Verification of selected calculation methods regarding shear strength in beams without web reinforcement

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Abstract. The purpose of the article was to compare selected calculation methods regarding shear strength in reinforced concrete beams without web reinforcement. Several calculation methods were tested. This included codes: PN-EN 1992-1-1:2008, ACI 318-14 and *fib* Model Code for Concrete Structures 2010. The analysis also consists of authorial methods published in technical literature. Calculations of shear strengths were made based on experimental works found in literature. The shear strength ratios V_{test}/V_{calc} were chosen to be the yardstick of comparison, where V_{test} is the experimental shear strength and V_{calc} is the calculated shear strength.. A wide range of variables including shear span/depth ratio, compressive strength of concrete, longitudinal steel percentage helped to verify the applicability of calculation methods. Although most of authorial techniques proved to be unstable, they succeeded to show that codes' formulas for shear strength may still be improved. The presented article is a part of Authors' long term research in the matter and a new chapter of their study now concerning beams without web reinforcement.

1 Introduction

Over the years much research and many debates have taken place all around the world to better understand the shear mechanism in beams both with and without web reinforcement [1, 2]. Although many experiments and analysis have been carried out, the provisions regarding calculating shear strength provide results that often differ from experimental data [3, 4, 5]. As it was prior proved by the Authors in the context of reinforced and prestressed concrete beams with shear reinforcement [6], the understanding of stress distribution in the support zone is still to be improved. Although the reliability of most calculation methods was undeniable, economics was lacking. However is this observation also valid for beams without shear reinforcement?

1.1 Primary assumptions

In order to ensure the coherence of the analysis, initial assumptions were taken:

- there was no shear reinforcement in beams,

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- longitudinal reinforcement was either steel or GFRP bars,
- all cross sections were either rectangular or T-beams,
- all beams failed in shear,
- all beams were single – span,
- no limitation on material properties was imposed,
- beams with fiber - reinforced concrete were excluded from the analysis,
- in order to compare analytical results with experimental data, all units were taken without reduction factors.

1.2 Nomenclature

In order to ensure the clear - cut of the analysis, main parameters are named below:

a	shear span
b_w	web width
d	effective beam depth
f_c	compressive stress in concrete
f_y	yield stress of steel
ρ_l	longitudinal reinforcement ratio
KHH	method of Kuo, Hsu, Hwang
RFF	method of Rebeiz, Fente, Frabizzio
AFR	method of Ahmad, Fareed, Rafeeqi

2 Calculation methods

The fundamental approach to the shear problem is the truss analogy model presented by Mörsch [7]. The Strut-and-Tie model was based on two basic types of elements: compression struts and tension ties. Being modified many times, mostly on the value of the inclination angle, it became a leading model in most European Codes. In the following analysis PN-EN 1992-1-1:2008 [8] (eq. (1)) was chosen to be its representative. Experiments held by members of the American Concrete Institute proved that the truss analogy model does not cover all the variables that were affecting shear strength during laboratory tests. Therefore a semi - empirical formula was developed and adopted in ACI 318 - 14 [9] (eq. (4)). The last standard that was taken into consideration in the following analysis is *fib* Model Code 2010 [10] with its levels of approximation that each base on different shear model. In the presented analysis the first (eq. (5)) and the second (eq. (8)) level of approximation were analyzed.

In addition to the standards, four authorial methods published in technical literature were taken into consideration. The first two pose an attempt to improve ACI 318 provisions. Frosch Method [11] (eq. (10)) modified Standard's provisions in two ways. First it replaced the effective beam depth (d) with a cracked transformed section neutral axis depth (c). Secondly it eliminated longitudinal reinforcement ratio (ρ_l) as it is already taken into account in computing the value of c. Another attempt to modify ACI 318 provisions was carried by Korean scientists Kuo, Hsu and Hwang [12] (eq.(13)). The KHH method adopted a function of a shear span to effective depth radio ($(a/d)^{-0,7}$) introduced at Huston University [12] that reflects the arch action in the beam. Following Frosch's expression it uses the depth of uncracked compression zone. A different method, introduced by Rebeiz, Fente, Frabizzio [13] (eq.(14)) was developed using the techniques of dimensional analysis and multiple regression analysis. The RFF method also takes into account the differences between short and long beams in terms of shear behavior. The last method by Ahmad, Fareed, Rafeeqi [14] (eq.(16)) for normal and light weight reinforced concrete slender beams without web reinforcement was based on the analysis of predictive accuracy of empirical equations used in different codes and also authorial methods.

