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Statistical Analysis and Robust Design of Circularly Polarized Antennas Using Sequential Approximate Optimization

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***Abstract*—In the paper, reliable yield estimation and tolerance-aware design optimization of circular polarization (CP) antennas is discussed. We exploit auxiliary kriging interpolation models established in the vicinity of the nominal design in order to speed up the process of statistical analysis of the antenna structure at hand. Sequential approximate optimization is then applied to carry out robust design of the antenna, here, oriented towards increasing the yield (defined with respect to design specifications imposed on the maximum in-band axial ratio level as well as antenna reflection). Demonstration example of a planar CP antenna with two feed line stubs is provided. Despite a large number of independent geometry parameters, accurate yield estimation is possible at the cost of a hundred EM simulations of the structure, whereas yield improvement (from the initial value of 63.5 percent at the nominal design to 99 percent) is achieved at the cost of 300 EM analyses. Comparison with conventional Monte Carlo analysis is also provided.**

Keywords—Antenna design; circularly polarized antennas; axial ratio; EM-driven design; statistical analysis; robust design

I. INTRODUCTION

Design closure of antennas is a process of adjusting the values of geometry and/or material parameters to ensure satisfaction of performance specifications imposed on the structure. In vast majority of reported works (e.g., [1]-[8]) nominal design is considered, i.e., assuming perfect agreement between nominal and actual (fabricated) structure dimensions. In practice, various uncertainties, either due to manufacturing tolerances or technological spread of material parameters, should be taken into account. In robust design (also referred to as tolerance-aware or yield-driven design) [9], [10], the objective is to maximize the probability of the fabricated component to satisfy given performance requirements under the assumed statistical deviations from the nominal parameter values [11].

Statistical analysis (in particular, yield estimation) is a fundamental step of robust design procedures [12]. Due to complexity of contemporary antenna structures, reliable performance evaluation requires expensive full-wave electromagnetic (EM) analysis. Consequently, conventional statistical analysis methods, primarily, Monte Carlo analysis may be computationally prohibitive. There have been various techniques proposed over the years to expedite the process. Some of the methods rely on replacing expensive EM simulations by auxiliary models (surrogates), e.g., response surface approximation models [13] or polynomial chaos expansion [14]. Here, a major disadvantage comes from the curse of dimensionality, i.e., a rapid growth of the number of training data samples for surrogate model construction with the increase of the number of parameters. Certain methods attempt to reduce the problem dimensionality, e.g., principal component analysis (PCA) [15]. Another approach is to utilize physics-based surrogate models, e.g., space mapping (SM) [16]. The limitation of space mapping is unavailability of fast coarse models (such as equivalent networks) for majority of antenna structures. To reduce the cost of building the surrogate model it is also possible to exploit a particular structure of the system response (through so-called feature-based modeling), as demonstrated in [17] for microwave filters, and in [18] for narrow-band antennas.

In this paper, a simple approach to statistical analysis and yield optimization of circular polarization antennas is discussed. A local kriging interpolation model is constructed in the vicinity of the nominal design that allows for fast yield estimation using Monte Carlo analysis. Robust design is then realized by means of sequential approximate optimization, i.e., optimizing the yield within the region of validity of the surrogate, and relocating the surrogate domain by centering it around the newly created approximation of the optimum. Here, the yield is calculated with respect to design specifications imposed on axial ratio of the antenna with constraints on its reflection response.

II. STATISTICAL ANALYSIS OF CP ANTENNAS USING SURROGATE MODELS

Let $\boldsymbol{x}^0$ be a nominal design, which is optimized for a specific set of performance requirements. In case of circular polarization (CP) antennas, one of the objectives is minimization of the axial ratio (AR) within a frequency range of interest, another one is ensuring sufficient impedance bandwidth. We will denote by $AR(\boldsymbol{x})$ the maximum AR within operational bandwidth, and by $S(\boldsymbol{x})$ the maximum $|S_{11}|$ within the same band. Thus, the nominal design is obtained as

$$\boldsymbol{x}^0 = \arg\min_{\boldsymbol{x}} AR(\boldsymbol{x}) \qquad (1)$$

subject to $S(\boldsymbol{x}) \leq -10$ dB.

