In many cases, much of research has focused on deriving such algorithms of the sensors placement that a presumptive contamination of water in the DWDS would be detected as soon as possible. Because of the threat of terrorist attacks this is still a timely issue. The proposed algorithms differ especially in a number and formulations of the indicators and the mathematical tools. A single as well as a multi-objective optimisation is one of the most popular techniques for solving the hard sensors placement problem. For example five objective functions: population exposed, time to detection, volume consumed, number of failed detections and an extent of contamination are defined in [17]. For each of them, a mixed-integer linear programming (MILP) is formulated. Next, an analysis of relationships between received results is performed and the compromised solution is chosen. An another approach can be found in [18]. There an expected fraction of the population that is at risk from an attack is minimised. The problem was formulated as the MILP and in order to solve it the CPLEX solver was used. Moreover, during formulation of the optimisation problem, several water demand scenarios at the DWDS nodes were taken into account. The methodology was further developed by using the stochastic models in [19]. In turn, the formulation of the sensors allocation problem as the multi-objective optimisation problem which was solved by using a solver for the single-objective can be found in [20]. Clearly, three objective functions refer to: expected time of detection, expected contaminated water demand prior to detection and expected likelihood of detection are defined. It is worth to add that the detailed formulating of the last two objective function was presented in [16]. Then, the multi-objective problem is reformulated by using an aggregation method. Afterwards, it is minimised subject to an available number of the hard sensors by using a dedicated genetic algorithm. Another approach based on the hydraulic analysis of the DWDS to construct a bipartite graph between intrusion points and the suitable nodes that can potentially be polluted by the contaminant was shown in [21].

Considering the short survey presented in the above paragraph, the following conclusions can be drawn. Firstly, the information is delivered from the hard sensors located in the DWDS in way that might be insufficient for the water quality monitoring purposes. Secondly, due to obvious reasons, the number of hard sensors and their cost was not minimised. Therefore, the algorithms for the hard sensors placement dedicated for the monitoring systems have appeared.

For instance a formulation based on maximising a demand coverage was presented in [22]. It was developed in [23]. A given area represents a percent of the total demand in the DWDS, which is monitored by placing sensor. A concept of pathways and a property referring to decay of the disinfectant concentration are used in the algorithm. Clearly, if the water quality is proper at the monitoring node and the outflow from a given upstream node (inflow to the monitoring node) covers an appropriate percent of the demand at this monitoring node, the water quality is also proper at the upstream node. This suitable percent results from the demand coverage parameter and typically is between 50 - 75%. Hence, the hydraulic solution of the DWDS as well as the flow directions are required by the algorithm. Additionally, during formulation of the maximisation problem, several water demand scenarios at the DWDS nodes can be taken into account. The optimisation problem was formulated as the integer programming which is, in this case, time-consuming. For this reason the more effective numerical procedures was shown in [24]. Whereas a genetic algorithm was used in [25]. However, the proposed algorithm is marked by the appearance of uncovered areas and the quantitatively information is available only at the monitoring nodes.

In previous research papers, an alternative approach to the optimised placement of the hard sensors of the chlorine concentration in the DWDS was presented. In general, this placement achieves a desired trade off between the sensors and their maintenance costs and the accuracy of estimation of the chlorine concentration. Moreover, several water demand scenarios are taken into account during the placing of the sensors in the DWDS. Each of the water demand scenario is composed of the water demand patterns at the nodes representing the users of water within a given area in the DWDS. Clearly, one of the different possibilities to assess the water demand pattern is based on dividing the DWDS into suitable parts and assigning one cumulated consumer to each of them [12] and [26]. A crucial part of the proposed algorithms is the interval observer used to produce the robust estimates of unmeasured chlorine concentrations [3]. The robustness has been achieved by employing the set bounded model of the uncertainty in: system dynamics, inputs, initial conditions and measurement errors. The results obtained by solving a bi-objective sensor allocation problem have been given in [27]. The approach was extended and introduced as a multi-objective allocation problem in [28]. The contribution of this work is a comprehensive approach to optimised sensor placement by addressing single, bi and multi-objective problem formulations. This includes also a comparison of the proposed methods in terms of the obtained results, namely the number of hard sensors placed within DWDS as well as the performance of the monitoring system (interval observer bounds). The detailed research on modelling, estimation with the interval observer and the optimised robust placement of the hard sensors of the chlorine concentration can be found in [29].

The paper is organised as follows. The model of the water quality for estimation purposes and the interval observer for the chlorine concentration are presented in section 2. Next, the single, bi and multi-objective formulations of the problem of the optimised placement of the hard sensors are derived in section 3. Next, the genetic solvers of the placement are described in section 4. In section 5 the proposed methodology is applied to the water quality estimation in the real DWDS and its performance is validated by simulation. The conclusions in section 6 complete this paper.

2. Optimised placement of the hard sensors - models

2.1. Model of the chlorine concentration in the DWDS

According to [9–12] the DWDS water quality model is composed of: a system of algebraic equations that describes changes of the chlorine concentration at the nodes, a system of ordinary differential equations that describes the chlorine decay in the tanks and a system of partial differential equations that describes distribution of the chlorine concentration along the pipes. There are several methods which can be used in order to solve this model [13]. One of them was comprehensively presented in [3]. By using this approach it is possible to obtain a time continuous lumped model of the chlorine concentration for the estimation purposes in the DWDS. Henceforth, the chlorine concentration dynamics throughout entire DWDS can be written as [3]:

$$\mathrm{d}_t\boldsymbol{x}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{b}(t), \tag{1}$$

where: t is the time instant; d_t denotes the derivative with respect to t; $\boldsymbol{x}(t) \in \mathbb{R}^n$ stands for the vector of the quality state variables representing the chlorine concentrations at all pipe segment ends and in the tanks; n denotes the number of quality state variables; $\boldsymbol{A}(t) \in \mathbb{R}^{n\times n}$ is the time varying state matrix which elements are composed of the hydraulic quantities, the lengths of pipe segments and the reaction rate coefficients; $\boldsymbol{b}(t) \in \mathbb{R}^n$ denotes the time varying vector of inputs which elements are dependent on the hydraulic quantities, the lengths of pipes segments, the quality quantities in the reservoirs and injection of the chlorine at the quality controlled nodes.

Remark 1. The chlorine concentrations at the nodes and in the tanks are seen as the most important and they are viewed as the DWDS quality outputs. However, only the state variables representing the chlorine concentrations at the nodes supplied by only one pipe and in the tanks are directly transferred to system output. In contrast, the chlorine concentrations at the nodes with several connected pipes are linear combinations of the appropriate state variables. Therefore, the chlorine concentrations at these nodes are calculated based on the suitable state variables [3].

2.2. Interval observer for the chlorine concentration in the DWDS

The detailed description of the set bounded estimation with the interval observer for the chlorine concentration assessment purposes in the DWDS was presented in [3]. The methodology utilises the water quality model (1) and takes into account the uncertainty in: the system dynamics (matrix $\boldsymbol{A}(t)$), the inputs (vector $\boldsymbol{b}(t)$), the initial conditions and the measurement errors. Hence, the following structure of the interval observer was derived [3]:

$$\left(\boldsymbol{S}^{\pm}\right) : \begin{cases} \mathrm{d}_t\boldsymbol{w}^{\pm}(t) = \boldsymbol{A}_{11}^{\pm}(t)\boldsymbol{w}^{\pm}(t) + \boldsymbol{N}_1\boldsymbol{A}_{12}^{\pm}(t)\boldsymbol{x}_2^{\pm}(t) + \boldsymbol{M}\boldsymbol{v}^{\pm}(t) \\ \boldsymbol{w}^{\pm}(0) = \boldsymbol{N}\boldsymbol{x}^{\pm}(0) \\ \hat{\boldsymbol{x}}_1^{\pm}(t) = \boldsymbol{N}_1^{-1}\boldsymbol{w}^{\pm}(t) \end{cases}, \tag{2}$$

