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The Implementation of Fuzzy Logic in Forecasting Financial Ratios

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ABSTRACT

This paper is devoted to the issue of forecasting financial ratios. The objective of the conducted research is to develop a predictive model with the use of an innovative methodology, i.e., fuzzy logic theory, and to evaluate its effectiveness. Fuzzy logic has been widely used in machinery, robotics and industrial engineering. This paper introduces the use of fuzzy logic for the financial analysis of enterprises. While many current phenomena in finance and economics are fuzzy, they are treated as if they are crisp. Fuzzy logic provides an appropriate tool for modeling imprecise, uncertain and ambiguous phenomena. Because the financial situation of a company is affected by many factors (economic, political, psychological, etc.) that cannot be precisely and unambiguously defined, the approach used in this paper greatly enhances the predictive power of financial analysis and makes it an economically useful tool for the management of enterprises. Empirically, this paper employs three testing samples: Central European enterprises, Latin American companies and global firms. From the verification of these models, it is evident that the refined processes are effective in improving the forecasting of financial situations of all three types of enterprises. The models created by the author are characterized by high efficiency. This study is one of the world's first attempts to combine ratio analysis with fuzzy logic to predict the financial situations of companies.

KEY WORDS: decision support systems, financial ratios, fuzzy logic, forecasting, financial crisis

JEL Classification: G01

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1. Introduction

Many economists specializing in issues of financial analysis entered the twenty-first century with illusory euphoria, hoping for an increased integration of financial markets, faster and more effective use of information and effective forecasting of economic phenomena using sophisticated statistical and econometric models and artificial intelligence. The global financial crisis

that began in mid-2008 refuted the myth of safe economic regions of the world, as well as countries and enterprises. We are currently witnessing unprecedented events, such as the reduction of the U.S. credit rating from AAA to AA+ for the first time in history and the collapse of various global enterprises, including many that previously had impeccable economic reputations. Questions arise about the causes of the crisis and the possibility of its earlier forecasting. Within this context, it is worth assessing the usefulness of early warning models, which aim to capture the early symptoms of the deteriorating economic situation of a country, enterprise or household.

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In this paper, the author refers to the issue of effectiveness of the methods of the economic analysis of enterprises to assess these companies' financial standing in the era of globalization, general uncertainty and risk as well as extremely rapid changes in these companies' environments. The author attempts to answer the question of whether the methods dating back to the second half of the nineteenth century are congruent with the present time.

The aim of the studies undertaken in this paper is to implement the fuzzy logic methodology in the ratio analysis used in enterprises and to verify the effectiveness of fuzzy logic models developed for the purposes of financial analysis. This verification was carried out with two types of enterprises: those in a bad financial situation and those in a good financial situation. A novel analytical approach proposed by the author is to adapt the ratio analysis to the conditions under which companies must currently operate. The traditional zero-one (good/bad) evaluation criteria for ratios have lost their relevance. The inadequacy of the traditional ratio analysis is expressed with the use of bivalent logic to describe and evaluate the phenomena that are fuzzy, vague and ambiguous. After more than one hundred years since the development of the first financial ratios, it is now worthwhile to consider the implementation of a different approach to the evaluation of these ratios, to which the criteria referring to the fuzzy set theory are applied. The financial situations of enterprises are affected by many internal (Asgharian, 2003; Kash & Darling, 1998; Stokes & Blackburn, 2002) and external (Bhattacharjee, Highson, & Holly, 2003; Dyrberg Rommer, 2005; Everett & Watson, 1998; Hunter & Isachenkova, 2006; Robson, 1996) factors, which cannot be defined precisely and unambiguously. In addition, these factors at different stages of the life cycle of a company may affect the company's economic situation or cause chain reactions (du Jardin & Severin, 2012). In addition, the finding that a company is in a "good" or "bad" financial situation is imprecise because in the current economic reality, analysts rarely analyze wholly "good" or "bad" companies. On the other hand, using the traditional financial ratios based on the classic set theory makes it difficult to determine the precise degree of risk or the advancement of a positive situation, whether it is a somewhat positive or highly positive situation. Additionally, the manner of interpretation of the calculated

values of the indicators is still controversial. First, it is difficult to determine the optimal ranges of values of financial indicators in a rapidly changing environment. For example, does the minimum value of 1.3 appointed by an analyst eight years ago during the global financial crisis remain valid as the limit for the current liquidity of a company? Even if an analyst designates the current limit for the investigated company, there is another controversial aspect related to ratio analysis; namely, when using the classic theory of sets to assess an indicator, there is a sharp boundary at the level of the limit value. For example, with the ratio recorded as 1.29, how is the financial situation of a company different from the situation in which it reaches a certain level of the critical value (1.3)? This "zero-one" approach (with a "good/bad" indicator for a company) seems to be heavily outdated and unfit to current economic conditions. Numerous phenomena in economics are "fuzzy" and are currently treated by economists as if they were "sharp," or bivalent. With fuzzy sets, we can formally define vague and ambiguous terms, such as the "high risk of bankruptcy," "low risk of bankruptcy," "good financial standing," etc. While the classic set theory assumes that any item (company) belongs or does not belong to a given set (a "bad" or "good" financial situation), in the fuzzy set theory, an item can partially belong to a set, and this belonging can be expressed by a real number from the interval $[0,1]$. In other words, using fuzzy logic, a company's financial situation can be assessed as partially "good" or partially "bad". This study is one of the world's first attempts to combine ratio analysis with fuzzy logic to predict the financial situations of companies.

In the studies of this paper, the following were examined:

- The effectiveness of the 13 fuzzy logic models developed for individual financial ratios compared to the traditional ratio analysis (**due to the limited length of this article, only two of these models are presented**),
- The behavior of these models, along with the extension of the period of analysis from one year to two years prior to the announcement that the company was in a bad financial situation,
- The effectiveness of the discussed models in three different test samples composed of companies from different regions of the world, which made it pos-

sible to draw conclusions about the universality of the fuzzy logic models and compare the aspects of these models with the versatility of the traditional financial ratios.

Based on a literature review and the conclusions of the author's research from over 14 years of academic study on the issue of forecasting the financial standing of companies, the author puts forward the following research hypothesis. The use of fuzzy logic in ratio analysis

- allows for a more effective assessment of the financial condition of enterprises than that of the traditional financial analysis,
- ensures the stability of forecasts by extending the analysis period to two years,
- increases the usefulness of individual financial ratios in the assessment of companies' financial standing,
- increases the universality (i.e., the predictive properties for a diverse population of companies) of ratio analysis.

The paper is organized as follows. Section 2 presents the major drawbacks of the traditional ratio analysis. Section 3 recalls the technical background of the fuzzy logic model. Section 4 presents the research assumptions of this study. Section 5 proposes forecasting models for two financial ratios: the current liquidity and the coverage of fixed assets by long-term capital. Section 6 concludes the paper.

2. Major drawbacks of financial analysis

In its most classic form, financial analysis is based on a set of financial ratios that allow four key aspects of company operations to be examined: liquidity, debt, efficiency of using assets, and profitability.

The use of ratio analysis to assess the financial situation of enterprises brings several disadvantages and limitations. A discussion of these disadvantages and limitations is necessary to understand the validity of the proposal of using fuzzy logic in this paper.

Three major objections are generally raised regarding ratio analysis:

- 1) *The obsolescence of optimal values of financial indicators over time.* While the methods for calculating ratios have remained virtually unchanged with the passage of time, there are new indicators in the

literature that explore new areas of business. However, the optimal ranges of the ratio values become obsolete as a result of various factors, including changes in the business cycle or changes in economic conditions.

- 2) *The need for determination and interpretation of the standards.* More specifically, there is a need for the designation of optimal ranges of values for various financial ratios. This issue is obviously related to the problem raised earlier, namely, the obsolescence of these standards. The calculation of financial ratios may be rendered useless and may lead to a wrong decision if the calculation lacks proper interpretation. The determination of the standards of values is more difficult in the case of financial ratios that are nominants. In practice, there is no single best value of an indicator. An indicator depends on the business strategy (liquidity, market position, etc.), the industry and the country in which a company operates, as well as external factors, such as rising energy costs.

- 3) *The static nature of ratio values.* Most financial ratios are calculated based on static values at a given moment (usually at the end of the year under review) from the balance sheet and the income statement. Such an analysis lacks a dynamic view of the indicators. The question arises of whether changes in indicators are relevant predictors of a company's coming financial crisis because declines or increases in values do not immediately mean the deterioration of the economic situation of the company. Nevertheless, by observing changes, we can distinguish between a company that has a low economic indicator values that improve each year and a company that has indicators at a similarly low level that become worse each year. The static model will not detect the difference between such companies. The dynamic model adds an element that differentiates companies with poor financial situations from companies that do not necessarily have good financial situations but have improving financial situations.

