Direct algorithm for optimizing robust MPC of drinking water distribution systems hydraulics

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***Abstract*— Model-based predictive control is an effective method for control the large scale systems. Method is based on on-lin solution of control task over the control horizon using current and past measurements as well as the system model. Because model and measurement uncertainty, predicted and plant outputs might be different and plant output may exceed plant output constraints. Generated control is not then robustly outputs feasible. In this paper robustly outputs feasible direct control algorithm, based on the robust output predictions and control vector are presented. Proposed robustly outputs feasible MPC is applied to drinking water distribution system (DWDS) of the Chojnice city.**

***Keywords*— robust control, predictive control, robust output prediction, genetic algorithms, drinking water distribution systems**

I. INTRODUCTION

Model-based predictive control is an effective method for control the large scale systems [1]-[6],[8],[16],[17]. Method is based on on-line solution of the control task over the control horizon using current and past measurements, as well as the system model. Only a first element of calculated control sequence is applied to the plant. At the next sampling instant, based on new process output measurements, control procedure is repeated.

In control process of large scale systems, two crucial factors exist: robust output feasibility control and computation time.

In the classical approach to the MPC, control is generated based on the nominal model of the system and nominal measurements [1]. This control does not take into account the uncertainties associated with the controlled system. Uncertainties are result of: structure and parameters of system model, measurements and estimates. Hence, generated control, used in the real system can generate output which exceed the system output constraints. Therefore, generating robust output feasibility control vector is an important aspect in control of large scale systems.

For large scale systems, determining of the control vector can take long time. It depends on complexity of the system and control algorithm. In classical approach of MPC, at every control/prediction step must be generated control vector based on: system model, current measurements and disturbance prediction. The computation time of the new control vector, depends strongly on the size of the system (size of vector control, number of variables in optimization task, numbers and type of system equations), so it is important to ensure that the new control vector is generated at time limits.

In the present paper, attention will be paid to these two aspects of MPC.

In articles [2] – [4], [6] presents the algorithms generating robust output feasibility control of DWDS. These algorithms generate the control of DWDS hydraulic based on a modification of original system constraints. This modification depending on value of the exceedances original constraints by boundary output trajectories of system [7]. The disadvantage of presented algorithms is difficulty in estimating of value of modified constraints at following iterations.

At this paper, robust output feasible MPC algorithm based on boundary trajectories and modification of the control vector, is presented. The solver, of presented algorithm, is based on the effective genetic algorithm (GA) with specialized genetic operators (SGO) ([3], [4], [8]) and the robust outputs prediction algorithm [9]. Presented robustly outputs feasible MPC algorithm is applied, the control of DWDS hydraulics of Chojnice city [6], [9], [13]-[15].

II. PROBLEM FORMULATION OF ROBUSTLY OUTPUTS FEASIBLE MPC

The MPC task (1) calculates control $\boldsymbol{u}(\Xi_u)$ over the control horizon $\Xi_u = [t_n, t_n + H_u]$ (H_u - control horizon length) with respect to: objective function J, nominal plant model F, output $\overline{\boldsymbol{y}} = [\boldsymbol{y}^{min}, \boldsymbol{y}^{max}]^T$ and input $\overline{\boldsymbol{u}} = [\boldsymbol{u}^{\min}, \boldsymbol{u}^{\max}]^T$ constraints.

$$\begin{aligned} &Find \quad \boldsymbol{u}^*(\Xi_u) = \min_{\boldsymbol{u}(\Xi_u)} J_k\left(\boldsymbol{u}(\Xi_u), \boldsymbol{y}_p(\Xi_p)\right) \\ &\text{satisfying :} \\ &\boldsymbol{y}_p(\Xi_p) = F(\boldsymbol{u}(\Xi_u)) \\ &\boldsymbol{y}_p(\Xi_p) \in \overline{\boldsymbol{y}} \\ &\boldsymbol{u}(\Xi_u) \in \overline{\boldsymbol{u}} \end{aligned} \tag{1}$$

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where, $\boldsymbol{y}_p(\Xi_p)$ - model output over the prediction horizon $\Xi_p = \lfloor t_n+1, t_n+H_p \rfloor$ (H_p - prediction horizon length).