where: the mark $\pm$ is to distinguish between the upper and the lower bounds, the former is given by taking variables and operators with upper parts of the mark while the latter by taking their lower parts; $\boldsymbol{x}(t) = [\boldsymbol{x}_1(t) \quad \boldsymbol{x}_2(t)]^{\mathrm{T}} \in \mathbb{R}^n$ and $\boldsymbol{x}_1(t) \in \mathbb{R}^s$, $\boldsymbol{x}_2(t) \in \mathbb{R}^m$ are the vectors of unmeasured and measured state variables, respectively and $s = n - m$; $\hat{\boldsymbol{x}}_1^{\pm}(t)$ denote the upper and the lower bounds on the estimated state variables; $\boldsymbol{w}(t)$ is the auxiliary variable, defined as: $\boldsymbol{w}(t) = \boldsymbol{N}\boldsymbol{x}(t)$; $\boldsymbol{N} = [\boldsymbol{N}_1 \quad \boldsymbol{0}]$; $\boldsymbol{N}_1 = \eta \boldsymbol{I} \in \mathbb{R}^{s\times s}$ denotes the invertible matrix proportional to the identity matrix and η is the real, arbitrary, positive and constant parameter; $\boldsymbol{M} = [\boldsymbol{N}_1 \quad \boldsymbol{0} \quad \boldsymbol{0}]$; $\boldsymbol{v}^{\pm}(t) = \left[\boldsymbol{b}_1^{\pm}(t) \quad \frac{1}{2}\boldsymbol{B}_1 \quad \pm\frac{1}{2}\boldsymbol{B}_2\right]^{\mathrm{T}}$; $\boldsymbol{B}_1 = \boldsymbol{b}_2^{+}(t) + \boldsymbol{b}_2^{-}(t)$; $\boldsymbol{B}_2 = \boldsymbol{b}_2^{+}(t) - \boldsymbol{b}_2^{-}(t)$; $\boldsymbol{A}_{11}(t) = \{a_{i,j}\}$, $\boldsymbol{A}_{12}(t) = \{a_{i,q}\}$, $\boldsymbol{A}_{21}(t) = \{a_{q,j}\}$, $\boldsymbol{A}_{22}(t) = \{a_{q,q}\}$, $\forall i, j \in \overline{1, s}$, $\forall q \in \overline{s+1, m}$ are suitable parts of the matrix $\boldsymbol{A}(t)$ structured by the measurement state variables: $\boldsymbol{A}(t) = \begin{array}{c} \textit{est.} \\ \textit{meas.} \end{array} \begin{bmatrix} \boldsymbol{A}_{11}(t) \in \mathbb{R}^{s\times s} & \boldsymbol{A}_{12}(t) \in \mathbb{R}^{s\times m} \\ \boldsymbol{A}_{21}(t) \in \mathbb{R}^{m\times s} & \boldsymbol{A}_{22}(t) \in \mathbb{R}^{m\times m} \end{bmatrix}_{n\times n}$; $\boldsymbol{b}_1(t) = \{b_i\}$, $\boldsymbol{b}_2(t) = \{b_q\}$ denote suitable parts of the vector $\boldsymbol{b}(t)$ structured by the measurement state variables: $\boldsymbol{b}(t) = \begin{array}{c} \textit{est.} \\ \textit{meas.} \end{array} \begin{bmatrix} \boldsymbol{b}_1(t) \in \mathbb{R}^{s} \\ \boldsymbol{b}_2(t) \in \mathbb{R}^{m} \end{bmatrix}_{n}$; $\boldsymbol{x}_2^{\pm}(t)$ denote the upper and the lower bounds of the measured state variables given by: $\boldsymbol{x}_2^{-}(t) \leq \boldsymbol{x}_2(t) \leq \boldsymbol{x}_2^{+}(t)$, where: $\boldsymbol{x}_2^{\pm}(t) = \boldsymbol{y}_{\mathrm{c_{out}}}(t) \pm \boldsymbol{\varepsilon}_{\mathrm{c_{out}}}^{\max}$, and: $\boldsymbol{y}_{\mathrm{c_{out}}}(t)$ is the vector of measurements and $\boldsymbol{\varepsilon}_{\mathrm{c_{out}}}(t)$ is the measurement error bound by: $|\boldsymbol{\varepsilon}_{\mathrm{c_{out}}}(t)| \leq \boldsymbol{\varepsilon}_{\mathrm{c_{out}}}^{\max}$; $\leq$ is understood element-wise.

The interval observer (2) produces stable and robust upper $\hat{\boldsymbol{x}}_1^{+}(t)$ and lower $\hat{\boldsymbol{x}}_1^{-}(t)$ envelopes, bounding the unmeasured state variables $\boldsymbol{x}_1(t)$ in spite of the uncertainty in the: inputs (chlorine measurements at reservoirs), initial conditions, state measurements (chlorine measurements at network nodes) and state matrix $\boldsymbol{A}(t)$ in a linear part of the system dynamics. The proof can be found in [3].

One should notice that, the chlorine concentration measurements are the state measurements only at certain DWDS nodes (see remark 1). Therefore, the interval observer (2) was further developed in order to handle the case where the chlorine concentration sensors are located also at the nodes with several connected pipes. In this case the vector of measurements is (3), where: $\overline{\boldsymbol{x}}_2(t) = [\boldsymbol{x}_2(t) \quad \widetilde{\boldsymbol{x}}_2(t)]^{\mathrm{T}}$, and $\widetilde{\boldsymbol{x}}_2(t)$ denotes the vector of indirectly measured state variables and is called the vector of pseudo measurements:

$$\boldsymbol{y}_{\mathrm{c_{out}}}(t) = \overline{\boldsymbol{x}}_2(t). \tag{3}$$

Hence, the letter structure of the interval observer was introduced [3]:

$$\left(\boldsymbol{S}^{\pm}\right) : \begin{cases} \mathrm{d}_t \boldsymbol{w}^{\pm}(t) = \boldsymbol{A}_{11}^{\pm}(t)\boldsymbol{w}^{\pm}(t) + \boldsymbol{N}_1\boldsymbol{A}_{12}^{\pm}(t)\overline{\boldsymbol{x}}_2^{\pm}(t) + \boldsymbol{M}\boldsymbol{v}^{\pm}(t) \\ \boldsymbol{w}^{\pm}(0) = \boldsymbol{N}\boldsymbol{x}^{\pm}(0) \\ \hat{\boldsymbol{x}}_1^{\pm}(t) = \boldsymbol{N}_1^{-1}\boldsymbol{w}^{\pm}(t) \end{cases}. \tag{4}$$

The interval observer (4) produces stable and robust upper $\hat{\boldsymbol{x}}_1^{+}(t)$ and lower $\hat{\boldsymbol{x}}_1^{-}(t)$ envelopes, bounding the unmeasured state variables $\boldsymbol{x}_1(t)$ in spite of the uncertainty in the: inputs, initial conditions, direct and indirect state measurements and state matrix $\boldsymbol{A}(t)$ in the linear part of the system dynamics. The proof can be found in [3].

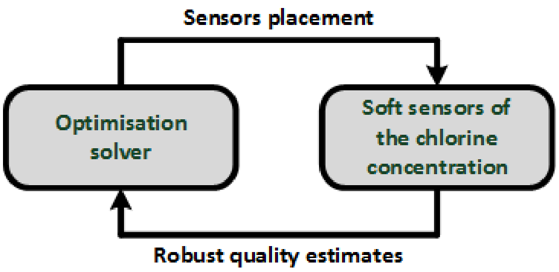

Figure 1: General structure of the optimised hard sensors placement algorithm in the DWDS.

Remark 2. In order to estimate the unmeasured state variables by using the interval observer (4), the necessity of calculating the pseudo measurements appears. Hence, the following procedure was proposed [3]:
S1: The estimation of unmeasured state variables by using the interval observer (2) is performed. During this estimation process only direct state measurements are used.
S2: The pseudo measurements of indirectly measured state variables are calculated. Next, the state variables that refer to pseudo measurements are removed from the estimated states and the vector of measurements $\boldsymbol{x}_2(t)$ is augmented by adding the vector of pseudo measurements $\widetilde{\boldsymbol{x}}_2(t)$ to produce the interval observer (4).
The pseudo measurements are the new, important source of information. Hence, the two-step estimation algorithm has better performance than the estimation process without the indirect state measurements.