The author conducted literature studies on the financial ratios used in credit scoring. After studying approximately 600 research papers on this subject, he chose 55 of them based on the three following criteria: the popularity of the authors and their research in the



Table 1. Overview of the most common financial ratios used in credit-scoring models

No.	Financial ratio	Used in studies
1.	Share of working capital in total assets (working capital / total assets)	[Ahn, Cho and Kim, 2000] [Back, Laitinen and Sere, 1996] [Baek and Cho, 2003] [Ignizio and Soltyas, 1996] [Karels and Prakash, 1987] [Lacher, Coats, Sharma and Fantc, 1995] [Lee, Han and Kwon, 1996] [Lee, Booth and Alam, 2005] [Leshno and Spector, 1996] [Lin and McClean, 2001] [Altman, 1993] [Lin and Piesse, 2004] [Hadasik, 1998] [Bandyopadhyay, 2006] [Galvão, Becerra and Abou-Seada, 2004] [Altman, Baidya and Ribeiro Dias, 1979] [Ginoglou, Agorastos and Hatzigagios 2002] [Boritz and Kennedy, 1995] [Serrano-Cinca, 1996] [Michaluk, 2003] [Wilson and Sharda, 1994] [Zapranis and Ginoglou, 2000] [Zhang, Hu, Patuwo and Indro, 1999] [Becerra, Galvao and Abou-Seada, 2005] [Rahimian and Singh, 1993]
2.	Encumbrance of cash surplus with liabilities [(net profit + depreciation) / total liabilities or EBIT / total liabilities]	[Ahn, Cho and Kim, 2000] [Bian and Mazlack, 2003] [Back, Laitinen and Sere, 1996] [Maczynska, 2004]
3.	Quick liquidity [(current assets - inventories) / current liabilities]	[Dimitras, Zanakis, & Zopounidis, 1996] [Fletcher and Goss, 1993] [Lee, Han and Kwon, 1996] [Leshno and Spector, 1996] [Lin and McClean, 2001] [Hadasik, 1998] [Emel, Oral, Reisman and Yolalan, 2003] [Park and Han, 2002] [Piramuthu, Ragavan and Shaw, 1998] [Sikora and Shaw, 1994] [Eklund, Back, Vanharanta and Visa, 2003]
4.	Current liquidity (current assets / current liabilities)	[Ahn, Cho and Kim, 2000] [Bian and Mazlack, 2003] [Bryant, 1998] [Dimitras, Zanakis, & Zopounidis, 1996] [Fletcher and Goss, 1993] [Back, Laitinen and Sere, 1996] [Lee, Han and Kwon, 1996] [Leshno and Spector, 1996] [Maczynska, 2004] [Hadasik, 1998] [Holda, 2001] [Boritz and Kennedy, 1995] [Kuruppu, Laswad and Oyelere, 2003] [McKee, 2003] [Pendharkar and Rodger, 2004] [Piramuthu, Ragavan and Shaw, 1998] [Shah and Murtaza, 2000] [Sikora and Shaw, 1994] [Witkowska, 2002] [Anandarajan, Lee and Anandarajan, 2001] [Zhang, Hu, Patuwo and Indro, 1999]
5.	Cash liquidity [(current assets - inventories - accounts receivables) / current liabilities]	[Back, Laitinen and Sere, 1996] [Lin and McClean, 2001] [McKee, 2003] [Witkowska, 2002] [Michaluk, 2003] [Laitinen and Kankaanpaa, 1999]
6.	Return on assets (net profit / total assets)	[Ahn, Cho and Kim, 2000] [Bian and Mazlack, 2003] [Bryant, 1998] [Dimitras, Zanakis, & Zopounidis, 1996] [Back, Laitinen and Sere, 1996] [Ignizio and Soltyas, 1996] [Lacher, Coats, Sharma and Fantc, 1995] [Lee, Han and Kwon, 1996] [Lee, Booth and Alam, 2005] [Leshno and Spector, 1996] [Lin and McClean, 2001] [Altman, 1993] [Lin and Piesse, 2004] [Gajdka and Stos, 1996] [Holda, 2001] [Yim and Mitchell, 2004] [Galvão, Becerra and Abou-Seada, 2004] [Ginoglou, Agorastos and Hatzigagios 2002] [Boritz and Kennedy, 1995] [McKee, 2003] [Min and Lee, 2005] [Pendharkar and Rodger, 2004] [Piramuthu, Ragavan and Shaw, 1998] [Serrano-Cinca, 1996] [Sikora and Shaw, 1994] [Witkowska, 2002] [Serrano-Cinca, 1997] [Michaluk, 2003] [Wilson and Sharda, 1994] [Zapranis and Ginoglou, 2000] [Anandarajan, Lee and Anandarajan, 2001] [Eklund, Back, Vanharanta and Visa, 2003] [Zhang, Hu, Patuwo and Indro, 1999] [Becerra, Galvao and Abou-Seada, 2005] [Rahimian and Singh, 1993]

Table 1. Overview of the most common financial ratios used in credit-scoring models (Continued)

No.	Financial ratio	Used in studies
7.	Relation of equity to total liabilities [equity / total liabilities]	[Ignizio and Soltyas, 1996] [Lee, Booth and Alam, 2005] [Altman, 1993] [Galvão, Becerra and Abou-Seada, 2004] [Altman, Baidya and Ribeiro Dias, 1979] [Ginoglou, Agorastos and Hatzigagios 2002] [Serrano-Cinca, 1996] [Sikora and Shaw, 1994] [Michaluk, 2003] [Wilson and Sharda, 1994] [Zapranis and Ginoglou, 2000] [Zhang, Hu, Patuwo and Indro, 1999] [Becerra, Galvao and Abou-Seada, 2005] [Rahimian and Singh, 1993]
8.	Period of repayment of short-term liabilities or rotation of liabilities [(current liabilities / operating costs) * 365 or operating costs / current liabilities]	[Lee, Han and Kwon, 1996] [Lin and McClean, 2001] [Gajdka and Stos, 1996] [Prusak, 2005] [Hořda, 2001] [Emel, Oral, Reisman and Yolalan, 2003]
9.	Inventory turnover [(inventories / sales) * 365 or sales / inventories]	[Ahn, Cho and Kim, 2000] [Bryant, 1998] [Back, Laitinen and Sere, 1996] [Karels and Prakash, 1987] [Lee, Han and Kwon, 1996] [Lin and McClean, 2001] [Hadasik, 1998] [Min and Lee, 2005] [Witkowska, 2002]
10.	Turnover of short-term receivables [(receivables/sales) * 365 or sales / receivables]	[Karels and Prakash, 1987] [Hadasik, 1998] [Kuruppu, Laswad and Oyelere, 2003] [Shah and Murtaza, 2000] [Witkowska, 2002] [Eklund, Back, Vanharanta and Visa, 2003]
11.	Turnover of total assets (sales / total assets)	[Bian and Mazlack, 2003] [Bryant, 1998] [Andres, Landajo and Lorca, 2005] [Baek and Cho, 2003] [Ignizio and Soltyas, 1996] [Lacher, Coats, Sharma and Fantc, 1995] [Lee, Han and Kwon, 1996] [Lee, Booth and Alam, 2005] [Leshno and Spector, 1996] [Lin and McClean, 2001] [Altman, 1993] [Gajdka and Stos, 1996] [Hořda, 2001] [Bandyopadhyay, 2006] [Galvão, Becerra and Abou-Seada, 2004] [Altman, Baidya and Ribeiro Dias, 1979] [Kuruppu, Laswad and Oyelere, 2003] [McKee, 2003] [Min and Lee, 2005] [Park and Han, 2002] [Serrano-Cinca, 1996] [Witkowska, 2002] [Michaluk, 2003] [Wilson and Sharda, 1994] [Zhang, Hu, Patuwo and Indro, 1999] [Becerra, Galvao and Abou-Seada, 2005] [Rahimian and Singh, 1993]
12.	Relation of gross profit or EBIT to total assets [EBIT / total assets or gross profit / total assets]	[Bryant, 1998] [Dimitras, Zanakis, & Zopounidis, 1996] [Atiya, 2001] [Baek and Cho, 2003] [Ignizio and Soltyas, 1996] [Lacher, Coats, Sharma and Fantc, 1995] [Lee, Booth and Alam, 2005] [Leshno and Spector, 1996] [Altman, 1993] [Altman, Baidya and Ribeiro Dias, 1979] [Ginoglou, Agorastos and Hatzigagios 2002] [Pendharkar and Rodger, 2004] [Serrano-Cinca, 1996] [Michaluk, 2003] [Wilson and Sharda, 1994] [Zhang, Hu, Patuwo and Indro, 1999] [Becerra, Galvao and Abou-Seada, 2005] [Rahimian and Singh, 1993]
13.	Share of total debt in total assets (total liabilities / total assets)	[Ahn, Cho and Kim, 2000] [Bryant, 1998] [Dimitras, Zanakis, & Zopounidis, 1996] [Karels and Prakash, 1987] [Leshno and Spector, 1996] [Lin and McClean, 2001] [Pang-Tien, Ching-Wen and Hui-Fun, 2008] [Gajdka and Stos, 1996] [Hadasik, 1998] [Gruszczynski, 2003] [Hořda, 2001] [Bandyopadhyay, 2006] [Boritz and Kennedy, 1995] [Kuruppu, Laswad and Oyelere, 2003] [Piramuthu, Ragavan and Shaw, 1998] [Shah and Murtaza, 2000] [Sikora and Shaw, 1994] [Witkowska, 2002] [Michaluk, 2003] [Zapranis and Ginoglou, 2000] [Anandarajan, Lee and Anandarajan, 2001] [Laitinen and Kankaanpaa, 1999] [Charalambous, Charitou and Kaourou, 2000]