Let $AR_{\max}$ be the acceptance threshold for the axial ratio. The function $H(\boldsymbol{x})$ is defined as follows

$$H(\boldsymbol{x}) = \begin{cases} 1 & \text{if } AR(\boldsymbol{x}) \leq AR_{\max} \text{ and } S(\boldsymbol{x}) \leq -10 \text{ dB} \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

We assume that the fabricated antenna exhibits certain deviations $d\boldsymbol{x}$ from $\boldsymbol{x}^0$ due to manufacturing tolerances. The yield Y can be estimated as [17]

$$Y(\boldsymbol{x}^0) = \frac{\sum_{j=1}^{N} H(\boldsymbol{x}^0 + d\boldsymbol{x}^j)}{N} \qquad (3)$$

where $d\boldsymbol{x}^j$, $j = 1, \ldots, N$, are random vectors sampled according to the assumed probability distribution. This could be, e.g., independent normal distributions with zero mean and certain variance σ (e.g., 0.017 mm for standard chemical etching process), or uniform with a specified maximum deviation. In general, other uncertainties can also be taken into account (e.g., those concerning substrate parameters).

Clearly, performing Monte Carlo analysis (3) directly on the EM simulation model of the antenna is prohibitive, especially if the number of geometry parameters is large (Monte Carlo is slowly convergent and reliable yield estimation requires a large number of samples). Here, in order to speed up the analysis, it is performed on a fast surrogate model, which is created in the vicinity of the nominal design. The modeling technique of choice is kriging interpolation [19], whereas the model domain is $[\boldsymbol{x}^0 - \boldsymbol{d}, \boldsymbol{x}^0 + \boldsymbol{d}]$, where components of the vector $\boldsymbol{d}$ are set to be 3σ. With this choice, almost all normally distributed deviation vectors $d\boldsymbol{x}$ will be allocated within the surrogate domain.

III. Robust Design Using Sequential Approximate Optimization

In this work, robust design is understood as maximization of the yield. It is realized by means of sequential approximate optimization, where the local kriging interpolation surrogate is optimized (in the statistical sense), and reconstructed in a relocated domain.

Let $Y_s(\boldsymbol{x};\boldsymbol{y})$ be the yield estimated at the design $\boldsymbol{x}$ using the surrogate model established at the design $\boldsymbol{y}$. The surrogate model domain has to be larger than $[\boldsymbol{y} - \boldsymbol{d}, \boldsymbol{y} + \boldsymbol{d}]$ (cf. Section II) in order to allow some movement of the yield evaluation region within the surrogate model domain. Here, we choose $[\boldsymbol{y} - 2\boldsymbol{d}, \boldsymbol{y} + 2\boldsymbol{d}]$ as the model domain, which means that the yield optimization can be done for the following lower and upper bounds: $\boldsymbol{y} - \boldsymbol{d} \leq \boldsymbol{x} \leq \boldsymbol{d} + \boldsymbol{y}$.

Yield maximization procedure works as follows ($\boldsymbol{x}^{(0)} = \boldsymbol{x}^0$ is the initial design):

1. Set $i = 0$;
2. Set up kriging interpolation surrogate at the design $\boldsymbol{x}^{(i)}$;
3. Maximize yield as
$$\boldsymbol{x}^{(i+1)} = \arg\min_{\boldsymbol{x}^{(i)}-\boldsymbol{d}\leq\boldsymbol{x}\leq\boldsymbol{x}^{(i)}+\boldsymbol{d}} \{-Y_s(\boldsymbol{x};\boldsymbol{x}^{(i)})\} \qquad (4)$$
4. Set up kriging surrogate model at the design $\boldsymbol{x}^{(i+1)}$;
5. If $Y_s(\boldsymbol{x}^{(i+1)};\boldsymbol{x}^{(i+1)}) > Y_s(\boldsymbol{x}^{(i)};\boldsymbol{x}^{(i)})$ set $i = i + 1$ and go to 2; else END.