3. Optimised placement of the hard sensors - problem formulation

The optimised placement of the chlorine concentration hard sensors enables to achieve the desired compromise between the overall sensors cost and the resulting accuracy of the robust quality estimates. The interval observer delivers the robust chlorine concentration estimates in a form of lower and upper bounds envelopes of the unknown chlorine trajectories. Hence, the tighter the bounding intervals are the more accurate the estimates are. The estimation accuracy needs to be traded off against the hard sensor costs. Therefore, a sensible approach is to formulate the sensors allocation problem as a multi-objective constrained optimisation task with the use of e.g. Pareto definition of optimality. Such formulations in the form of bi and multi-objective optimisation tasks are proposed and further discussed. The difference between bi and multi-objective formulation is that the bi-objective optimised hard sensors placement problem was formulated and solved under the one water demand scenario. However, the formulation with one performance function can be also useful. Clearly, the objective function expressing the hard sensor costs by specifying how many of them are located in the DWDS can be applied. Nevertheless, a proper quality of estimates should be guaranteed by a suitable choice of the estimation accuracy. This so-called single-objective formulation approach is shown in section 3.1.

A general structure of the algorithm solving the problem is illustrated in Fig. 1. The algorithm starts from an initially chosen hard sensors placement pattern in the DWDS with a given water demand scenario. The measurements of chlorine concentrations at the measurement nodes and in the reservoirs as well as the hydraulic solution are delivered by a well-known EPANET simulator. The EPANET DWDS quantity-quality model equations can be considered as a faithful representation of reality. Next, the obtained values of the chlorine concentrations and the hydraulic quantities from the EPANET are distorted by about $\pm\Delta\%$. Clearly, the above mentioned operation is performed for the purpose of simulating the water quantity monitoring system and drawing the chlorine concentrations at the measurement nodes and in the reservoirs (with the set-membership description of uncertainty). They are necessary for monitoring of water quality in the DWDS. Hence, they are the input data to the interval observer of the chlorine concentration in the DWDS. The interval observer produces the estimates of the chlorine concentrations over a whole DWDS. These estimates are used by the optimisation solver. The optimiser can then evaluate the fitness function of the hard sensors placement and produce a better one or stop the algorithm.

3.1. Single-objective formulation

The hard sensors allocation problem is formulated as the single-objective constrained optimisation task to minimise the number of sensors placed at the feasible nodes of the DWDS. The mathematical formulation of this problem is:

$$\min \left\{ Z = \sum_{sfr=1}^{|SFR|} g_{sfr} \right\}, \tag{5a}$$

$$\text{subject to:} \qquad \sum_{sfr=1}^{|SFR|} g_{sfr} \leq ASR, \tag{5b}$$

$$\begin{aligned} \left[c^{+}_{\mathrm{out},r}(k) - c^{-}_{\mathrm{out},r}(k)\right] &\leq X_{1,\max,r}, \\ \left[c^{+}_{\mathrm{f},h}(k) - c^{-}_{\mathrm{f},h}(k)\right] &\leq X_{2,\max,h}, \end{aligned} \tag{5c}$$

where: g_{sfr} is a decision variable allocating hard sensor to the sensor feasible nodes, $sfr \in SFR$, $g_{sfr} \in \{0,1\}$ and if $g_{sfr} = 1$ the sensor is placed at the sfrth node; SFR denotes a set of nodes where the sensors can be located (sensor feasible nodes) and $|SFR|$ stands for a number of nodes in SFR, $SFR \subset \Omega_1$; Ω_1 is a set of all nodes in the DWDS and $|\Omega_1|$ signifies a number of nodes in Ω_1; Ω_2 denotes a set of all tanks in the DWDS and $|\Omega_2|$ is a number of tanks in Ω_2; Ω_{E} stands for a set of monitored nodes where there are not hard sensors and $|\Omega_{\mathrm{E}}|$ signifies the number of nodes in Ω_{E}, $\Omega_{\mathrm{E}} \subset \Omega_1$; ASR is a number of available sensors; $c^{+}_{\mathrm{out},r}(k)$, $c^{-}_{\mathrm{out},r}(k)$ denote the upper and lower envelopes bounding the unknown chlorine concentration at the rth node at the discrete time instant k, respectively, $r \in \Omega_{\mathrm{E}}$; $c^{+}_{\mathrm{f},h}(k)$, $c^{-}_{\mathrm{f},h}(k)$ are the upper and lower envelopes bounding the unknown chlorine concentration in the hth tank at the discrete time instant k, respectively, $h \in \Omega_2$; $k = 1, 2, ..., K$ is a discrete time instant imposed by the quality sampling interval (T_{QP}) to produce the estimates at these time instances, $K = \frac{T}{T_{\mathrm{QP}}}$, T denotes a considered time horizon; $X_{1,\max,r}$, $X_{2,\max,h}$ stand for the upper limits on estimation accuracy (a maximal allowed width of the bounding intervals) at the rth node and in the hth tank, respectively.

The choice of the upper limits $X_{1,\max}$ and $X_{2,\max}$ on the estimation accuracy is very important. Moreover, they can be the same or differ depending on how significant the water user at a particular demand node is. Determining the upper limits is not always an obvious task. Too small values of $X_{1,\max}$ and $X_{2,\max}$ may lead to a lack of solution of (5), because the constraints (5c) will not be fulfilled. In turn, too relaxed limits may cause that the estimation accuracy will not be acceptable despite the hard sensors placement will be desirable (a few sensors will be located in the DWDS).

Moreover, limiting the SFR set to the most important nodes e.g. from the chlorine propagation point of view significantly increases the computational efficiency. It can be done by using the operator experience or by applicability different tools e.g. fractal geometry [30].

During formulation of the single-objective optimisation problem only one water demand scenario is taken into account. Hence, in this case, the best results are obtained in the DWDSs where the water demand scenarios are repeatable. Clearly, for a definite hard sensors placement if the water demand patterns at DWDS nodes widely change the quality of information from the chlorine concentration monitoring system might be unsatisfying. In the following part of the text, the main considerations are focused on the bi and multi-objective formulations. Hence, only reformulating of the single-objective optimisation task for genetic algorithm purposes is shown is section 4.

3.2. Bi-objective formulation

In this formulation, the objective function related to the estimation accuracy is traded off against the overall sensor costs maintaining the same priority of importance. The mathematical formulation of this problem is:

$$
\begin{aligned}
&\min \left\{ Z_0 = \sum_{sfr=1}^{|SFR|} g_{sfr} \right\}, \\
&\min \left\{ Z_1 = \sum_{r=1}^{|\Omega_{\mathrm{E}}|} \sum_{k=1}^{K} \left[c^{+}_{\mathrm{out},r}(k) - c^{-}_{\mathrm{out},r}(k) \right] + \sum_{h=1}^{|\Omega_2|} \sum_{k=1}^{K} \left[c^{+}_{\mathrm{f},h}(k) - c^{-}_{\mathrm{f},h}(k) \right] \right\}, \qquad (6) \\
&\text{subject to:} \quad \sum_{sfr=1}^{|SFR|} g_{sfr} \leq ASR.
\end{aligned}
$$

One property of the bi-objective formulation is that the upper limits $X_{1,\max}$ and $X_{2,\max}$ are not needed *a priori*. It is because, the estimation accuracy (the width of estimated intervals) is one of the objective functions. Therefore, this formulation has only one constraint referring to the number of available sensors (ASR). However, one needs to be aware that the computational time might be longer than in the single-objective optimisation task. A dedicated solver is employed in order to solve the bi-objective constrained optimisation problem e.g. [31]. The Pareto definition of the optimality is utilised in this approach, hence, the final solution will be selected from the Pareto front, where there are solutions representing the particular hard sensors placement patterns. The system user makes a decision on which of the achievable solutions will be preferred.

Because the decision making process is almost always difficult the decision support tools are very useful for the decision makers. In this paper the following procedure is taken for these purposes [32]:

Step 1: Relative values of the objective functions for a given Pareto solution are calculated:

$$\delta_{\mathrm{rv},l} = \frac{\delta_l(\upsilon_\gamma)}{\max\limits_{\gamma}\left(\delta_l(\upsilon_\gamma)\right)}, \tag{7}$$

where: $\delta_{\mathrm{rv},l}$ denotes the relative value of the lth objective function, $l \in L$; L is a set of the objective functions; υ_γ stands for the γth Pareto solution, $\gamma \in \Gamma$; Γ is a set of the Pareto solutions; $\delta_l(\upsilon_\gamma)$ signifies the value of lth objective function at the γth Pareto solution.