Calculated control $\boldsymbol{u}(\Xi_u)$ can be applied to the plant. Because the system model includes an uncertainty, a differences between a model output prediction $\boldsymbol{y}_p$ and a plant output $\boldsymbol{y}$, might exist. It means that plant output may not fulfil the system output constraints $\boldsymbol{y}(\Xi_p) \not\in \bar{\boldsymbol{y}}$ (Fig. 1).

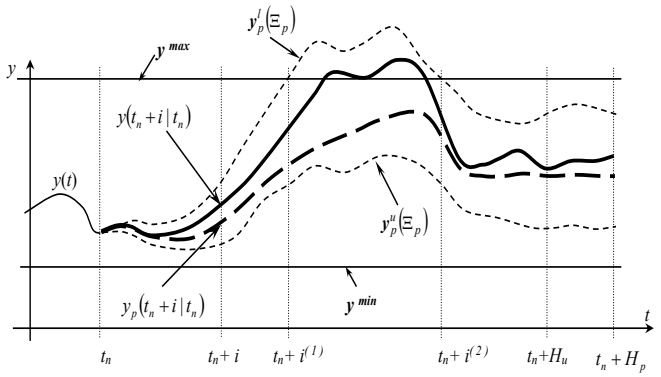

Fig. 1. Illustration of output constraints violation by upper output prediction trajectory.

To verify fulfilling of the output constraints by the plant output, robust output prediction (ROP) is necessary. In presented paper, set-bounded uncertainty model [10], is used. Robust output prediction generates lower $\boldsymbol{y}_p^l(\Xi_p)$ and upper $\boldsymbol{y}_p^u(\Xi_p)$ bounds of output trajectory over Ξ_p. All plant outputs, generated by all possible uncertainty scenarios, are contained between this two trajectories. That means

$$\boldsymbol{y}_p^l(\Xi_p) \le \boldsymbol{y}(\Xi_p) \le \boldsymbol{y}_p^u(\Xi_p) \tag{2}$$

Let define bound trajectory vector as

$$\bar{\bar{\boldsymbol{y}}}_p(\Xi_p) = \left[\boldsymbol{y}_p^l(\Xi_p), \boldsymbol{y}_p^u(\Xi_p)\right] \tag{3}$$

,where: $\boldsymbol{y}_p^l = \left[y_p^l(t_n+1\,|\,t_n), \ldots, y_p^l(t_n+H_p\,|\,t_n)\right]^T$ and $\boldsymbol{y}_p^u = \left[y_p^u(t_n+1\,|\,t_n), \ldots, y_p^u(t_n+H_p\,|\,t_n)\right]^T$.

Robust output prediction generates boundary trajectories, upper $\boldsymbol{y}_p^u$ and lower $\boldsymbol{y}_p^l$, based on: control vector $\boldsymbol{u}(\Xi_u)$, set-bounded initial state and plant model $\bar{\bar{F}}$ with set-bound uncertainty model [7].

The set-bounded output prediction, of the *j*-th components of the output vector $\boldsymbol{y}_p$ are determined by solving the following optimization tasks:

$$\begin{gathered} y_p^{l,j}(t_n+i\,|\,t_n) = min\, y_p^l \\ y_p^{u,j}(t_n+i\,|\,t_n) = max\, y_p^u \\ \text{satisfying:} \\ y_p^{l,j}(t_n+i\,|\,t_n), y_p^{u,j}(t_n+i\,|\,t_n) \in \bar{\bar{\Gamma}}(t_n+i\,|\,t_n); i \in \Xi_p \end{gathered} \tag{4}$$

,where: $; j \in J$ predicted variable index set.

The $\bar{\bar{\Gamma}}$ represent the set of all possible trajectories of system outputs, generated with the bounded system model $\bar{\bar{F}}$ by the control vector $\boldsymbol{u}(\Xi_u)$. This model take into account uncertainty associated with system model presented in [2] – [4], [6]. Details of generating bounded trajectories are shown in [9].