Note that that surrogate model domain is relocated to the new design $\boldsymbol{x}^{(i+1)}$ if the iteration (4) is successful. Figure 1 shows the conceptual illustration of the robust design using sequential approximate optimization.

IV. Case Studies and Results

The operation and performance of the statistical analysis and robust design procedure of Sections II and III is demonstrated using the antenna shown in Fig. 2. The structure is implemented on a Taconic RF-35 ($\varepsilon_r = 3.5$, $h = 0.762$ mm). The antenna is based on the design of [20], where CP operation is obtained by means of two stubs connected to the feed line. Here, to improve the AR bandwidth, the low-impedance load is attached to one of the stubs. Also, the rectangular ground plane slots are removed. The antenna geometry is parameterized by nine variables $\boldsymbol{x} = [l_f\ l_3\ l_5\ w_{11}\ w_2\ w_4\ g_1\ g_{11}\ g_3]^T$, whereas $w_f = 1.7$ to ensure 50 ohm input impedance. Relative variables are $l_1 = 0.25\cdot2^{1/2}\cdot l_f$, $l_2 = 0.5\cdot2^{1/2}\cdot l_f$, $w_1 = 0.5\cdot2^{1/2}\cdot w_2$, $w_3 = 2.2l_4$, $l_4 = 0.55w_3$, $o = w_{11}$. The unit for all dimensions is mm.

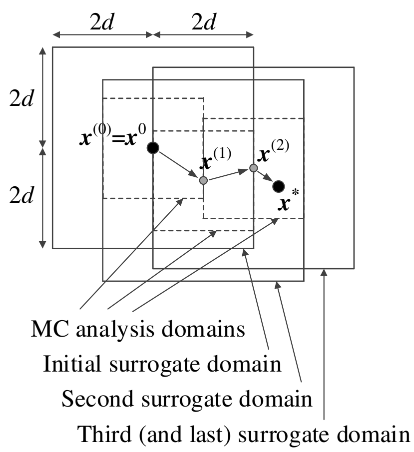
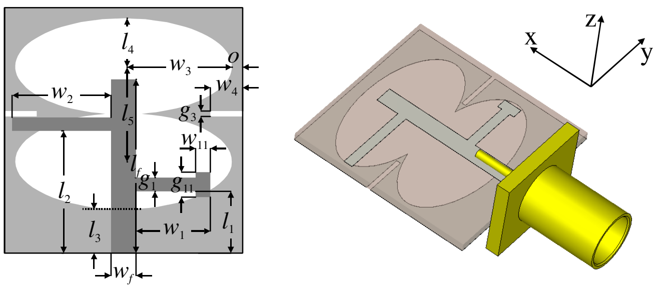


Fig. 1. Sequential approximate optimization for robust design. The surrogate model domain of the size $4\boldsymbol{d}$ (cf. Section II) is relocated after each iteration to the new design $\boldsymbol{x}^{(i+1)}$. The Monte Carlo (MC) analysis domain is $[\boldsymbol{x} - \boldsymbol{d}, \boldsymbol{x} + \boldsymbol{d}]$, whereas components of $\boldsymbol{d}$ are 3σ (variance of the normal distribution pertinent to assumed manufacturing tolerances). The maximum shift of the MC domain (therefore, the design) is $\boldsymbol{d}$ so that the analysis is still performed within the surrogate model domain. In the example shown here, the optimization process is accomplished after three iterations.

The computational model of the antenna is implemented in CST Microwave Studio [21] and simulated using its time domain solver (1,700,000 mesh cells, evaluation time 10 min). The model contains an SMA connector. The operational bandwidth (reflection- and AR-wise) is 5 GHz to 7 GHz.