Step 2: Relative distances of the Pareto solutions from the coordinate system origin are calculated:

$$dis_{\mathrm{rv},\upsilon_\gamma} = \|\boldsymbol{\delta}_{\mathrm{rv}}\|, \tag{8}$$

where: $dis_{\mathrm{rv},\upsilon_\gamma}$ is the relative distance of the γth Pareto solution from the coordinate system origin; $\|\cdot\|$ denotes the Euclidean norm.

Step 3: The best solution among the Pareto solutions is chosen:

$$\upsilon^* = \min_{\gamma}(dis_{\mathrm{rv},\upsilon_\gamma}). \tag{9}$$

One should notice that analogously as in the single-objective formulation also in the bi-objective formulation of the hard sensors allocation problem only one water demand scenario is taken into account. Hence, from this point of view, the bi-objective approach has the same features.

3.3. Multi-objective formulation

The optimised hard sensors placement problem is formulated as the multi-objective constrained optimisation task subject to several water demand scenarios:

$$\begin{aligned}
&\min\left\{Z_0 = \sum_{sfr=1}^{|SFR|} g_{sfr}\right\},\\
&\min\left\{Z_{ob} = \sum_{r=1}^{|\Omega_{\mathrm{E}}|}\sum_{k=1}^{K}\left[c^{+}_{\mathrm{out},r,ob}(k) - c^{-}_{\mathrm{out},r,ob}(k)\right] + \sum_{h=1}^{|\Omega_2|}\sum_{k=1}^{K}\left[c^{+}_{\mathrm{f},h,ob}(k) - c^{-}_{\mathrm{f},h,ob}(k)\right]\right\},\\
&\text{subject to:}\quad \sum_{sfr=1}^{|SFR|} g_{sfr} \le ASR, \quad \forall ob \in SC,
\end{aligned} \tag{10}$$

where: ob is an individual water demand scenario; $SC = \overline{1, sc}$ signifies a set of all considered water demand scenarios and in consequence $L = SC \cup \{0\}$; $c^{+}_{\mathrm{out},r,ob}(k)$, $c^{-}_{\mathrm{out},r,ob}(k)$ denote the upper and lower envelopes bounding the unknown chlorine concentrations at the rth node at the discrete time instant k for obth water demand scenario, respectively; $c^{+}_{\mathrm{f},h,ob}(k)$, $c^{-}_{\mathrm{f},h,ob}(k)$, are the upper and lower envelopes bounding the unknown chlorine concentrations in the hth tank at the discrete time instant k for obth water demand scenario, respectively.

Because several water demand scenarios are taken into account the hydraulic conditions in the DWDS change. Therefore, the necessity of using more than one interval observer during estimation process appears. Clearly, the appropriate interval observer is used for a given water demand scenario. The structures of the chlorine concentration interval observers are similar, but the values of the hydraulic quantities are different [3].

Analogously as in the bi-objective formulation also in the multi-objective formulation of the hard sensors allocation problem the optimality is defined in the Pareto sense and the final solution is chosen by using the procedure (7)-(9). However, one needs to be aware that the computational time will be the longest of all approaches presented in this paper.

Obviously, the allocation of the hard sensors being the result of the multi-objective approach is not robust with respect to change of the DWDS structure. For example, if a new control algorithm is applied in the DWDS or the chlorine booster stations are located, the given number and/or locations of sensors may turn out to be too small for obtaining the proper water quality monitoring purposes. Of course, as an opposite situation, it might happen that the sensors number will be excessive. Such cases might require solving of the hard sensors allocation problem once again.

4. Solvers for the sensors placement tasks

The binary decision variables and the real as well as integer valued constraints mark the optimisation tasks corresponding to the: single, bi and multi-objective formulations of the hard sensors allocation problem defined by (5), (6) and (10). Moreover, the optimisation tasks defined by (6) and (10) have two or more objective functions. Hence, e.g. evolutionary algorithms can be used in order to solve this problem. Therefore, a genetic algorithm NSGA-II (Non-dominated Sorting Genetic Algorithm II) has been chosen as a solver in this work. Applied to the bi and multi-objective optimisation tasks, the algorithm determines a solution set optimal in the Pareto sense [31]. NSGA-II has been already successfully applied for solving the problem of sensor network design [33]. Moreover, it was used for an optimised placement of the water quality control system actuators (the chlorine booster stations) in exemplary DWDSs as well as in the model of a real DWDS [34] and [32], respectively.

In order to apply the NSGA-II the bi-objective optimisation task (6) has to be reformulated:

$$
\begin{aligned}
Z_0^{'} &= \sum_{sfr=1}^{|SFR|} g_{sfr} + f_{\mathrm{p},1} + f_{\mathrm{p},2}, \\
Z_1^{'} &= \sum_{r=1}^{|\Omega_{\mathrm{E}}|} \sum_{k=1}^{K} \left[c_{\mathrm{out},r}^{+}(k) - c_{\mathrm{out},r}^{-}(k) \right] + \sum_{h=1}^{|\Omega_2|} \sum_{k=1}^{K} \left[c_{\mathrm{f},h}^{+}(k) - c_{\mathrm{f},h}^{-}(k) \right] + f_{\mathrm{p},1} + f_{\mathrm{p},2},
\end{aligned}
\tag{11}
$$

where: $Z_0^{'}$, $Z_1^{'}$ are the objective functions to be minimised in the Pareto sense; $f_{\mathrm{p},1}$, $f_{\mathrm{p},2}$ denote the penalty functions.

The first penalty function $f_{\mathrm{p},1}$ handles the constraint on the number of available hard sensors and it is defined as:

$$
f_{\mathrm{p},1} = \begin{cases} 0 & \textit{if} \quad \sum_{sfr=1}^{|SFR|} g_{sfr} \leq ASR \\ P_1 & \textit{if} \quad \sum_{sfr=1}^{|SFR|} g_{sfr} > ASR \end{cases}, \tag{12}
$$

where: P_1 is a positive, real number.

Next, the penalty function $f_{\mathrm{p},2}$ forces placement of at least one hard sensor and it is defined as:

$$
f_{\mathrm{p},2} = \begin{cases} 0 & \textit{if} \quad \sum_{sfr=1}^{|SFR|} g_{sfr} \neq 0 \\ P_2 & \textit{if} \quad \sum_{sfr=1}^{|SFR|} g_{sfr} = 0 \end{cases}, \tag{13}
$$

where: P_2 is a positive, real number.

For the purpose of applying the NSGA-II the multi-objective optimisation task (10) has to be reformulated:

$$
\begin{aligned}
Z_0^{'} &= \sum_{sfr=1}^{|SFR|} g_{sfr} + f_{\mathrm{p},1} + f_{\mathrm{p},2}, \\
Z_{ob}^{'} &= \sum_{r=1}^{|\Omega_{\mathrm{E}}|} \sum_{k=1}^{K} \left[c_{\mathrm{out},r,ob}^{+}(k) - c_{\mathrm{out},r,ob}^{-}(k) \right] + \sum_{h=1}^{|\Omega_2|} \sum_{k=1}^{K} \left[c_{\mathrm{f},h,ob}^{+}(k) - c_{\mathrm{f},h,ob}^{-}(k) \right] + f_{\mathrm{p},1} + f_{\mathrm{p},2},
\end{aligned}
\tag{14}
$$

where: $Z_0^{'}$, $Z_{ob}^{'}$ are the objective functions to be minimised in the Pareto sense.

As it has been stated in the previous section, the selection of the final hard sensors placement from the Pareto front will be done by a decision maker. In this paper, the decision making process is supported by the procedure of a minimum distance from the coordinate system origin (7)-(9).