Let define overflow vector as:

$$\begin{gathered} \bar{\bar{\boldsymbol{v}}}(\Xi_p) = \left[\boldsymbol{v}^{\min}(\Xi_p)^T, \boldsymbol{v}^{\max}(\Xi_p)^T\right] = \\ \left[\max\left(0, \boldsymbol{y}^{\min}(\Xi_p) - \boldsymbol{y}_p^l(\Xi_p)\right)^T, \max\left(0, \boldsymbol{y}_p^u(\Xi_p) - \boldsymbol{y}^{\max}(\Xi_p)\right)^T\right] \end{gathered} \tag{5}$$

If boundary trajectories fulfils plant constraints $\left(\bar{\bar{\boldsymbol{v}}}(i) = 0, i \in \Xi_p\right)$ generated control is robustly output feasible and it can be applied to the plant. Else, if $\bar{\bar{\boldsymbol{v}}}(i) > 0, \left(i \in \Xi_p\right)$ the fulfilling of the output constraints by plant output, cannot be guaranteed (Fig. 1). It means that ,the generated control vector can't be applied to the plant.

Given the overflow vector $\bar{\bar{\boldsymbol{v}}}$ for checking the control robustness, it can be formulated optimization task of determining the robustly output feasible control as follows:

$$\begin{gathered} \boldsymbol{u}^{**}(\Xi_u\,|\,t_n) = \underset{\boldsymbol{u}(\Xi_u|t_n)}{arg\,min}\, J_k\left(\boldsymbol{u}(\Xi_u\,|\,t_n), \bar{\bar{\boldsymbol{y}}}_p(\Xi_p\,|\,t_n)\right) \\ \text{satisfying} \\ F\left(\boldsymbol{u}(\Xi_u\,|\,t_n), \boldsymbol{y}_p(\Xi_p\,|\,t_n)\right) + \varepsilon = 0 \\ \varepsilon^{min} \le \varepsilon \le \varepsilon^{max} \\ \bar{\bar{\boldsymbol{v}}}(i) = 0 \\ \boldsymbol{u}(\Xi_u\,|\,t_n) \in \bar{\bar{\boldsymbol{u}}} \end{gathered} \tag{6}$$

where, ε - system uncertainty vector; $\varepsilon^{\min}, \varepsilon^{\max}$ - bounds of system uncertainty.

The uncertainty vector ε includes all the uncertainties associated with the system: measurements, model structure, parameters etc.

III. DIRECT ALGORITHM SOLVING ROBUSTLY OUTPUTS FEASIBLE MPC PROBLEM

In the present paper, it is propose an algorithm determining a robust output feasibility control vector. This algorithm is based on the bounded output trajectories by directly modifying the control vector. In paper, to finding this control vector, two-step algorithm, (Fig. 2), is proposed.

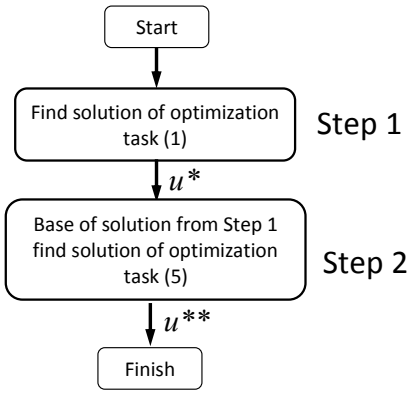

Fig. 2. Scheme of two-step algorithm for finding robust output feasibillity control.

In the first step, by solving optimization task (1), the optimum control vector is determined using the nominal system model. The optimization task (1) is nonlinear with mixed variable, that why genetic algorithm solver, presented in [3],[4],[6],[8], is used. GA based on the value of the objective function, hence the optimization problem (1) is reformulated to optimization problem without constraints to the form:

$$\begin{aligned} &\boldsymbol{u}^{*}(\Xi_u \mid t_n) = \\ &= \underset{\boldsymbol{u}(\Xi_u \mid t_n)}{arg\,min} \begin{pmatrix} J_k\left(\boldsymbol{u}(\Xi_u \mid t_n), \boldsymbol{y}_p(\Xi_p \mid t_n)\right) \\ + J_{bounds}\left(\boldsymbol{u}(\Xi_u \mid t_n), \boldsymbol{y}_p(\Xi_p \mid t_n)\right) \end{pmatrix} \\ &\text{satisfying} \\ &\boldsymbol{u}(\Xi_u \mid t_n) \in \overline{\overline{\boldsymbol{u}}} \end{aligned} \tag{6}$$

where, J_{bounds} - penalty function for breaking optimization task constraints [3].