Table 1. Overview of the most common financial ratios used in credit-scoring models (Continued)

No.	Financial ratio	Used in studies
14.	Share of equity in total assets (equity / total assets)	[Ahn, Cho and Kim, 2000] [Leshno and Spector, 1996] [Sandin and Porporato, 2007] [Maczynska, 2004] [Yim and Mitchell, 2004] [Emel, Oral, Reisman and Yolalan, 2003] [Kuruppu, Laswad and Oyelere, 2003] [Min and Lee, 2005] [Park and Han, 2002] [Shah and Murtaza, 2000] [Eklund, Back, Vanharanta and Visa, 2003]
15.	Net return on sales (net profit / total revenues)	[Ahn, Cho and Kim, 2000] [Bryant, 1998] [Lee, Han and Kwon, 1996] [Pang-Tien, Ching-Wen and Hui-Fun, 2008] [Min and Lee, 2005] [Piramuthu, Ragavan and Shaw, 1998] [Shah and Murtaza, 2000] [Sikora and Shaw, 1994]
16.	Operating profit margin or gross profit margin [operating profit / sales or gross profit / sales]	[Bian and Mazlack, 2003] [Karels and Prakash, 1987] [Leshno and Spector, 1996] [Lin and McClean, 2001] [Sandin and Porporato, 2007] [Gajdka and Stos, 1996] [Eklund, Back, Vanharanta and Visa, 2003]
17.	Return on equity (net profit / equity)	[Bian and Mazlack, 2003] [Karels and Prakash, 1987] [Kuruppu, Laswad and Oyelere, 2003] [Shah and Murtaza, 2000] [Witkowska, 2002] [Serrano-Cinca, 1997] [Eklund, Back, Vanharanta and Visa, 2003]
18.	Coverage of fixed assets with long-term capital or equity [(equity + non-current liabilities) / fixed assets or equity / fixed assets]	[Min and Lee, 2005] [Park and Han, 2002]

scientific community, the degree of innovation of the research (duplications of studies showing only adaptations of existing models of low importance were avoided), and the diversification of the methods used. Table 1 shows the results of the query. The query contains the 18 financial ratios that were most frequently used in the research on forecasting the financial situations of companies worldwide.

Based on the results of the query, the frequency of use of each indicator in the 55 aforementioned studies was calculated. Table 1 shows that the following six financial ratios occurred in at least 30% of the studies: the share of working capital in total assets, current liquidity, net return on total assets, turnover of total assets, return on assets measured by profit before taxation and repayment of interest and the share of total debt in total assets. Two of these ratios are liquidity ratios, two are profitability ratios, one is an indicator of debt and one is an indicator of efficiency. The most

common (occurred in 63.6% of studies) was the net return on total assets. The second and third most common ratios were the total turnover of assets (50.9%) and the share of working capital in total assets (47.3%), respectively.

3. Application of fuzzy logic to ratio analysis

With regard to the impressive development of forecasting models for companies' financial situations and to the development of ratio analysis itself, the inadequacy of most models and financial indicators for the phenomena occurring in the current business environment should be recognized. This inadequacy is expressed in the use of bivalent logic to describe and evaluate fuzzy, vague and ambiguous phenomena.

The use of statistical methods to forecast a company's risk of experiencing bankruptcy, such as multivariate discriminant analysis, does not change the

situation. When the value of the discriminant function for the studied company is less than the limit, the company is at risk of bankruptcy. Here, too, an analyst faces three similar problems, as follows, as in the case of ratio analysis:

- ✓ First, even though the assessment of a company's financial condition should be based on a simultaneous calculation of a series of often indirectly related financial ratios without individual interpretations, statistical models are based on the use of traditional financial indicators (both static and using classic logic),
- ✓ Second, the threshold of the discriminant function is the value of bivalent logic (the company situation is classified as "good" when the value of the function is above the estimated limit [for example, for the Altman model the limit is 2.99] and "bad" when it is less than this value),
- ✓ Third, the term "bankruptcy risk" is ambiguous from both a legal and an economic standpoint (for example, a company may be at risk of bankruptcy, but its situation steadily improves, so it cannot be classified as a future bankruptcy; even defining the situation as "bad" would be inaccurate because the condition of the company is improving steadily).

With fuzzy sets, we can formally define vague and ambiguous terms, such as the "high risk of bankruptcy" and "low risk of bankruptcy." Fuzzy set A in a certain non-empty space X ($A \subseteq X$) can be defined as follows (Wu, Zhang, Wu & Olson, 2010):

$$A = \{(x, \mu_A(x)) \mid x \in X\} \quad (1)$$

where $\mu_A: X \rightarrow [0,1]$ is a function that specifies the extent to which each element in X belongs to set A . Function μ_A is the so-called membership function of fuzzy set A .

While the classic set theory assumes that any item (company) belongs or does not belong to a given set (a "bad" or "good" financial situation), in fuzzy set theory, an item can partially belong to a set, and this belonging can be expressed by a real number from the interval $[0,1]$. Thus, the membership function $\mu_A(x): U \rightarrow [0,1]$ is defined as follows:

$$\forall_{x \in U} \mu_A(x) = \begin{cases} f(x), & x \in X \\ 0, & x \notin X \end{cases} \quad (2)$$

where $\mu_A(x)$ is a function specifying the membership of x in set A , which is a subset of U , and $f(x)$ is a function of values in the range $[0,1]$. The values of this function are called degrees of membership.

The membership function of each element $x \in X$ assigns a degree of membership to fuzzy set A , in which we can distinguish three situations:

- $\mu_A(x) = 1$ means full membership of element x in fuzzy set A ,
- $\mu_A(x) = 0$ means no membership of element x in fuzzy set A ,
- $0 < \mu_A(x) < 1$ means partial membership of element x in fuzzy set A .

Membership functions are usually presented in graphical form (Nakandala, Samaranayake, & Lau, 2013). The trapezoid function $\mu_A(x)$ is frequently used; the graph of this function is shown in Figure 1. This figure also includes the accepted standards for the current ratio, as reported in the literature. The correct value of this index is a value in the range $[1.2; 2]$; the incorrect value belongs to the range $(0; 1.2) \cup (2; \infty)$. When this ratio is less than 1.2, the company's current liquidity is considered to be too low. Conversely, when this value is greater than 2.0, it is said that the company has excess liquidity (in the case of excess liquidity, such companies may have too much inventory, indicating inefficient management of the company), which is also considered a negative phenomenon.

In such a situation, when using the classic set theory to assess this indicator, there is a sharp boundary between the two sets for the ratio values of 1.2 and 2.0. If one company reported a current ratio, for example, at 1.19, it would be classified as an invalid value, or negative, while if the second company reported this ratio of 1.2, it would be considered as a valid, positive value in an assessment of the company's risk of bankruptcy, even though the values of the two firms differ by only 0.01. The interpretation of the values of these indicators (e.g., liquidity) is further complicated by the fact that different literature sources give different reference limits for each indicator.