In order to be consistent with section 3.1, the single-objective optimisation task (5) can be reformulated as follows:

$$Z' = \sum_{sfr=1}^{|SFR|} g_{sfr} + f_{\mathrm{p},1} + f_{\mathrm{p},2} + f_{\mathrm{p},3} + f_{\mathrm{p},4}, \tag{15}$$

where: Z' is the objective function to be minimised; $f_{\mathrm{p},3}$, $f_{\mathrm{p},4}$ denote the penalty functions.

The penalty functions $f_{\mathrm{p},3}$ and $f_{\mathrm{p},4}$ handle the constraints (5c) on guaranteed estimation accuracy at the rth node and in the hth tank, respectively and they can be defined as:

$$\begin{aligned} f_{\mathrm{p},3} &= \sum_{r\in\Omega_{\mathrm{E}}} P_{r,3}\left(\min\left[0,\left(X_{1,\max,r} - \max_{k}\left[c^{+}_{\mathrm{out},r}(k) - c^{-}_{\mathrm{out},r}(k)\right]\right)\right]\right)^{2} \\ f_{\mathrm{p},4} &= \sum_{h\in\Omega_{2}} P_{h,4}\left(\min\left[0,\left(X_{2,\max,h} - \max_{k}\left[c^{+}_{\mathrm{f},h}(k) - c^{-}_{\mathrm{f},h}(k)\right]\right)\right]\right)^{2} \end{aligned}, \tag{16}$$

or

$$\begin{aligned} f_{\mathrm{p},3} &= \sum_{r\in\Omega_{\mathrm{E}}}\sum_{k=1}^{K} P_{r,3}\left(\min\left[0,\left(X_{1,\max,r} - \left(c^{+}_{\mathrm{out},r}(k) - c^{-}_{\mathrm{out},r}(k)\right)\right)\right]\right)^{2} \\ f_{\mathrm{p},4} &= \sum_{h\in\Omega_{2}}\sum_{k=1}^{K} P_{h,4}\left(\min\left[0,\left(X_{2,\max,h} - \left(c^{+}_{\mathrm{f},h}(k) - c^{-}_{\mathrm{f},h}(k)\right)\right)\right]\right)^{2} \end{aligned}, \tag{17}$$

where: $P_{r,3}$, $P_{h,4}$ denote positive, real numbers for the rth node and hth tank, respectively.

5. Application in Chojnice DWDS case study

The algorithms of the hard sensors allocation were applied to the model of a real DWDS. The considered DWDS (Fig. 2) is located in Chojnice [35]. This model has been also successfully applied for other research purposes e.g. an allocation of the chlorine booster stations, leakage detection and their localisation in the DWDS by using the multiregional PCA or the kernel PCA [32], [36] and [37], respectively. Chojnice DWDS delivers water to about 40,000 inhabitants. The number of particular elements in Chojnice DWDS is: 177 nodes, 271 pipes, 2 reservoirs, 1 tank and 3 pumps. The water is provided to the network from two sources, which are modelled as the reservoirs of treated water with constant chlorine concentration. There are no water quality controlled nodes. During the modelling process, the Chojnice DWDS was divided into seven parts and one cumulated consumer was assigned to each part. Consequently, there are seven water demand nodes in the presented model. These water demand nodes are clearly marked in Fig. 2. The demand values as well as the demand patterns at these nodes are based on real data and are distinguished by two types of demand pattern at nodes: 31, 39, 67, 83, 88 and 60, 70 (Figs. 3 and 4). Moreover, these patterns constitute two water demand scenarios: 1 and 2, respectively. The Chojnice DWDS model was implemented in the EPANET to produce the input data to the optimised hard sensors placement algorithms. Clearly, the demand patterns (Figs. 3 and 4) are used

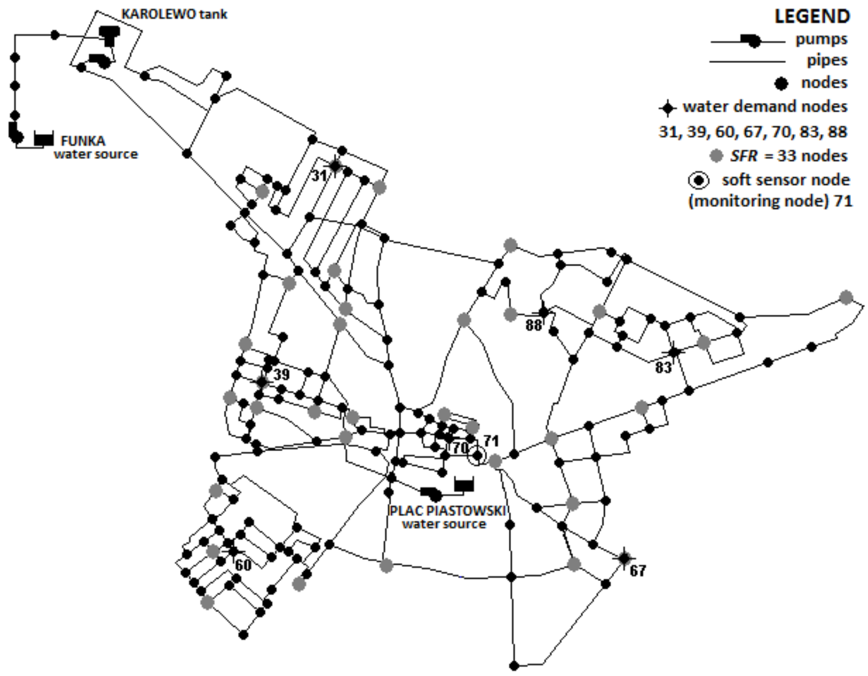

Figure 2: Chojnice drinking water distribution system.

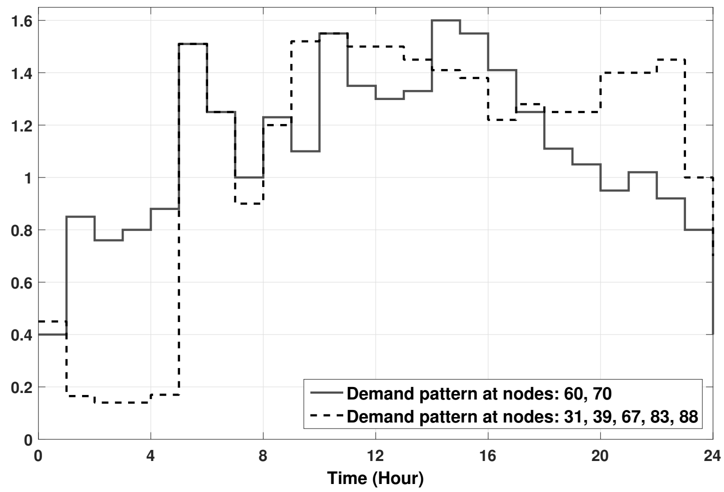

Figure 3: The water demand scenario 1.

in order to obtain the nominal values of hydraulic quantities from the EPANET and then they are distorted by ±2% (this spans a range which encloses a typical hydraulic quantities measurement error in DWDSs). The chlorine concentration measurements and the chlorine concentrations in the reservoirs are provided by the EPANET also. However, reaction rate coefficients were changed. The chlorine concentration measurements are contaminated by the measurement error of ±2% (similarly, it is a typical error value for chlorine concentration

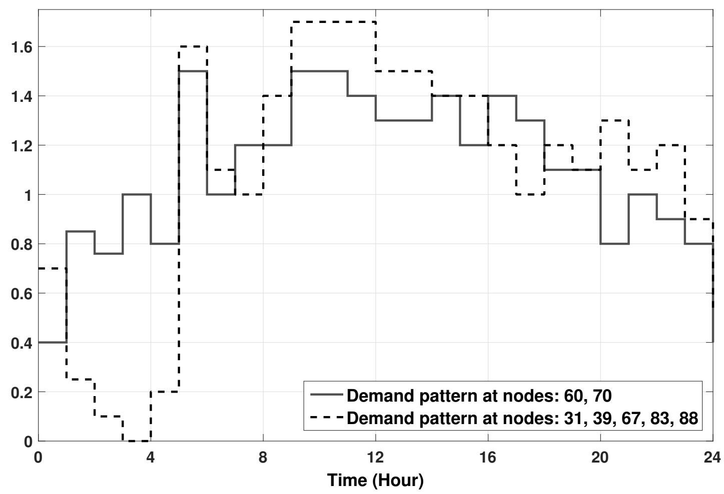

Figure 4: The water demand scenario 2.

measurements). The influence of uncertainty values of the particular quantities on estimates of unmeasured chlorine concentrations was examined for exemplary DWDS in [14]. According to section 3.1, T as well as T_{QP} are introduced in the algorithms and they are equal to 24 [h] and 5 [min], respectively. Moreover, the EPANET Chojnice DWDS model was also used to validate the optimised allocation results. The EPANET was coupled with MATLAB in order to create a computational environment for the chlorine concentration soft sensors and NSGA-II based optimisation solver.