At this step, the efficient solver of optimization task (1) is used, to find the starting control vector u* for the algorithm at second step.

In the second step, similarly to the first, to determine control modified AG is used. Control task (1) must be modified to optimization task without constraints in the form:

$$\begin{aligned} &\boldsymbol{u}^{**}(\Xi_u \mid t_n) = \\ &= \underset{\boldsymbol{u}(\Xi_u \mid t_n)}{arg\,min} \begin{pmatrix} J_k\left(\boldsymbol{u}(\Xi_u \mid t_n), \overline{\overline{\boldsymbol{y}}}_p(\Xi_p \mid t_n)\right) \\ + \overline{\overline{J}}_{bounds}\left(\boldsymbol{u}(\Xi_u \mid t_n), \overline{\overline{\boldsymbol{y}}}_p(\Xi_p \mid t_n)\right) \end{pmatrix} \\ &\text{satisfying} \\ &\boldsymbol{u}(\Xi_u \mid t_n) \in \overline{\overline{\boldsymbol{u}}} \end{aligned} \tag{7}$$

where, $\overline{\overline{J}}_{bounds}$- penalty function for breaking optimization task (1) constraints [3].

In both optimization tasks, the same function determines the cost of control, are used. While, there are various penalty functions of exceeding the limits of optimization task (1). This is due to the fact that for the assessment of proposed control vector, in task (6) is used a nominal output trajectory, while in task (7) bounded outputs trajectories. Hence penalty function J_{bounds} or $\overline{\overline{J}}_{bounds}$ is the sum of four elements:

1) penalty of exceeding system model constraints;

2) penalty of exceeding output/state constraints;

3) penalty of exceeding of the restrictions on the state of the system at the end of prediction horizon;

4) penalty of excessive speed change control.

An example component of penalty functions J_{bounds} or $\overline{\overline{J}}_{bounds}$, for the optimization task (6) and (7) can be presented as:

-for tasks (6)

$$\begin{aligned} J_y = \sum_{i=0}^{N_p} \Big[&\left\{\max\left(0, y_p(t_n+i \mid t_n) - y^{\max}\right)\right\}^2 + \\ &\left\{\max\left(0, y^{\min} - y_p(t_n+i \mid t_n)\right)\right\}^2 \Big] \end{aligned} \tag{8}$$

where, N_p - length of prediction horizon.

-for tasks (7)

$$\begin{aligned} J_y = \sum_{i=0}^{N_p} \Big[&\left\{\max\left(0, y_p^{u}(t_n+i \mid t_n) - y^{\max}\right)\right\}^2 + \\ &\left\{\max\left(0, y^{\min} - y_p^{l}(t_n+i \mid t_n)\right)\right\}^2 \Big] \end{aligned} \tag{9}$$

To finding bounded output trajectories, optimization algorithm presented in [9], is used.

There are several reasons of solving optimization task (1) in two steps:

1) Assume that, the optmization task (1), can be solved by optimization solver presented in [3],[4],[6],[8] and the

algorithm presented in [9]. The optimization solver, at the beginning of its action, starts at random starting point. This point may not fullfil the system model constraints. In consequency, it will not be possible to designate boundary trajectories, based at this model and solution of this task will be empty.