The application of fuzzy sets changes the assessment of this problem. A current ratio with a value of 1.19 is considered partially correct and partially invalid. The degree of membership in the two sets depends on the shape of the membership function.



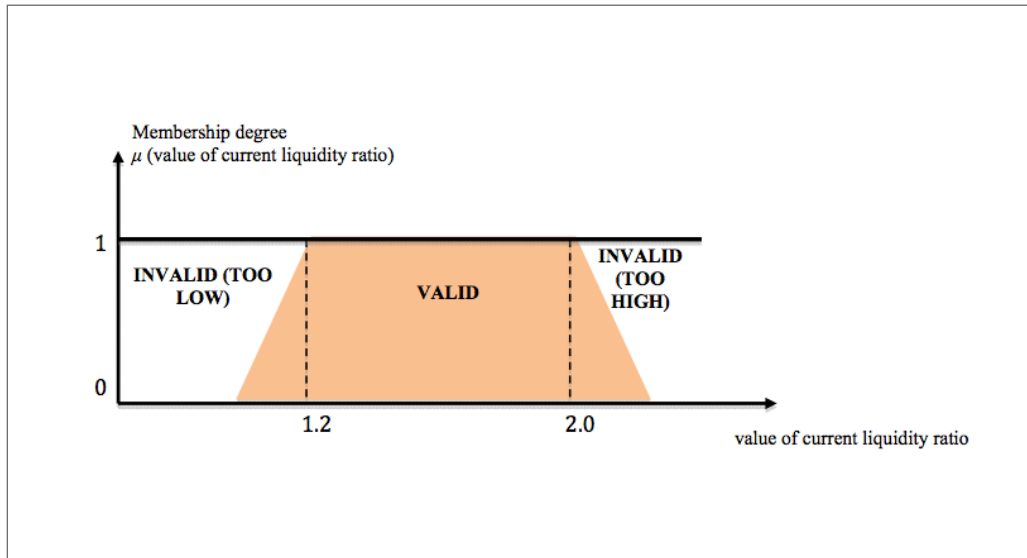


Figure 1. Example trapezoid membership function for current liquidity ratio

4. Assumptions of this study

In this study, the author developed a population of 166 companies from the service and manufacturing sectors. This study disregarded companies from the financial sector (banks and insurance companies) due to their different characteristics and the different financial indicators used in the ratio analysis of these companies.

Within this population of companies, the following characteristics were identified:

- ✓ Test sample no. 1 consisted of 60 Polish joint stock companies listed on the Warsaw Stock Exchange, wherein 30 of them were at risk of bankruptcy. The remaining 30 companies were enterprises in good financial condition. The surveyed companies came from different sectors, such as construction, the metal industry, food processing, chemicals, telecommunications, etc.;
- ✓ Test sample no. 2 consisted of 60 companies from Latin America. The sample was also a balanced sample, including 30 “good” and 30 “bad” companies. These companies came from different sectors of the economy, in such countries as Mexico, Argentina, Brazil, Chile, Peru, and Venezuela;

- ✓ Test sample no. 3 was comprised of 46 international and/or global companies (e.g., Coca-Cola). The test sample included 23 “bad” companies that were at risk of bankruptcy and 23 companies with impeccable financial records. The locations of these companies include countries such as the USA, Germany, France, Great Britain, Sweden, Japan, Finland, Taiwan, South Korea, and the Netherlands.

No training sample was used in this work, as the author tested the fuzzy logic models developed on the basis of his knowledge and previous 14 years of work experience. Models based on the use of fuzzy logic do not require any assumptions about the learning process, as they are developed on the basis of expertise.

For all 166 companies, the author calculated and used 34 financial variables (Table 2) to be analyzed for the period of one year and two years before the recognition of a company as “good” or “bad.” However, the study used financial statements for three years (498 balance sheets and 498 income statements) because some of the variables in the dynamic approach were studied between the first and second year and then between the second and third year. In test sample no. 1, the financial data were from 1999-2007; in test sample

Table 2. Financial variables used in the study

No.	Variable symbol	Description of financial variable
1.	CA	Current assets
2.	INV	Inventory
3.	STL	Short-term liabilities
4.	STR	Short-term receivables
5.	CSH	Cash and cash equivalents
6.	STI	Short-term investments
7.	GP	Gross profit
8.	OP	Operating profit
9.	TRS	Revenues from sales
10.	TR	Total revenues
11.	$(TRS / TA) * 100\%$	Turnover of total assets
12.	$(STL_t / STL_{t-1}) * 100\%$	Dynamics of short-term liabilities
13.	$(LTL_t / LTL_{t-1}) * 100\%$	Dynamics of long-term liabilities
14.	$[(NP_t + A_t) / (NP_{t-1} + A_{t-1}) * 100\%$	Dynamics of cash surplus (net profit and depreciation)
15.	$(NP + A) / TA$	Share of cash surplus in total assets
16.	$STL / (NP + A)$	Number of years of repayment of short-term liabilities
17.	$LTL / (NP + A)$	Number of years of repayment of long-term liabilities
18.	$(TA_t / TA_{t-1}) * 100\%$	Dynamics of total assets
19.	$(EQ_t / EQ_{t-1}) * 100\%$	Dynamics of equity
20.	$(LTC_t / LTC_{t-1}) * 100\%$	Dynamics of long-term capital (equity + long term liabilities)
21.	$(FA_t / FA_{t-1}) * 100\%$	Dynamics of fixed assets
22.	$(FA / TA) * 100\%$	Share of fixed assets in total assets
23.	$(CA / TA) * 100\%$	Share of current assets in total assets
24.	$(EQ / TA) * 100\%$	Share of equity in total assets
25.	$(INV / CA) * 100\%$	Share of inventories in current assets
26.	$(STR / CA) * 100\%$	Share of short-term receivables in current assets
27.	LTC / EQ	Ratio of fixed capital to equity
28.	$(TRSt / TRSt-1) * 100\%$	Dynamics of revenues from sales
29.	$(INvt / INvt-1) * 100\%$	Dynamics of inventories
30.	TRS / INV	Inventory turnover ratio
31.	$(STRt / STRt-1) * 100\%$	Dynamics of short-term receivables
32.	TRS / STR	Short-term receivables turnover ratio
33.	TRS / STL	Short term liabilities turnover ratio
34.	CA / STL	Current liquidity ratio



no. 2, the data were from 2000-2009; and in test sample no. 3, the data were from 1999-2009.

In addition, each company was described with a zero-one output variable, which grouped the populations in the two groups of companies as those with bad financial situations (variable = zero) and those with good economic conditions (variable = one).

In total, the author tested 13 models within the time periods of one year and two years prior to the categorization of the company as "good" or "bad," with three test samples (a total of 78 tests). Due to the limited length of this article, only two of these models are presented, which are the disaggregated current liquidity ratio model and the disaggregated ratio of coverage of fixed assets with long-term capital model.

The quality of the classification of all the models was evaluated based on the overall performance as well as Type I and Type II errors. Thus, the following formulas were used:

$$\square \text{ Type I error: } E_1 = (D_1 / BR) \cdot 100\%$$

where D_1 represents the number of companies with bad financial situations classified by the model or ratio as "good," and BR represents the number of companies in the test sample with poor economic conditions;

$$\square \text{ Type II error: } E_2 = (D_2 / NBR) \cdot 100\%$$

where D_2 represents the number of companies with impeccable financial records classified by the model or ratio as "bad," i.e., at risk of bankruptcy, and NBR represents the number of "good" companies in the test sample;

$$\square \text{ Overall model effectiveness: } S = \{1 - [(D_1 + D_2) / (BR + NBR)]\} \cdot 100\%.$$

5. Forecasting financial ratios

In this study, the author proposes the use of fuzzy logic for disaggregated financial ratios. The disaggregation of the indicators was aimed at not only the method of calculating the value of the index but also, and above all, the correct assessment of the factors affecting its level. Due to the limited length of this article, the author will present only two of the 13 models to show the development and operation of these models.

The first sample model is based on the disaggregation of the current liquidity ratio carried out by the author. In the author's opinion, in assessing the liquidity of companies, one should focus more on establishing

the critical factors affecting the level of liquidity and less on fixing their limits. This disaggregation was as follows:

$$CR = \frac{CSH}{STL} \times \frac{STI}{CSH} \times \frac{(CA - INV)}{(CA - INV - STR)} \times \frac{CA}{(CA - INV)} \quad (3)$$

Based on this disaggregation, we can conclude that the current ratio (CA / STL) is dependent on the quick ratio [(CA - INV) / STL] and the ratio of long-term current assets to medium-term current assets, i.e., exchangeable to cash in reasonable time [CA / (CA - INV)]. On the other hand, the quick ratio is dependent on two other measures, i.e., the ratio of the value of medium-term current assets to the value of short-term current assets [(CA - INV) / (CA - INV - STR)] and cash liquidity [(CA - INV - STR) / STL], which can be further disaggregated into the ratio of short-term investments to cash held by the company (STI / CSH) and the ratio of cash to short-term liabilities (CSH / STL).