The optimised placement obtained by the algorithm based on the bi-objective optimisation problem formulation are shown in Figs. 5 and 6. The Pareto front is illustrated in Fig. 5 where the best chromosome is marked and the optimised placement referring to this solution is presented in Fig. 6. The algorithm parameters are: population of 80 chromosomes, SFR of 33 nodes and ASR equals to 20 sensors. During the allocation process water demand scenario 1 (Fig. 3) was taken into account. With the optimally placed hard sensors the estimation results at node 71 are shown in Fig. 7. Three trajectories are presented in this figure: the chlorine concentration from the EPANET and the bounds on the estimated chlorine concentration.

In turn, results obtained by the algorithm of the hard sensors allocation based on the multi-objective optimisation problem formulation under the water demand scenarios 1 and 2 (Figs. 3 and 4) are illustrated in Figs. 8 and 9. The values of the algorithm parameters as well as the interpretation of Figs. 8 and 9 are analogous to the one in the previous paragraph. With the optimally placed hard sensors the estimation results at node 71 under the water demand scenario 1 are shown in Fig. 10. Three trajectories are presented in this figure: the chlorine concentration from the EPANET and the bounds on the estimated chlorine concentration.

One should notice that the simulation results presented above confirm that the allocation problem formulation has crucial impact on the location as well as the number of hard

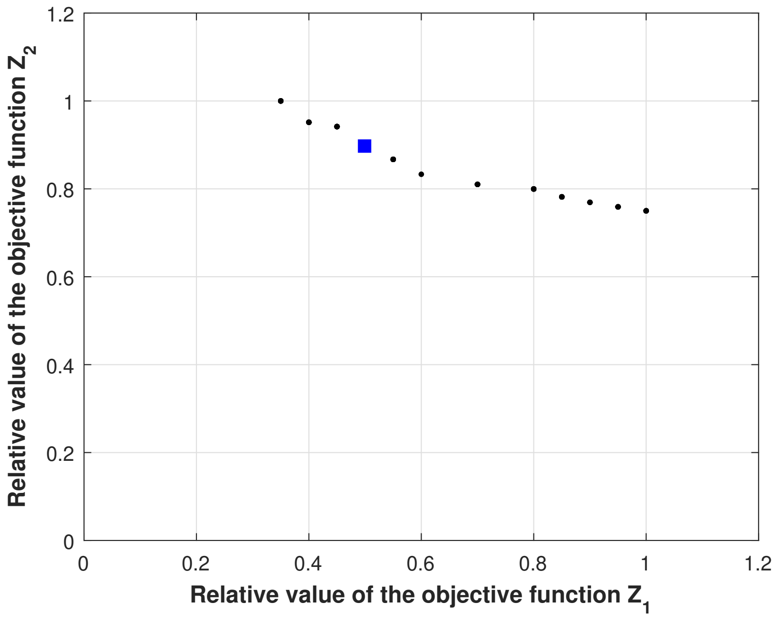

Figure 5: The bi-objective allocation algorithm - Pareto front.

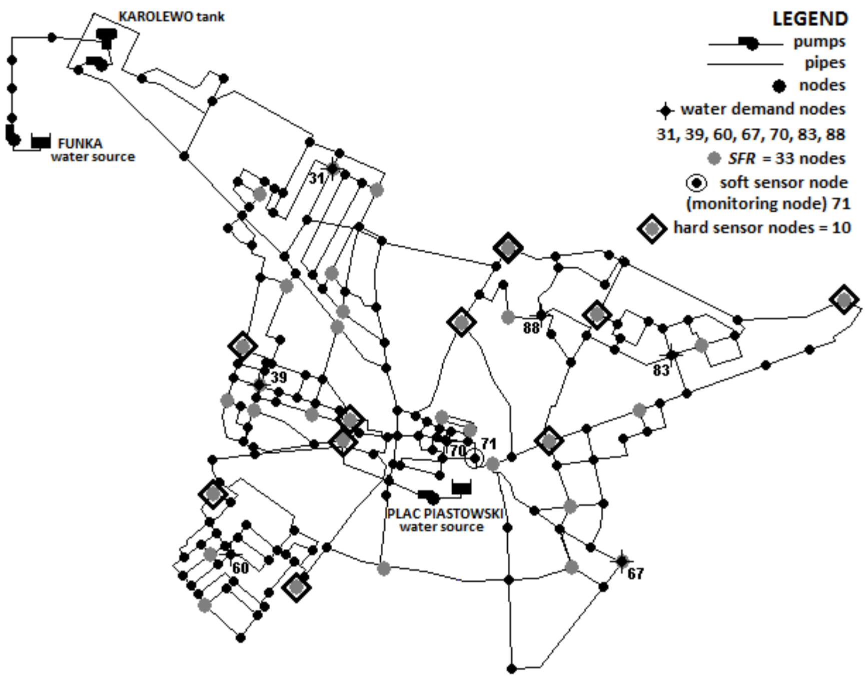

Figure 6: The bi-objective allocation algorithm - sensors placement.

sensors in the DWDS. For example, the hard sensors placement differs in the bi and multi-objective formulations. It is worth adding, that there are nodes where the hard sensors are always placed independent of the approaches (Figs. 6 and 9). As it was mentioned, if

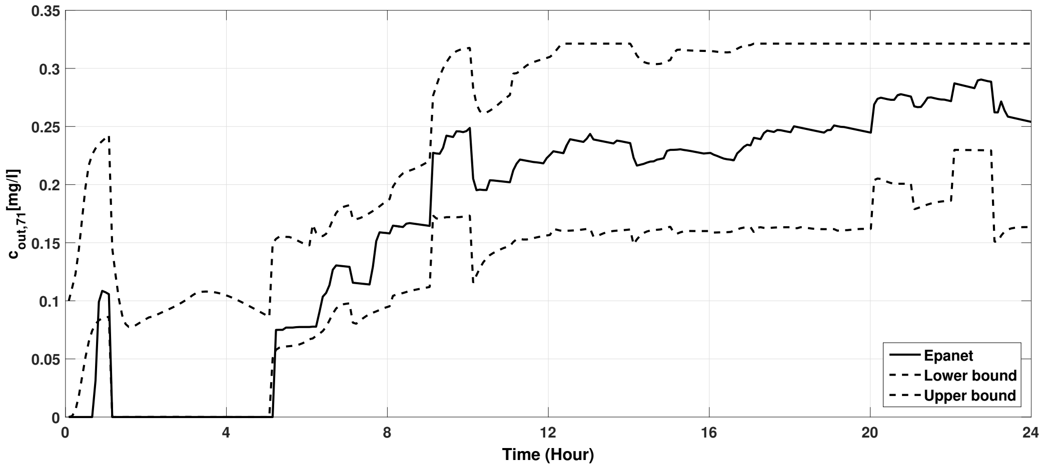

Figure 7: The bi-objective allocation algorithm - interval estimation results at node 71.