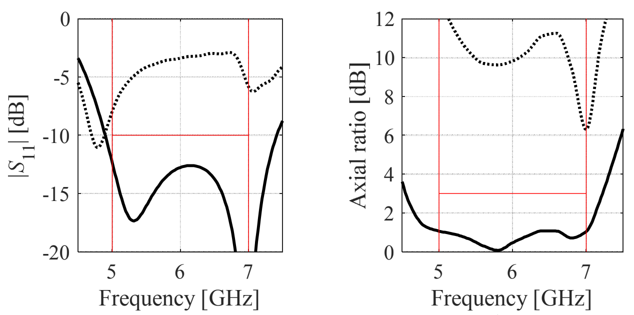
The initial design is $\boldsymbol{x}^{\text{init}}$ = [15 3 6.5 1 7.5 7.5 0.85 1.7 0.4]^T mm. The nominal design $\boldsymbol{x}^0$ = [13.05 3.15 6.40 0.96 7.24 2.28 1.93 2.52 0.33]^T mm is obtained by solving the problem (1) using a trust-region-based gradient search algorithm. Figure 3 shows the reflection and axial ratio characteristics for the initial and the nominal designs. The maximum in-band AR at $\boldsymbol{x}^0$ is about 1.1 dB.

For the purpose of statistical analysis, we assume the maximum acceptable in-band axial ratio of 1.2 dB. The kriging interpolation model has been constructed using 100 data samples with the model domain of the size $\boldsymbol{d}$ = [0.1 … 0.1] mm. Table I shows the yield estimated using the surrogate model and 5000 random samples, assuming Gaussian distribution with the variance of 0.017 mm, and uniform distribution (with the maximum deviation of 0.05 mm). For the sake of comparison, conventional Monte Carlo analysis has been performed using 500, 1000, and 2000 samples. It can be observed that surrogate-assisted yield estimation is accurate, and, clearly, dramatically less expensive than EM-based MC. Furthermore, MC realized with small number of samples (here, 500) is not particularly reliable due to relatively high variance of yield estimation. 2,000 samples are needed to obtain stable results. Figure 3 shows visual comparison of yield estimation using the surrogate and MC with 2,000 samples.



(a) (b)

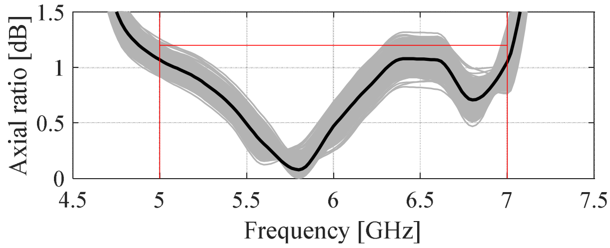
Fig. 2. Compact circular polarization antenna for statistical analysis and robust design demonstration: (a) structure geometry, (b) visualization of the antenna EM model.



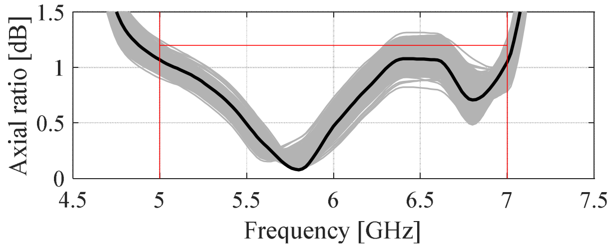
(a) (b)

Fig. 3. Compact CP antenna of Fig. 2 at the initial design $\boldsymbol{x}^{init}$ (- - -) and the nominal design $\boldsymbol{x}^0$ (—): (a) reflection characteristics, (b) axial ratio.

Subsequently, yield optimization has been performed using the methodology of Section III. Table II shows the results indicating that the yield can be improved significantly both in case of Gaussian and uniform distributions. Visualization of the yield estimation at the nominal and optimized designs is shown in Figs. 4 and 5.



(a)



(b)

Fig. 3. Monte Carlo analysis of the antenna of Fig. 2 assuming Gaussian distribution with σ = 0.017 mm, using (a) EM simulations and (b) kriging surrogate. The black plot indicates the response at the nominal design. Horizontal line denotes 1.2 dB level (acceptance level for axial ratio), whereas vertical lines indicate the frequency range of interest (5 GHz to 7 GHz).