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2.1 Empirical equations for shear capacity in concrete

$$V_{PN\text{-}EN} = max\{[C_{rd,c}\ k\ (100\ \rho_l)^{1/3}+k_1\ \sigma_{cp}]\ b_w\ d;\ (v_{min}+ k_1\ \sigma_{cp})\ b_w\ d\} \quad (1)$$

where:

$$C_{rd,c} = 0.18/\gamma_c \quad (2)$$

(note: following primary assumptions all units were taken without reduction factors)

$$k = min\{1+(200/d)^{1/2};\ 2\} \quad (3)$$

$$V_{ACI} = 0.17\ f_c^{1/2} b_w\ d \quad (4)$$

$$V_{MC,I} = k_v\ f_c^{1/2} b_w\ z \quad (5)$$

where:

$$k_v = 180/(1000+1,25z) \quad (6)$$

$$z = 0,9d \quad (7)$$

$$V_{MC,II} = k_v\ f_c^{1/2} b_w\ z \quad (8)$$

where:

$$k_v = (0.4/(1+1500\varepsilon_x))/(1300/(1000+k_{dg}\ z)) \quad (9)$$

$$V_{Frosch} = 0.42\ f_c^{1/2} b_w\ c \quad (10)$$

where:

$$c = (2\ \rho_l\ n+(\rho_l\ n)^2)^{1/2} - \rho_l\ n \quad (11)$$

$$n = E_S/E_C \quad (12)$$

$$V_{KHH} = min\{1.17(a/d)^{-0,7}\ f_c^{1/2} b_w\ c;\ 0.83\ f_c^{1/2} b_w\ c\} \quad (13)$$

$$V_{RFF}=(0,4+(f_c\ \rho_l(d/a))^{1/2}(10 - 3A_d))b_w\ d \quad (14)$$

where:

$$A_d = \{a/d \text{ for } 1.0<a/d<2.5;\ 2.5 \text{ for } a/d \geq 2.5\} \quad (15)$$

$$V_{AFR}=\xi\ 0,35(\ a/d\ f_c\ c)^{0.33}\ \rho_l^{0.1} \quad (16)$$

where:

$$\xi = \{1 \text{ for } d < 300\text{ mm};\ 17.32/(d)^{1/2} \text{ for } d \geq 300\text{ mm}\} \quad (17)$$

3 Beams database

The assessment of the effectiveness of the selected calculation methods was based on shear test results of beams published in technical literature. A numbet of 76 beams without shear reinforcement was considered. Table 1. presents properties of the analysed beams that were used while computing shear strength.

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Table 1. Properties of specimens.

Author	**No.**	**f_c[MPa]**	**P_l [%]**	**b_w [cm]**	**a/d**
Sokołowski [15]	3	55.0	4.49	10	1.5 - 3.5
Thamrin et. al [16]	14	13.0 - 33.5	0.6-2.5	12.5 - 13	2.3 - 3.8
Palaskas et. al[17]	3	32.0 - 32.8	0.5 - 0.7	19	3.9 - 4.1
Hanoon et. al[18]	4	19.9 - 60.1	1.1 - 3.5	18	1.9 - 2.1
Słowik [19]	25	35.0	0.9 - 1.8	12	1.8 - 4.1
Bakhmari, Ahmad [20]	27	44.8 - 52.7	0.6 - 1.1	15	2.0 - 6.0

4 Shear strength analysis

Fig. 1 presents distributions of shear strength ratios V_{test}/V_{calc} where V_{test} is the experimental shear strength and V_{calc} is the calculated shear strength by Codes' design procedures. Table 2 presents statistic data of the results. It can be seen that statistically ACI 318 - 14 [9] was the most accurate, however as the only method it happened to underestimate shear strengths. Underestimation means that the calculated value of shear strength was higher than the actual shear strength that was observed in laboratory tests. Therefore looking at the presented data it would be most reasonable to calculate shear strengths based on PN-EN 1992-1-1:2008 [8] or second level of approximation of *fib* Model Code 2010 [10]. The first level of approximation of *fib* Model Code 2010 [10] highly overestimated (calculated shear strength much lower than tested shear strength) most of the results.

Table 2. Statistic data for Codes.

method	PN-EN	ACI	MC I	MC II
average	2.03	1.34	3.04	2.29
standard diviation	0.78	0.52	1.14	0.86
variation coefficient [%]	38.59	38.50	37.72	37.62