The model developed by the author consists of the following:

- ✓ Six input variables: current assets (CA), inventories (INV), short-term receivables (STR), short-term investments (STI), cash (CSH) and short-term liabilities (STL). The model for each input variable identifies three fuzzy sets (which are subsets of the set field of values of a given input) and the corresponding membership functions:
 - "small" for low-level variables tested,
 - "medium" for medium-level variables,
 - "high" for high-level variables;
- ✓ Five decision centers, K1, K2, K3, K4 and K5, which are shown in Figure 2;
- ✓ One output, which represents the assessment of the current liquidity ratio on the basis of four factors affecting its level: the ratio of long-term current assets to medium-term current assets, i.e., exchangeable to cash within a reasonable time (K1 decision center), the ratio of the value of medium-term current assets to short-term current assets (K2 decision center), the ratio of short-term investments to cash held by the company (K3 decision center) and the ratio of cash to current liabilities (K4 decision center). Because this indicator is a nominant, the final assessment of the current liquidity ratio (K5 decision center)

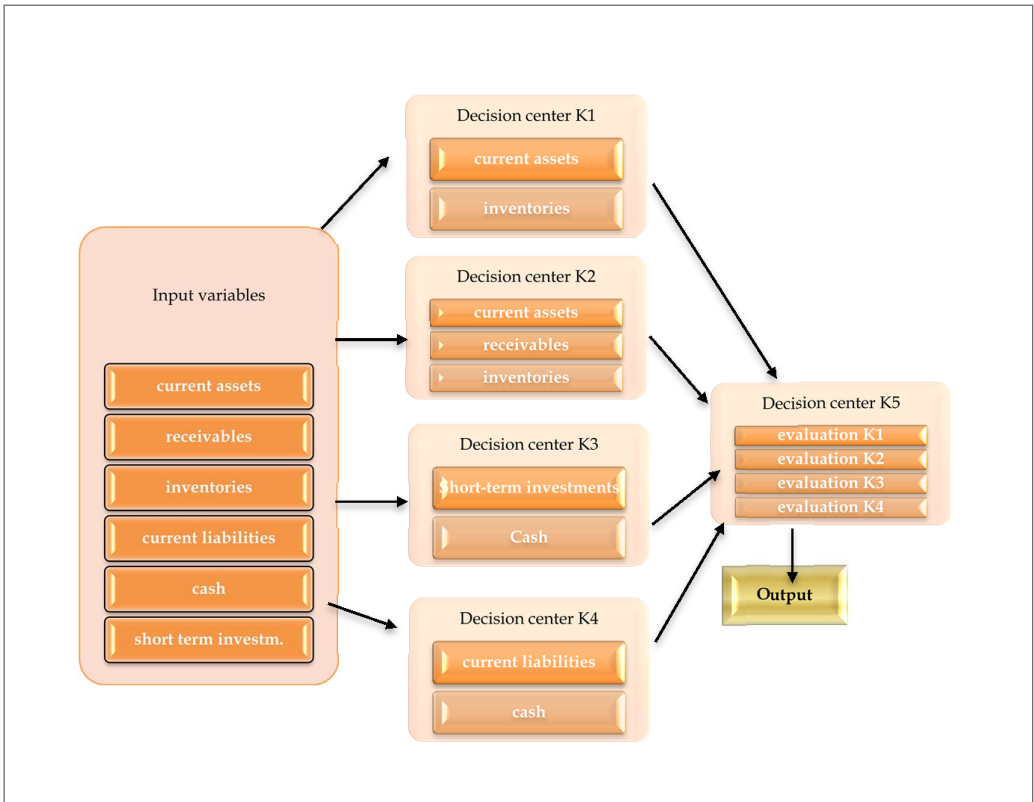


Figure 2. Structure of the fuzzy logic model for the disaggregated current liquidity ratio

is presented on a scale of 0 to 5, with five membership functions, as shown in Figure 3:

- “neg_low,” representing information about negative (dangerously low) levels of current liquidity,
- “low,” representing information about a low but safe level of this indicator,
- “medium,” indicating the optimal level of the indicator,
- “high,” representing a high, but still within optimal range, level of current liquidity,
- “neg_high,” describing a situation of too high a degree of liquidity of a company (indicating its mismanagement, affecting the decrease in company efficiency).

The levels of input variables are fuzzy values. Fuzzy sets and the shape of the membership function have been arbitrarily set by the author. The rating of the CA and STL

variables (“small,” “medium” or “high”) is based on an analysis of their share in total assets (from 0% to 100%). On the other hand, the variables INV and STR are presented as a percentage of current assets from 0% to 80%, and the CSH and STI variables are presented as a percentage of current assets from 0% to 50%, as shown in Figure 4. These variables have to be presented in relative form. If they are presented in the monetary form, these values, such as a value of assets of one million USD, could be high for one company but low for another.

The outputs of the decision centers K1, K2, K3 and K4 are also inputs of the K5 decision center. In other words, the assessment of the current liquidity ratio is based on the partial evaluation of the various factors affecting liquidity. The assessment of these disaggregated factors takes one of three states: a low score, a medium score, or a high score.

Table 3 shows the developed decision rules for the decision blocks K1, K2, K3 and K4. The decision centers K1, K3 and K4 consist of 9 decision rules, and block K2 consists of 27 rules. For example, when the share of short-term investments in current assets is at a high level (membership function “high”), and the share of cash in current assets is at a low level (membership function “small”), the assessment generated for this disaggregated current liquidity factor will be of a high value (“high”), which is decision rule No. 7 for block K3. This high rating is also an input of decision block K5, where the final assessment of the current liquidity ratio is conducted (due to the limited length of this paper, the table with 81 decision rules developed for decision block K5 can be sent to readers as supplementary data).

Based on 81 decision rules, the final assessment of the current liquidity ratio (“result”) is assigned to one of five membership functions (Figure 3) from the situation indicating a liquidity shortage (“neg_low”) through optimal conditions, low, medium, high, or excess financial liquidity (“neg_high”).

Figure 4 shows the fuzzy sets with membership functions for the sample variable, cash. The share of cash in current assets in excess of 36% is classified as absolutely high (membership function “high”). The share of this variable in current assets less than 13% is considered absolutely low (membership function “small”). Values between 13% and 36% partially belong to individual states from low to high through medium.

The second sample model is developed for the ratio of the coverage of fixed assets (FA) by long-term capital to equity and noncurrent liabilities (LTC). The model uses the following input variables:

- dynamics of the equity in the current period t ($EQ_t / EQ_{(t-1)}$),
- dynamics of long-term capital in the current period t ($LTC_t / LTC_{(t-1)}$),
- dynamics of fixed assets in the current period t ($FA_t / FA_{(t-1)}$),
- dynamics of total assets in the current period t ($TA_t / TA_{(t-1)}$),
- share of fixed assets in total assets in the previous period $(t-1)$ ($FA_{(t-1)} / TA_{(t-1)}$),
- share of equity in financing total assets in the previous period $(t-1)$ ($EQ_{(t-1)} / TA_{(t-1)}$),

- ratio of long-term capital to equity in the previous period $(t-1)$ ($LTC_{(t-1)} / EQ_{(t-1)}$).

With such disaggregation of the ratio, the author obtained the following formula:

$$\frac{LTC}{FA} = \frac{\left(\frac{EQ_{(t-1)}}{TA_{(t-1)}} \times \frac{EQ_t}{TA_t} \right) \times \left(\frac{LTC_{(t-1)}}{EQ_{(t-1)}} \times \frac{LTC_t}{EQ_t} \right)}{\frac{FA_{(t-1)}}{TA_{(t-1)}} \times \frac{FA_t}{TA_t}} \quad (4)$$

Using disaggregation, an analyst can compare the growth rate of equity and long-term capital with the growth rate of fixed assets and total assets. For example, a situation in which equity is maintained at a constant level and the value of long-term capital grows may mean that there is an increase in long-term capital due to the growth of long-term liabilities. In addition, by analyzing the growth rate of the dynamics of fixed assets and total assets, we can assess the degree of danger of a situation in which assets are not fully financed by long-term, stable capital.