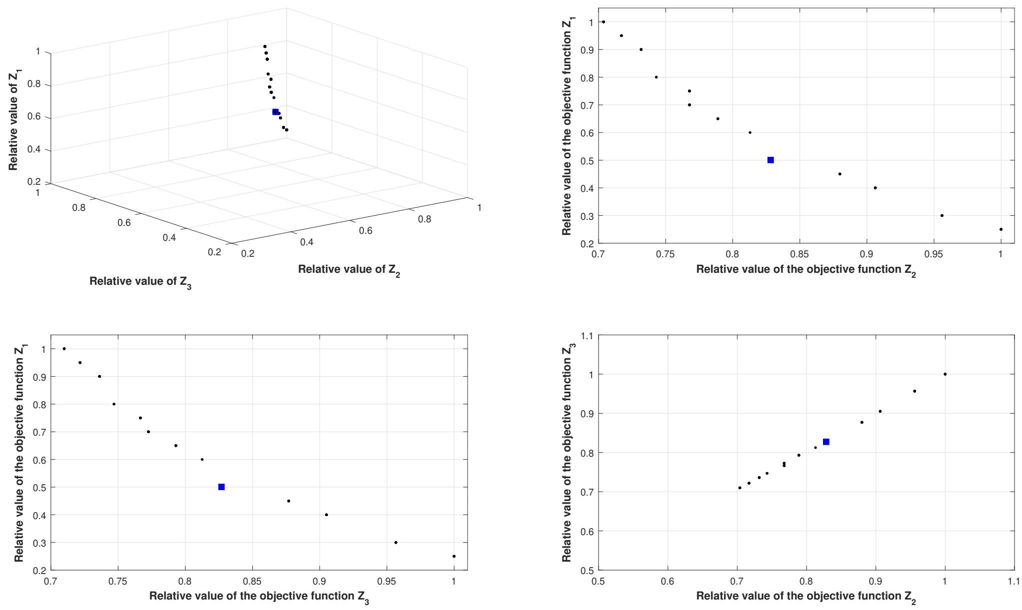

Figure 8: The multi-objective allocation algorithm - Pareto front.

the hydraulic conditions change, another allocation might be desirable. For example, if the third water demand scenario is added in the considered DWDS the hard sensors placement is as in Fig. 11. Therefore, it is important to identify a representative set of water demand scenarios in order to cope with the related uncertainty. Moreover, enlarging the set of considered water demand scenarios yields the increase in the computational time. It is because every additional water demand scenario associates with additional objective function

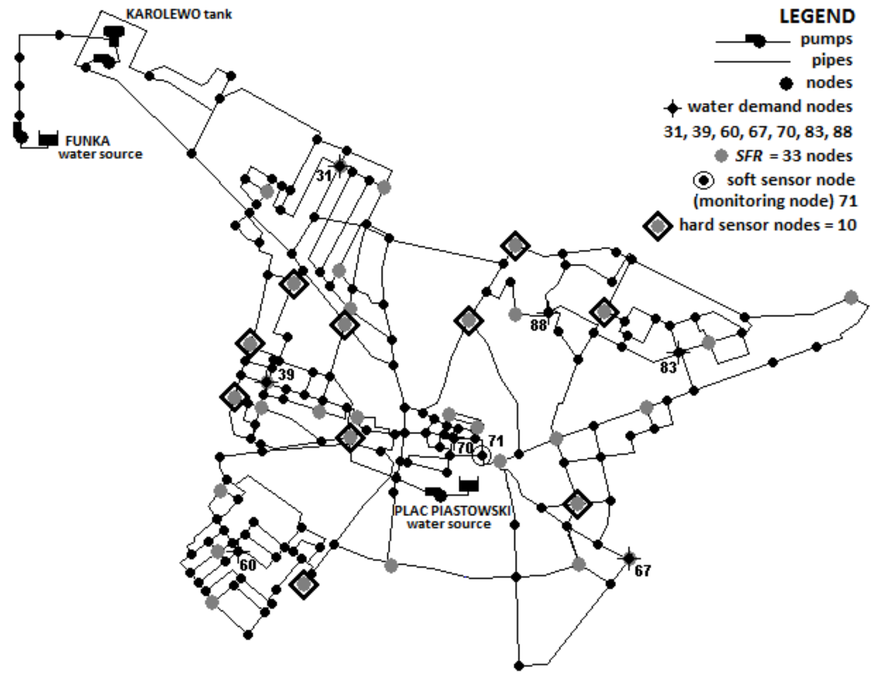

Figure 9: The multi-objective allocation algorithm - sensors placement.

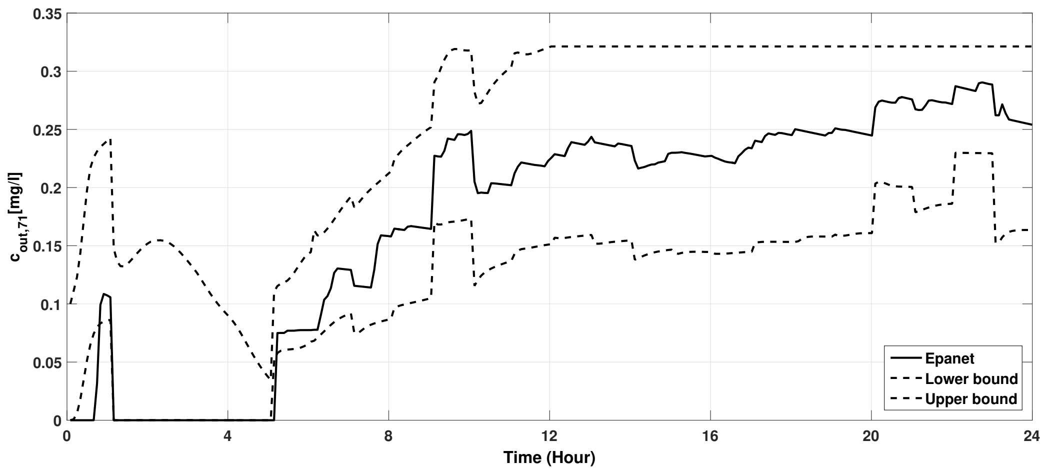

Figure 10: The multi-objective allocation algorithm - interval estimation results at node 71.

(see section 3.3). Hence, on one hand the wider hydraulic conditions are taken into account, but on the other hand solving the allocation problem lasts longer. Moreover, as it was mentioned in section 3.1, limiting the SFR set to the most important nodes also increases the computational efficiency. However, it may cause the worse performance of estimation. This issue might be one of direction of future work, but the preliminary results are presented in Figs. 12 and 13. Clearly, for SFR that equals 24 nodes the optimised placement obtained by the algorithm based on the bi-objective optimisation problem formulation is shown in

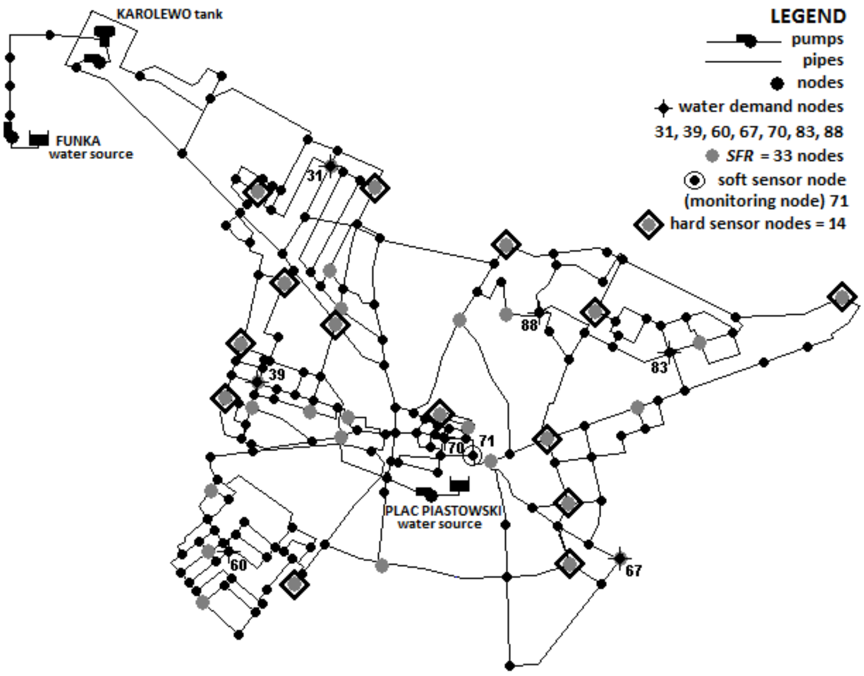

Figure 11: The multi-objective allocation algorithm - sensors placement under three water demand scenarios.