2) It can be assume that we create the algorithm that generates model acceptable solution u that will be the starting point for the second step of the algorithm. The starting point for the second step of the algorithm, is random point from a limited set. This set contain every soluton, of optimization task (1), model acceptable. Thus, better solution (from the point of view of efficiency of presented algorithm) is a optimal solution of the optimization problem (1). This solution is one and it is definitely a solution closer to optimal solution of the optimiaztion task (1) than random point from the model acceptance set.
3) Another aspect, is differences in computation time of determining nominal trajectory and bounded trajectories. To determine nominal trajectory simulator Epanet [10], is used. At this simulator effective algorithm of solving DWDS model [11], is implemented. To determine bounded output trajectories it is necessary to known nominal trajectory and it is need to solve multiple optimization problems (4). Thus, computation time of determining bounded output trajectories are much longer than computation time of determining nominal trajectories [9].

A similar way (multistep algorithm) to solve the optimization task can be found in the paper [15].

IV. BENCHMARK SYSTEM

The effectiveness of presented algorithm is presented based on DWDS of Chojnice city. Model structure of DWDS Chojnice city is shown in Fig. 3.

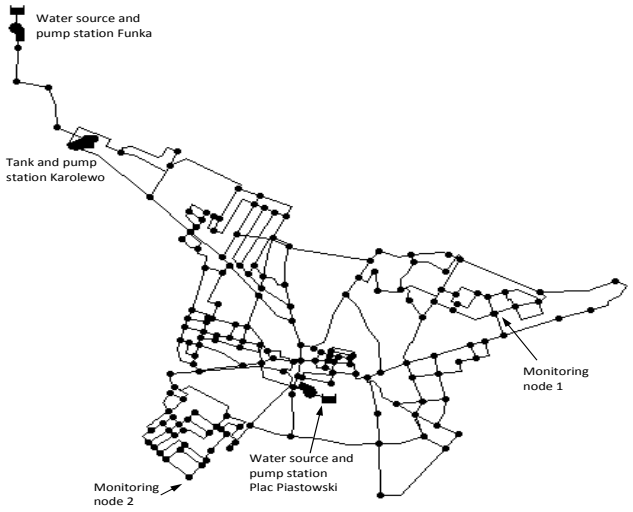

Fig.3 DWDS Chojnice full model diagram

Test model consists of: 177 nodes, 271 pipelines, 2 water sources, 3 pump stations and 1 water tank. Hydraulic step is 1 hour and length of prediction and control horizon are 24 hours. Pump stations "Karolewo" and "Plac Piastowski" supply water to area with "Monitoring nodes 1 and 2 respectively". Limits on pressure and its increment at "Monitoring nodes 1 and 2" are on value from 185 up to 210 [m] and 3 [m/h], respectively. Pressure in tank "Karolewo" is limited to the value from 167.2 up to 170.8 [m]. During 8 am – 12 am and 4 pm – 10 pm electric energy cost 0.24 [zl/kWh] and rest of a day is 0.12 [zl/kWh].

A. Nominal DWDS model

DWDS hydraulic model can be described in a form of differential-algebraic equation set over modelling horizon $\Xi_m = [t_n, t_n + N_m]$ (N_m - length of the modelling horizon) with discretization step T_h equal to hydraulic step (sampling step). Model consists of three parts: (i) linear static – conservations of water mass in nodes; (ii) nonlinear static – conservation of energy on connection elements (pipelines, valves, pumps); (iii) nonlinear dynamic – conservations of water mass tanks. Model can be presented in a form:

$$\begin{cases} A_L \cdot \boldsymbol{s}(k) = 0 \\ f_{NL}(\boldsymbol{s}(k)) = 0; \qquad \mathrm{k} = 0,1,\ldots \\ h_z(k+1) = \Psi(s(k)) \\ h(t_n) = h_{z,0} \end{cases} \tag{10}$$

where, s - characteristic variable vector of the system compound of subvectors: u - control vector (pumps and valves), d - unmeasured disturbance vector (water consumption), y - output vector (pressure in node h, flow in pipelines q, water velocity in pipelines v), h_z - state vector (water level in tanks); $h_{z,0}$ - tanks water level at initial t_n, k - discreet sampling instant $k = T_h \cdot t$.