For each input variable in the dynamic aspect, three membership functions were developed representing the decline in value (“decrease” membership function) and the relative stability (“stable” membership function) or an increase in value (“increase” membership function). For the input variables in the static aspect, the same conditions were used (low, medium, high). It should also be noted that each of these variables has a different course in the membership function. A comparison of Figures 5 and 6 shows that for the share of fixed assets in total assets, a wider range of values of functions was used that belonged to the “medium” fuzzy set, representing the average state of this variable in the case of the share of equity in total assets. The author proceeded on the basis that the proportion of equity, with an average value of 50% and fluctuations from 30% to 70% (depending on the sector of the economy and the business cycle), represents the average level of the index, whereas in the case of the share of fixed assets, it is much more difficult to determine the level of this variable as medium (“medium” membership function) because it depends heavily on the type of business. Therefore, the set of values equivalent



Table 3. The set of decision rules for individual decision centers of the current liquidity ratio model (K1, K2, K3, K4)

Decision center K1				Decision center K2				
No.	If "CA" is:	If "INV" is:	Then "K1" is:	No.	If "CA" is:	If "STR" is:	If "INV" is:	Then "K2" is:
1.	Small	Small	Low	1.	Small	Small	Small	Medium
2.	Small	Medium	Low	2.	Small	Small	Medium	Medium
3.	Small	High	Medium	3.	Small	Small	High	Medium
4.	Medium	Small	Low	4.	Small	Medium	Small	Medium
5.	Medium	Medium	Medium	5.	Small	Medium	Medium	Medium
6.	Medium	High	High	6.	Small	Medium	High	Low
7.	High	Small	Medium	7.	Small	High	Small	Medium
8.	High	Medium	High	8.	Small	High	Medium	Low
9.	High	High	High	9.	Small	High	High	Low
Decision center K3				10.	Medium	Small	Small	High
No.	If "STI" is:	If "CSH" is:	Then "K3" is:	11.	Medium	Small	Medium	Medium
1.	Small	Small	Low	12.	Medium	Small	High	Medium
2.	Small	Medium	Low	13.	Medium	Medium	Small	Medium
3.	Small	High	Low	14.	Medium	Medium	Medium	Medium
4.	Medium	Small	High	15.	Medium	Medium	High	Medium
5.	Medium	Medium	Medium	16.	Medium	High	Small	Medium
6.	Medium	High	Low	17.	Medium	High	Medium	Medium
7.	High	Small	High	18.	Medium	High	High	Low
8.	High	Medium	High	19.	High	Small	Small	High
9.	High	High	Medium	20.	High	Small	Medium	High
Decision center K4				21.	High	Small	High	High
No.	If "CSH" is:	If "STL" is:	Then "K4" is:	22.	High	Medium	Small	High
1.	Small	Small	Medium	23.	High	Medium	Medium	High
2.	Small	Medium	Low	24.	High	Medium	High	Medium
3.	Small	High	Low	25.	High	High	Small	Medium
4.	Medium	Small	Medium	26.	High	High	Medium	Medium
5.	Medium	Medium	Low	27.	High	High	High	Low
6.	Medium	High	Low					
7.	High	Small	High					
8.	High	Medium	Medium					
9.	High	High	Low					

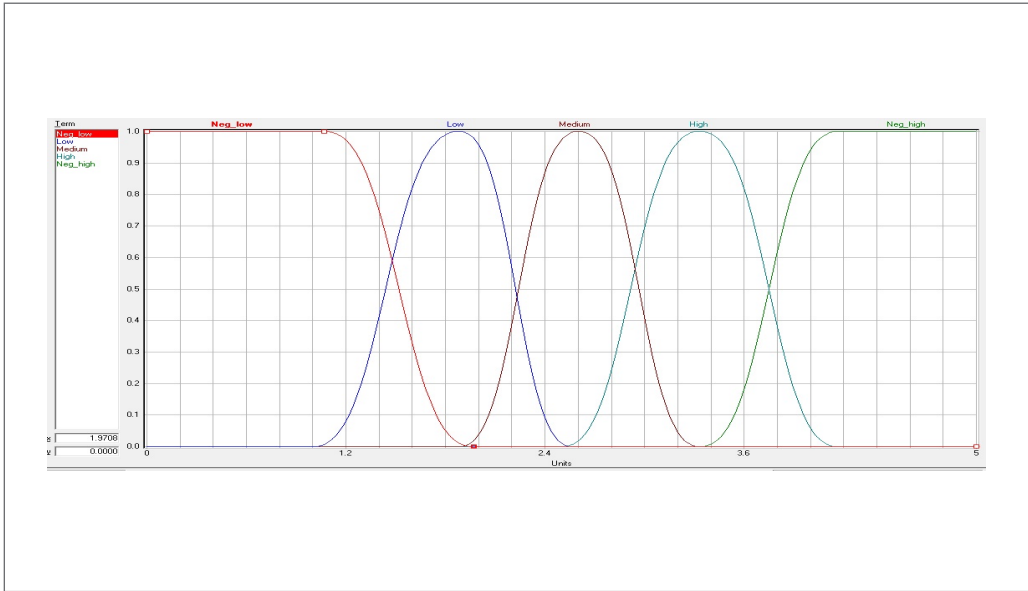


Figure 3. Membership functions for the disaggregated current liquidity ratio
Source: The author's own study (FuzzyTech software).

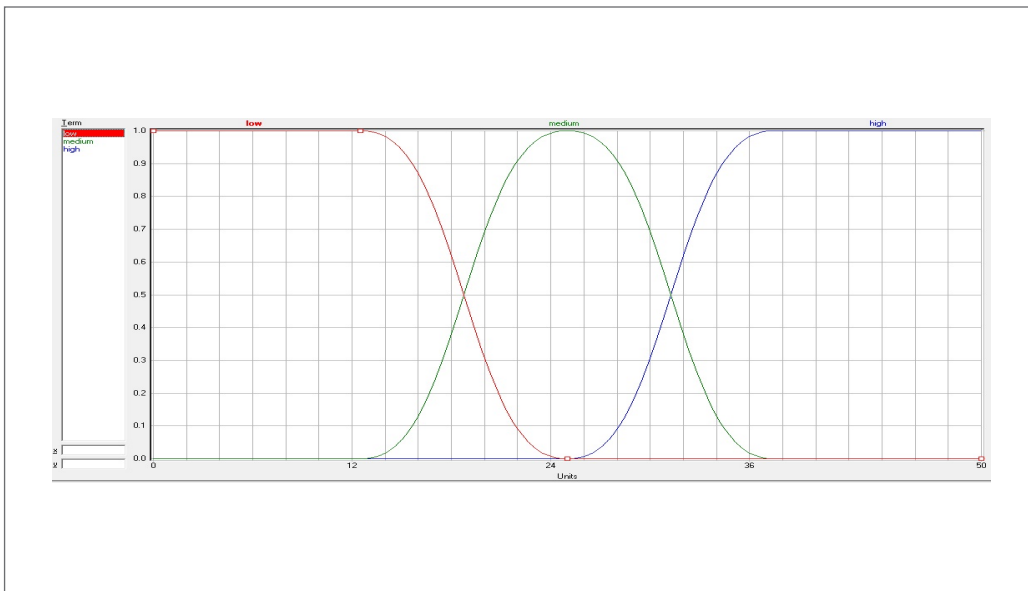


Figure 4. Fuzzy sets for the input variable "cash" with the membership functions
Source: The author's own study (FuzzyTech software).