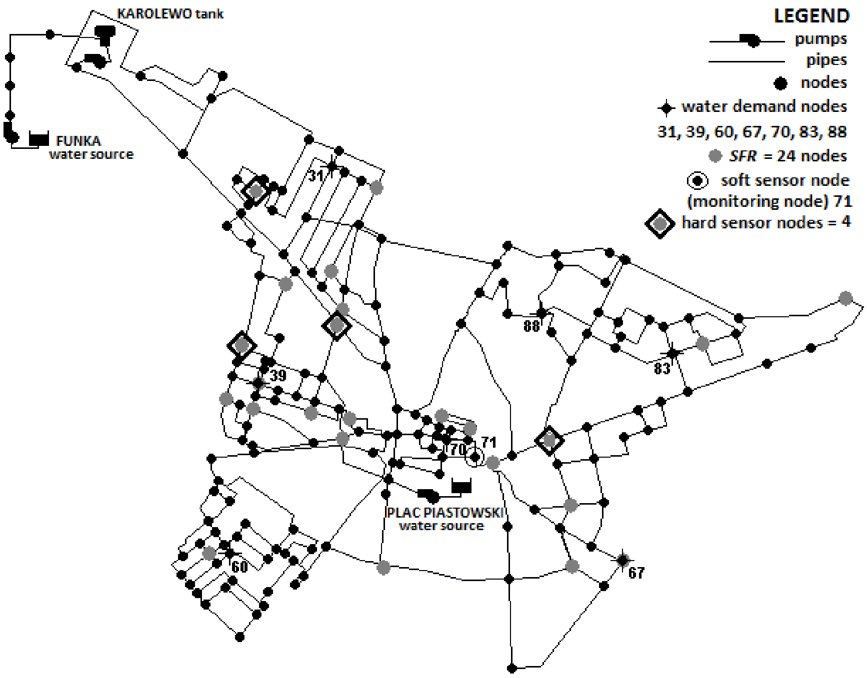

Figure 12: The bi-objective allocation algorithm - sensors placement for $SFR = 24$ nodes.

Fig. 12. A comparison between the estimation results at node 71 in case when SFR equals 33 and 24 nodes is presented in Fig. 13. Five trajectories are presented in this figure:

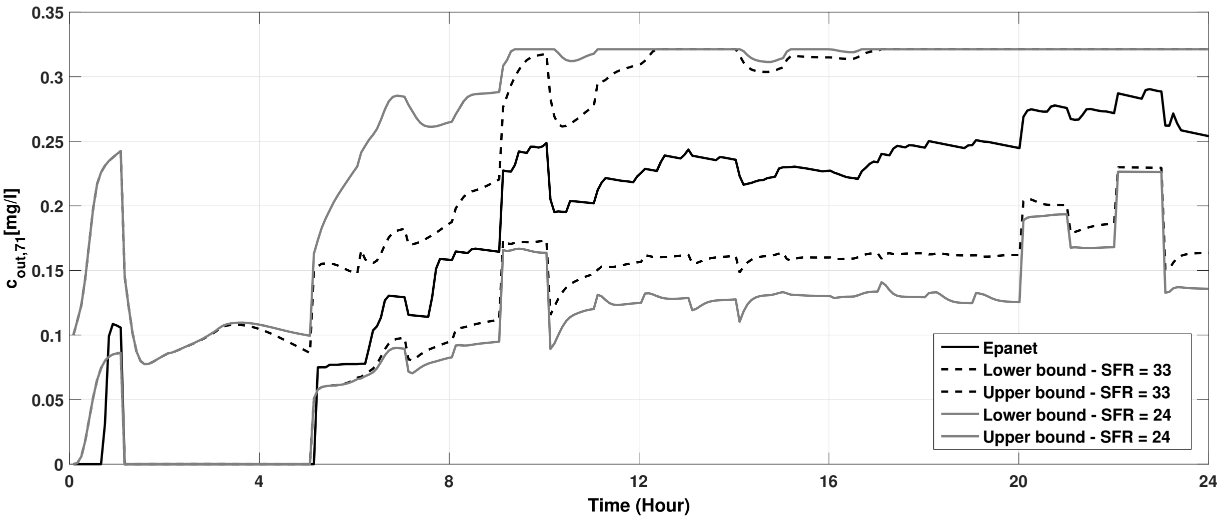

Figure 13: A comparison of the estimation performance, for different SFR sets, at node 71.

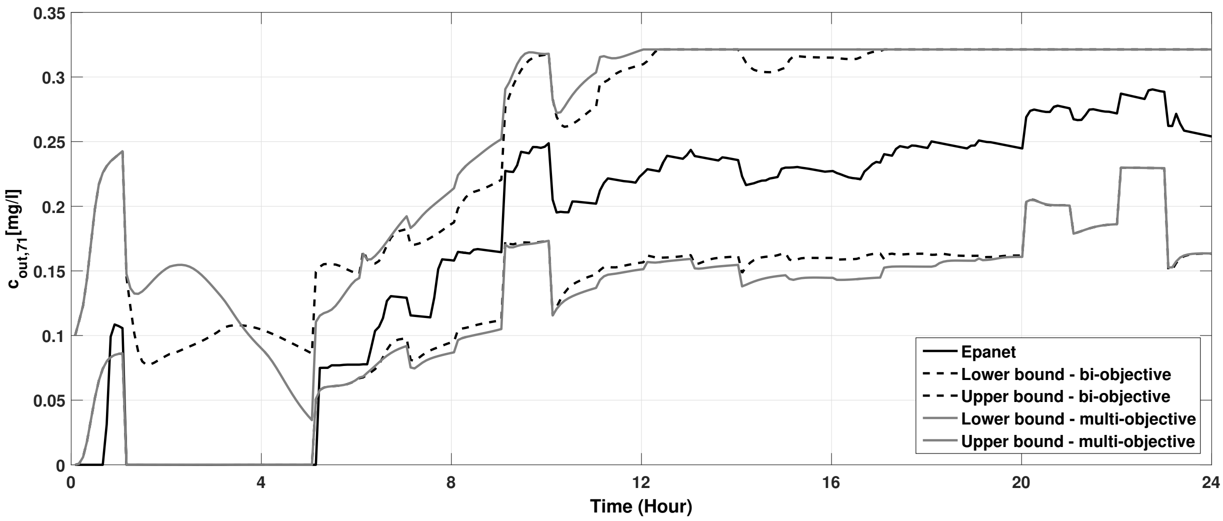

Figure 14: A comparison of the estimation performance at node 71.

the chlorine concentration from the EPANET and the bounds on the estimated chlorine concentration in both above cases.

Moreover, the presented simulation results show that the chlorine concentration trajectories are always inside the estimated bounds. However, the abrupt changes of the trajectories during transients can be observed. This is the effect of initial and boundary conditions on the chlorine concentration being equal to zero at the beginning of the observation. Additionally, the location and the number of the hard sensors have an important impact on estimation performance in the DWDS. A comparison (Figs. 7 and 10) of the estimation results at node

71 is presented in Fig. 14. Five trajectories are presented in this figure: the chlorine concentration from the EPANET and the bounds on the estimated chlorine concentration from the bi and multi-objective formulations.

6. Conclusions

In this paper the problem of an optimised placement of the hard sensors in the DWDS for a robust monitoring of the chlorine concentration has been derived and implemented in the MATLAB-EPANET environment.

Numerical algorithms to solve this problem have been presented. The optimality is understood as achieving a desired trade off between the sensors and their maintenance costs and the accuracy of estimation of the chlorine concentration.

The robust estimation algorithm recently developed by the authors has been applied as a soft sensors for the chlorine concentration monitoring purposes.

The results have been successfully validated in Chojnice DWDS case study. The sensors placements produced by the derived algorithms have been validated for several water demand scenarios, hopefully adequately representing the DWDS disturbing inputs.

The undergoing as well as future research can be focused on certain interesting issues. The utilisation of dedicated water demand patterns for certain nodes, connecting single and multi-objective formulations in order to capture specified requirements for estimation accuracy in certain nodes or parts of the DWDS and further considering the influence of the set of feasible nodes on the sensor allocation belonging to them. Moreover, the presented algorithms do not consider the hard sensor faults. Hence, the required accuracy of the estimates is not guaranteed under the sensor faults. This issue is the next of important topics for further research.

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