TABLE I: YIELD ESTIMATION RESULTS FOR CP ANTENNA

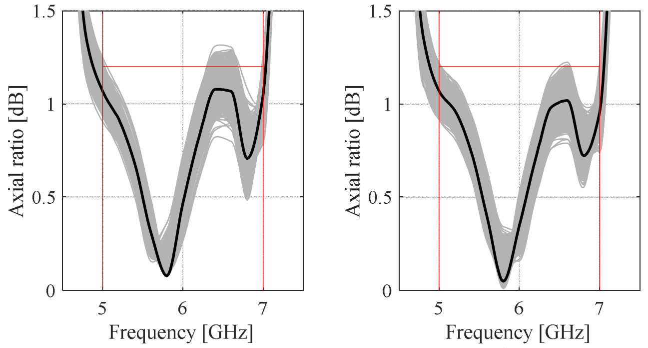
Case	Geometry Parameter Deviations	Yield Estimation Method	Estimated Yield	CPU Cost[1]
I	Gaussian σ = 0.017 mm	Surrogate modeling (this work)	0.937	100
		EM-based Monte Carlo	0.921	500
		EM-based Monte Carlo	0.912	1,000
		EM-based Monte Carlo	0.907	2,000
II	Uniform max. deviation 0.05 mm	Surrogate modeling (this work)	0.635	100
		EM-based Monte Carlo	0.610	500
		EM-based Monte Carlo	0.618	1,000
		EM-based Monte Carlo	0.617	2,000

[1] Estimation cost in number of EM analyses. Feature-based yield estimation utilizes N = 5000 random samples.

TABLE II: YIELD OPTIMIZATION OF CP ANTENNA

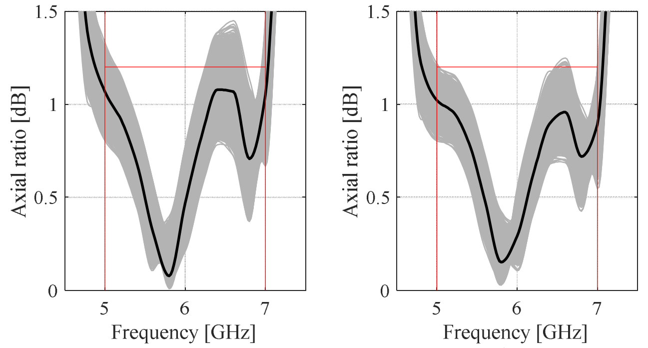
Case	Geometry Parameter Deviations	Yield Status	Estimated Yield	Optimization cost[1]
I	Gaussian σ = 0.017 mm	Initial	0.937	200
		Optimized	0.998	
II	Uniform max. deviation 0.05 mm	Initial	0.635	300
		Optimized	0.990	

[1] Number of EM simulations of the antenna structure.



(a) (b)

Fig. 4. Monte Carlo analysis of the antenna of Fig. 2 using kriging surrogate and assuming Gaussian distribution with $\sigma = 0.017$ mm: (a) at the nominal design, (b) at yield-optimized design. The black plot indicates the response at the nominal design. Horizontal line denotes 1.2 dB level (acceptance level for axial ratio), whereas vertical lines indicate the frequency range of interest (5 GHz to 7 GHz).



(a) (b)

Fig. 5. Monte Carlo analysis of the antenna of Fig. 2 using kriging surrogate and assuming uniform distribution with maximum deviation of 0.05 mm: (a) at the nominal design, (b) at yield-optimized design. The black plot indicates the response at the nominal design. Horizontal line denotes 1.2 dB level (acceptance level for axial ratio), whereas vertical lines indicate the frequency range of interest (5 GHz to 7 GHz).

V. Conclusion

Reliable statistical analysis and robust design of circularly polarized antennas has been proposed using sequential approximate optimization. Our technique is simple to implement and permits considerable reduction of the computational cost compared to conventional EM-driven Monte Carlo analysis. For a demonstration example of a compact microstrip CP antenna, yield improvement (assuming uniform distribution of manufacturing tolerances) from 63.5 percent at the nominal design to 99 percent at yield-optimized design has been obtained at the cost of about 300 full-wave EM antenna simulations.

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