B. Control of DWDS

There are two major aspects in DWDS control: quantity (hydraulics) and quality over prediction horizon $\Xi_p = \lfloor t_n, t_n + N_p \rfloor$ (where N_p - length of prediction horizon). Because of differences in dynamics of hydraulics and quality, effective DWDS control is realized in a frame of suboptimal two layer hierarchical control structure [12]. At the upper control layer hydraulics control and coarse values of quality control are appointed. In the lower (correction) layer a correction of quality control, obtained from upper layer is realized. In the paper only hydraulics control at the upper layer is considered over control horizon Ξ_u, where $\Xi_u = \Xi_p$ and $H_u = H_p = 24\,[\mathrm{h}]$.

C. System constraints

In DWDS control problem there exist four major constraints on:

- head at water monitoring nodes $h_d(\Xi_p) \in \left[h_d^{min}(\Xi_p), h_d^{max}(\Xi_p)\right]$: $d \in D$, where D – water consumptions nodes index set;
- head change at water monitoring nodes $\underset{k \in 1, H_p - 1}{\forall} \Delta h_d(k, k+1) = \left|h_d(t_n + k) - h_d(t_n + k + 1)\right| \le \Delta h^{\max}$ where Δh^{max} - maximum pressure change;
- water tanks level – $h_z(\Xi_p) \in \left[h_z^{\min}(\Xi_p), h_z^{\max}(\Xi_p)\right], z \in Z$ (Z – water tanks index set);
- initial $h_z(t_n)$ and final $h_z(t_n + H_p)$ water tanks level must be equal $\Delta h_z(t_n, t_n + H_p) = h_z(t_n) - h_z(t_n + H_p) = 0$

Set of system constraints is given as Ω.

Output and state trajectories $\boldsymbol{y}(\Xi_p) = \left[\boldsymbol{h}_d(\Xi_p)^T, \boldsymbol{h}_z(\Xi_p)^T\right]$ must satisfy the system constraints:

$$\boldsymbol{y}(\Xi_p) \in \Omega \tag{11}$$

Input control variables constraints are given as:

$$\boldsymbol{u}(\Xi_u) \in \left[\boldsymbol{u}^{min}(\Xi_u); \boldsymbol{u}^{max}(\Xi_u)\right]; \ \boldsymbol{u} \in R_+ \tag{12}$$

In this paper the pump control is only take into account.

D. Formulation of DWDS optimizing control problem

Effective DWDS hydraulics control is based on optimizing predictive control algorithm. In this paper control problem is formulated as follows:

$$\begin{aligned} &Find \quad \boldsymbol{u}(\Xi_u) = \underset{U(\Xi_u)}{\operatorname{argmin}} \mathrm{E}(\boldsymbol{u}(\Xi_u)) \\ &\text{subject to}: (11) - (12) \end{aligned} \tag{13}$$

,where: E – pumping energy cost.

V. SIMULATION RESULTS

To demonstrate the effectiveness of presented in paper, control cost and mean computation time will be compared with two other algorithms. First algorithm is optimized algorithm presented i [8], [9] and second is algorithm realized control of real DWDS of Chojnice.

Comparing that three control solutions it must be noted that outputs generated using a robust and optimized algorithms are within system constraints but real output can exceed system constraints (Fig. 4 and 5).

Table 1 shows that control cost of generated by robust output feasible algorithm is higher that control costs of optimized algorithm and real system. Higher control cost is price for taking into account uncertainty associated with system. This due mainly is from the need to raise pressure at monitoring nodes (and others) in order to reduce adverse impact of system uncertainty on real system outputs.

Table 1. Comparison of control costs and the computation average time (at one simulation step) at simulation horizon 720 [h].

Algorithm	***Simulation average time***	***Control cost [zl]***
Optimized	4 min 45 s	4886
Direct	12 min 37 s	6364
Real system	-	5515

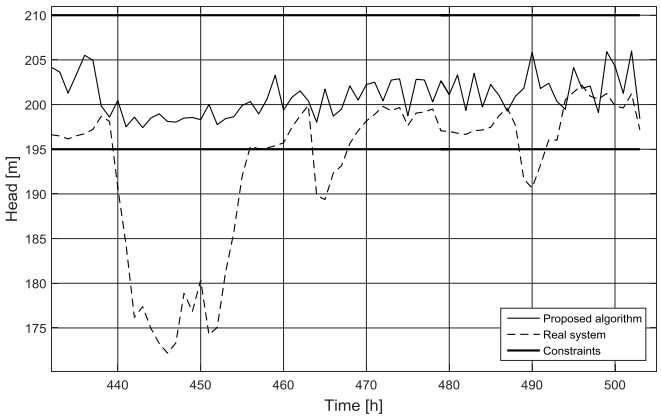

Fig. 4 Comparison of head trajectories at "Monitoring node 1" for proposed algorithm and real system control.