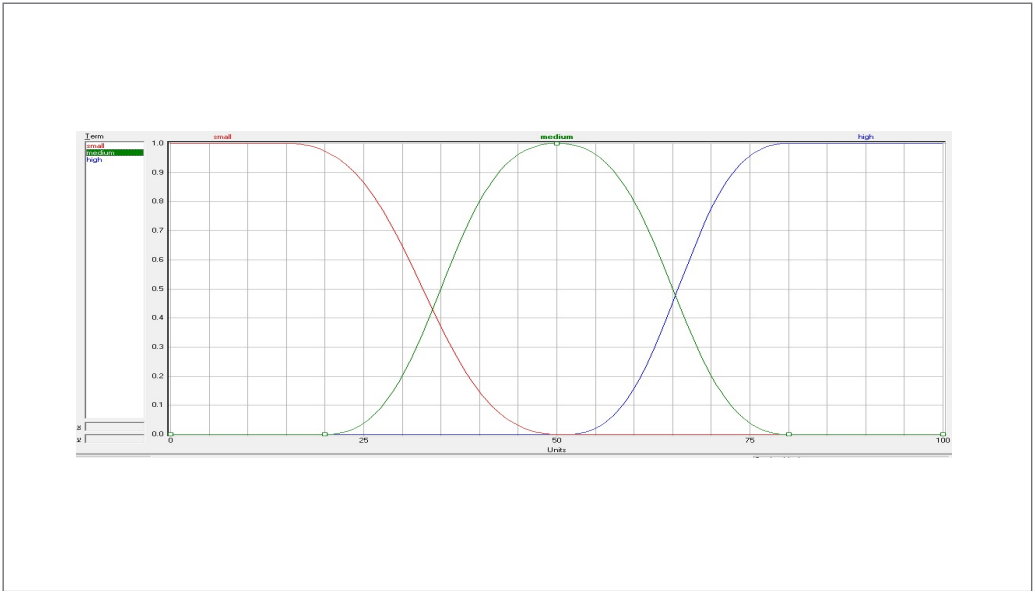


Figure 5. Membership functions for the input variable “share of fixed assets in total assets” (FuzzyTech software)
 Source: The author’s own study (FuzzyTech software).

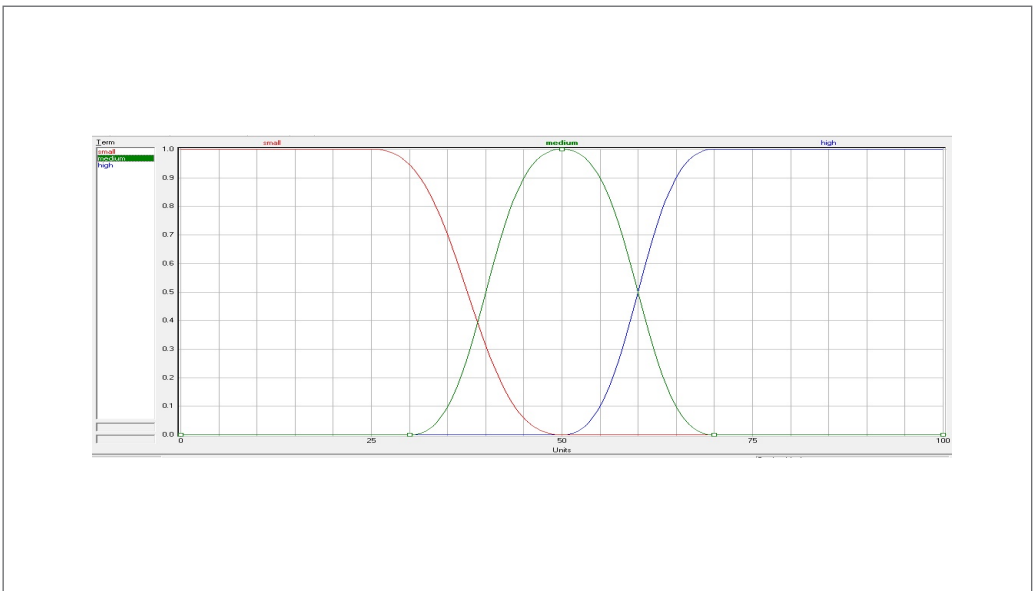


Figure 6. Membership functions for the input variable “share of equity in the financing of total assets” (FuzzyTech software)
 Source: The author’s own study (FuzzyTech software).

to the medium level was fuzzified, assuming a range of values from 20% to 80% (Figure 5).

For the last variable (ratio of long-term capital to equity), it was assumed that a value below 115% fully represents a low level (100 percent belonging to the set of values) because the ratio does not take values lower than 100%. When this variable has a value above 170%, it is classified as possessing full membership in the set, showing a high-ratio status.

The decision process of this model also consists of three stages.

In the first stage, the decision centers K1, K2 and K5 allow the assessment of the variables in the dynamic aspect. More specifically, block K1 compares the dynamics of equity to the dynamics of total assets, block K2 evaluates the dynamics of long-term capital in relation to the dynamics of equity, and in block K5, the dynamics of fixed assets are assessed against the background of the dynamics of total assets. These centers generate four different evaluations: the state of decline ("decrease"), the state of slow decline ("stable_dec"), the state of slow growth ("stable_inc") and the state of growth ("increase"), depending on the rate of increase or decrease in the numerator and denominator in the disaggregation formula of the fixed assets coverage ratio by long-term capital.

The second stage assesses the impact of a decrease, slow decrease, slow increase, or increase in the individual variables on the value of the share of equity in the financing of total assets (K3 decision center), the share of fixed assets in total assets (K6 decision center) and the ratio of the amount of long-term capital to equity (K4 decision center). This assessment takes four possible states: low ("low"), medium low ("low_med"), medium high ("high_med") or high ("high"), according to the share of equity in the financing of total assets (K3 decision center), the ratio of long-term capital to equity (K4 decision center) and the share of fixed assets in total assets (K6 decision center). It is worth noting that in this model, the higher the rating of K3 and K4 and the lower the level of assessment of K6, the lower the risk for the company resulting from the structure of financing of fixed assets through relative stability, i.e., safe, long-term capital.

This assessment is made in the K7 decision center in the third and final stage. For example, according to decision rule No. 4 (in block K7), when the share of equity

in the financing of the total assets and the ratio of fixed capital to equity remain at a low level ("low" rating), and the share of fixed assets in total assets remains at a high level ("high" rating), then the entire ratio of the coverage of fixed assets by long-term capital receives a "low" score, which indicates the low share of long-term capital in the financing of illiquid assets, such as fixed assets. This situation is synonymous with a company's high risk of liquidity problems in financing these assets. Another example involves decision rule No. 41, presenting a situation in which the company has a medium high share of equity and a medium ratio of long-term capital to equity (both ratings "high_med"), and the share of fixed assets in total assets was rated as low ("low" rating). In this case, the ratio score is high ("high" rating), which is equivalent to a high level of coverage of fixed assets through long-term capital (low risk of bad financing of the company).

The evaluation of the two above models is shown below in Tables 4 and 5.

The model examining the current liquidity of the company generates an assessment of the ratio from the range of values from 0 to 5. The membership function informing the too low level ("Neg_low") of this measure ranges from 0 to 1.9, whereas values from 0 to 1.2 indicate full (100%) belonging to the set "Neg_low," and values above 1.9 (it should be noted that this is the scope of the generated ratio rating and not the value of the current liquidity ratio) indicate zero affiliation with the set. The author adopted the intersection point (value of 1.5) of the "Neg_low" with the "Low" function, representing a low but safe level of current liquidity, as the limit point, ranking companies as "good" and "bad" for all three test samples, i.e., regardless of the region of the enterprise's operations, the classification criterion was set the same.

Table 4 shows the resulting overall one-year and two-year efficiencies of the fuzzy logic model for the three test samples ("fuzzy" S1 and S2). These efficiencies were compared with the results obtained from the tests of the examined population of companies using the traditionally calculated current liquidity ratio ("traditional" S1 and S2). In all three test samples for the two-year analysis, the application of a model based on fuzzy logic clearly had a positive impact on improving the efficiency of the assessment of the companies' current liquidity. The greatest improvement

Table 4. Overall efficiency of the current liquidity model one year (S1) and two years (S2) prior to the classification of the companies' financial condition, including Type I (E1) and Type II (E2) errors

"Fuzzy"	ONE-YEAR ANALYSIS			TWO-YEAR ANALYSIS		
	Central European	Latin America	Global companies	Central European	Latin America	Global companies
S	90.0%	88.33%	76.09%	88.33%	81.67%	71.74%
E1	6.67%	10.0%	21.74%	10.0%	16.67%	26.09%
E2	13.33%	13.33%	26.09%	13.33%	20%	30.43%
"Traditional"	Central European	Latin America	Global companies	Central European	Latin America	Global companies
S	81.67%	81.67%	67.39%	80.0%	75.0%	60.87%
E1	20.0%	20.0%	34.78%	23.33%	26.67%	39.13%
E2	16.67%	16.67%	30.43%	16.67%	23.33%	39.13%

in performance was observed in test sample no. 3 with global companies in the two-year analysis (from 60.87% to 71.74%). The smallest increase in efficiency due to the use of fuzzy logic can be seen in test sample no. 2 (companies from Latin America). For the two-year analysis, the effectiveness increased by approximately 6 percentage points. The highest efficiency of the fuzzy logic model was observed in test sample no. 1 with Polish companies in the one-year analysis (90% efficiency).