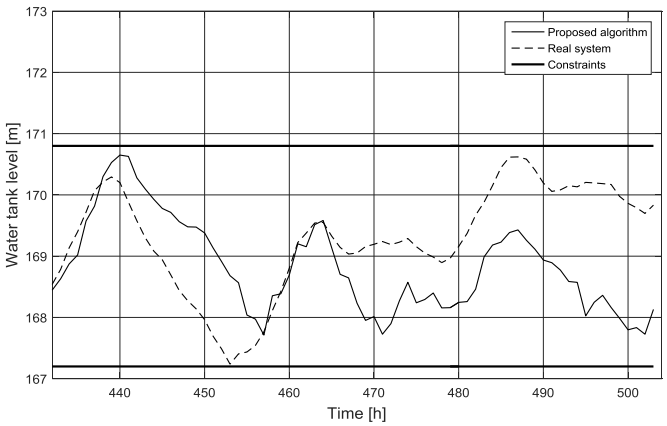

Fig. 5 Comparison of water tank level trajectories at "Karolewo tank" for proposed algorithm and real system control.

An important aspect of optimization algorithms used in the MPC is computation time, which must be less than control/prediction step time. For DWDS of Chojnice city control/prediction step time is 60 minutes.

For presented algorithm computation time for evry simulation step is shown in Fig. 6 showing the calculation time for each step of the simulation.

For presented algorithm only in 15 of 720 cases (~ 2% of cases) computation time has been exceeded the limit 60 minutes.

The calculations were performed on a PC with operating system Windows 10 (Intel i7, 16GB RAM).

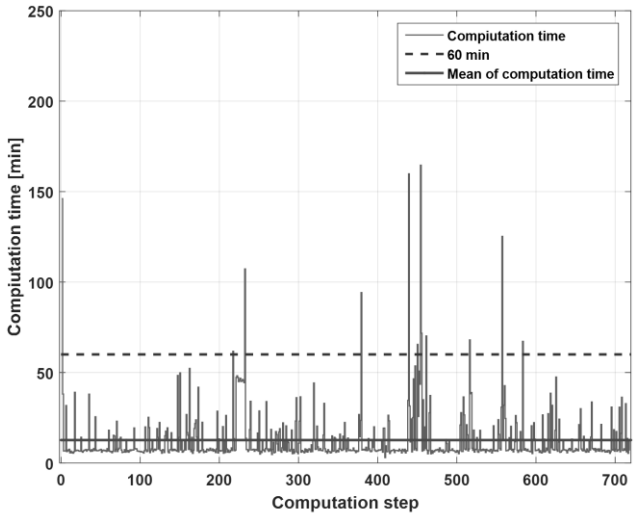

Fig. 6 Computation time over simulation horizon (720 h) for direct robust output feasible algorithm.

VI. SUMMARY

In paper, algorithm determining robust output feasible control of DWDS hydraulics, was proposed. This algorithm, in contrast to the algorithms presented in literature, modifying directly determined control. To increase efficiency two-step algorithm, was proposed. At first step determines optimal control based on nominal model. Solution, from first step, is the starting point for second step of the algorithm. In the second step, with starting from solution from first step, robust output feasible control is determined. To solve optimization problems (at first and second step of proposed algorithm) efficient optimization algorithm based on genetic algorithm and specialized genetic operators, is used. Simulations results, showed that control determined by proposed, is more expensive than optimized and real control. Increase of control cost is a price to pay for control robustness. But, it can be guarantee that used in the actual system control will fulfilling system constraints with uncertain system model.

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