Another element that is worth noting is the level of Type I and II errors generated by the fuzzy logic model and by the traditional ratio analysis. In both years analyzed, not only did the use of fuzzy logic result in a reduction in both types of errors but in addition, when applying the traditional ratio analysis, Type I errors were greater than Type II errors. Using fuzzy logic, the situation is reversed, i.e., Type I errors are reduced much more than Type II errors, which means that the level of Type I error was less than the level of Type II error. This positive trend occurred for both years in all three test samples analyzed. In the literature, a Type I error is considered more expensive than a Type II error. This conclusion is based on the classification of companies with a poor financial situation as a company with a positive financial situation. In this case, the purchase of the "bad" company's shares (Type

I error) indicates future losses initially resulting from a decline in trading of that company and ultimately from the company's bankruptcy. On the other hand, a Type II error indicates the loss of potential profits when resigning from the purchase of the "healthy" company's shares when believing that the company is "bad."

The highest Type I and Type II errors in both analyzed years in the traditional and fuzzy approaches were reported for the test sample containing global companies from different countries. These errors were at a level over 30% using the traditional ratio and over 20% with the use of fuzzy logic. On the other hand, the lowest errors were generated when testing the sample of companies from Central Europe.

For the ratio of the coverage of fixed assets by long-term capital, the membership function representing a high financial risk due to the low level of funding for these assets by long-term capital is in the range of 0 to 0.5. Ratings for the values of this ratio below 0.3 indicate full belonging to the set; therefore, these values were considered the companies' classifying values. However, for the traditional ratio analysis, the value of 100% was adopted for all three test samples. The value of 100% indicates a situation in which assets are fully financed by equity and long-term liabilities. Table 5 shows the efficiency obtained from the present model with Type I and Type II errors.



Table 5. Overall efficiency of the model of coverage of fixed assets with long-term capital one year (S1) and two years (S2) prior to the classification of the companies' financial condition, including Type I (E1) and Type II (E2) errors

"Fuzzy"	ONE-YEAR ANALYSIS			TWO-YEAR ANALYSIS		
	Central European	Latin America	Global companies	Central European	Latin America	Global companies
S	83.33%	80.0%	73.91%	83.33%	73.33%	63.04%
E1	16.67%	23.33%	21.74%	13.33%	26.67%	39.13%
E2	16.67%	16.67%	30.43%	20.0%	26.67%	34.78%
"Traditional"	Central European	Latin America	Global companies	Central European	Latin America	Global companies
S	81.67%	76.67%	67.39%	78.33%	71.67%	58.70%
E1	20.0%	26.67%	26.09%	20.0%	26.67%	43.48%
E2	16.67%	20.0%	39.13%	23.33%	30.0%	39.13%

The highest model efficiency was achieved with test sample no. 1, which consisted of companies from Central Europe, and the lowest efficiency was achieved with the sample of multinationals (test sample no. 3). All three test samples noted improvements in the accuracy of the assessment with the use of fuzzy logic (the greatest improvement was shown in the one-year analysis of test sample no. 3, with 6.52 percentage points).

A positive aspect of the fuzzy logic model is also a reduction in the number of errors of both types for the one-year and two-year analysis in all the test samples. In the one-year analysis of companies, the greatest improvement was observed for Type II errors in test sample no. 3 (from 39.13% to 30.43%). In the analysis covering two years, the largest decrease in the number of Type I errors was reported in test sample no. 1 (from 20% to 13.33%).

To enrich the conclusions of this research, the author presented additional information from the study application portion (Table 6). In Table 6, there is information about the mean and median values of two analyzed ratios (CR and LTC/FA) for nonbankrupt and bankrupt enterprises (separately) in all three test samples for the two-year period and one-year period of analysis. The data shows that Central European nonbankrupt enterprises were characterized with much higher current ratio (CR) values than compa-

nies from Latin America and global enterprises. However, in the case of companies at risk of bankruptcy in all three test samples, the values were similarly low, which means that Polish firms recorded a much higher decrease in current liquidity with an increasing risk of bankruptcy than the enterprises from other parts of the world.

In the case of the ratio of the coverage of fixed assets by long-term capital (LTC/FA), it can be noted that Latin American companies and global enterprises were characterized by higher values of this ratio than the firms from Central Europe (both "good" and "bad" companies). It can be seen that the companies with high risk of bankruptcy in Poland registered negative values of this ratio, which means that the firms generated profound losses causing negative values in stockholders' equity.

6. Conclusions and recommendations

This paper presents the efficiency problems related to the oldest and most popular method of financial analysis, i.e., ratio analysis. In the introduction, questions were raised about the topicality and, thus, the effectiveness of traditional ratio analysis in the current environment of the increasing globalization and computerization of enterprises. In addition, the global financial crisis that began in mid-2008 resulted in in-

Table 6. Historical data for the analyzed ratios (CR, LTC/FA)

RATIO		CR							
PERIOD		ONE-YEAR ANALYSIS				TWO-YEAR ANALYSIS			
		Median		Mean		Median		Mean	
Central European	NON	BANKR.	NON	BANKR.	NON	BANKR.	NON	BANKR.	
	1.647	0.569	2.838	0.779	2.309	0.701	3.041	0.744	
		Median		Mean		Median		Mean	
Latin American	NON	BANKR.	NON	BANKR.	NON	BANKR.	NON	BANKR.	
	1.321	0.452	1.853	0.644	1.541	0.65	1.975	0.843	
		Median		Mean		Median		Mean	
Global companies	NON	BANKR.	NON	BANKR.	NON	BANKR.	NON	BANKR.	
	1.454	0.502	2.035	0.705	1.884	0.818	2.273	0.85	
RATIO		LTC/FA							
PERIOD		ONE-YEAR ANALYSIS				TWO-YEAR ANALYSIS			
		Median		Mean		Median		Mean	
Central European	NON	BANKR.	NON	BANKR.	NON	BANKR.	NON	BANKR.	
	1.123	0.407	1.243	-3.517	1.101	0.804	1.251	-0.184	
		Median		Mean		Median		Mean	
Latin American	NON	BANKR.	NON	BANKR.	NON	BANKR.	NON	BANKR.	
	1.508	0.557	1.352	0.275	1.621	0.712	1.452	0.595	
		Median		Mean		Median		Mean	
Global companies	NON	BANKR.	NON	BANKR.	NON	BANKR.	NON	BANKR.	
	1.735	0.571	1.825	0.459	1.795	0.592	1.894	0.655	

creased uncertainty and complexity of the phenomena occurring in the global economy. Events such as the emergence of countries' bankruptcy risk (e.g., Greece) or the mass decline of their credit ratings directly and indirectly influenced companies' financial situations. The author hopes that these studies will be of interest to economic analysts, entrepreneurs, students and researchers in economic sciences because of the following contributions:

1. **The proposal of disaggregation, which allows not only the calculation of a financial indicator but**

also the further assessment of the impact of various financial measures on the size of the indicator. For example, by introducing a dynamic approach, an analyst can compare the dynamics of the factors (e.g., dynamics of selected liabilities in relation to the dynamics of selected assets) and evaluate them on the basis of their values from the previous period, specifically, how much an increase or decrease in these components affects the current state of the examined ratio. Thus, for example, breaking down the ratio of the cover-

age of fixed assets by long-term capital into the factors of the dynamic and static approaches allowed the author to clearly indicate the risks resulting from an inadequate capital structure in a studied company. Importantly, these risks were observed two years prior to the analysis. In the traditional approach (i.e., in the static approach and without the use of fuzzy sets), this indicator remained at a high, safe level; therefore, an analyst could not have noticed the warning signs of imminent risk.

2. The ranking of financial ratios in order of their most frequent use. This query was based on the literature studies involving 600 articles.
3. The presentation of the theoretical basis of the method, taking fuzzy logic into account.
4. The development of 13 fuzzy logic models for the most popular financial indicators with which companies' financial conditions were assessed.
5. The testing of the fuzzy logic models with three different test samples involving companies from the following countries: Poland, Germany, France, Japan, Taiwan, Sweden, Finland, South Korea, United Kingdom, Mexico, Argentina, USA, Brazil, Chile, Peru, and Venezuela.
6. The verification of the models' universality, i.e., the predictive properties of the proposed models based on a diverse population of 166 companies.
7. A check of the behavior of the models through an extension of the analysis period to two years.
8. The comparison of the effectiveness of ratio analysis based on fuzzy sets with the effectiveness of traditional (zero-one) ratio analysis.

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