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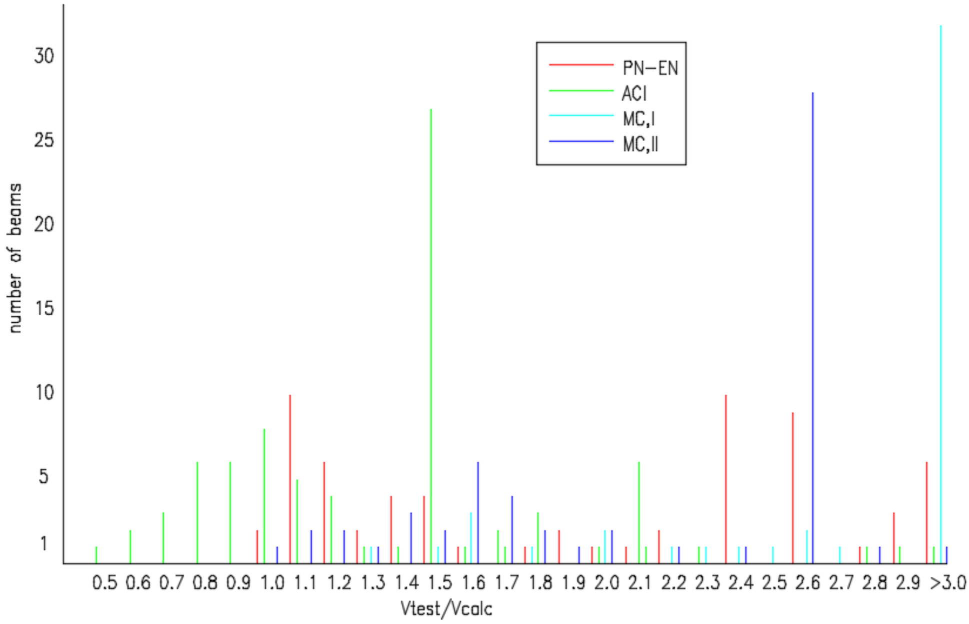

Fig. 1. Distributions of shear strength ratios for Codes.

Fig. 2 presents distributions of shear strength ratios V_{test}/V_{calc} where V_{test} is the experimental shear strength and V_{calc} is the calculated shear strength by Authorial design procedures. Table 3 presents statistic data of the results. The average ratios V_{test}/V_{calc} were not so much improved in comparison with Codes' results. Moreover the variation coefficients were higher comparing with Codes. Frosch method [11] and KHH method [12] led to a high number of uneconomical overestimations ($V_{test}/V_{calc} > 3.0$). In the Authors' opinion the only Authorial method that seems promising was the RFF method [13].

Table 3. Statistic data for Authorial Methods.

method	Frosch	RFF	KHH	AFR
average	3.64	1.79	2.92	1.25
standard diviation	1.37	0.72	1.18	0.56
variation coefficient [%]	37.62	40.38	40.44	44.79

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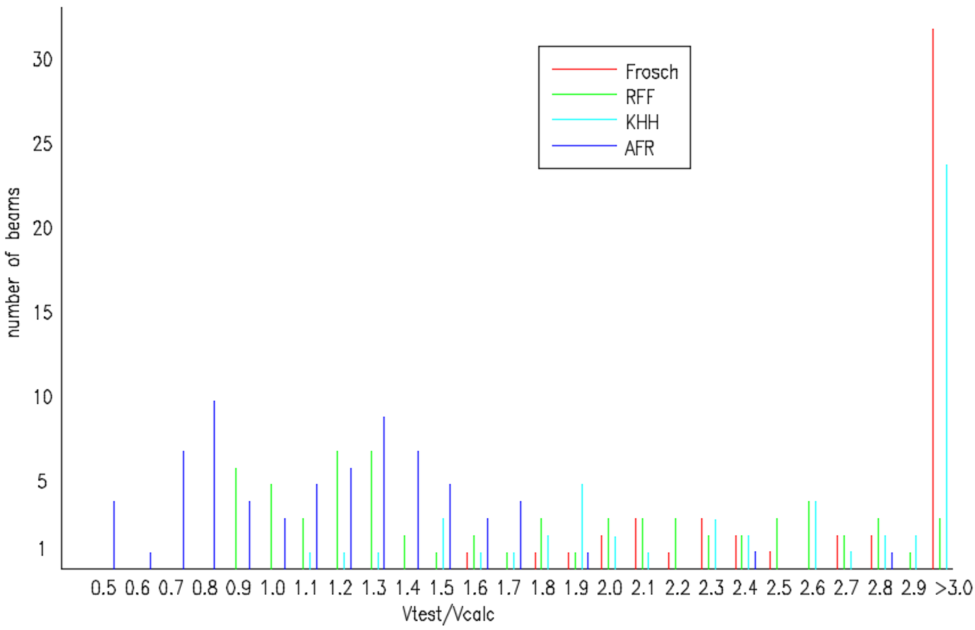

Fig. 2. Distributions of shear strength ratios for authorial methods.

5 Conclusions

The aim of the article was to compare selected calculation methods regarding shear strength in beams without web reinforcement. The analysis was limited due to the number of beams (76) and variety of properties affecting shear strength. Nevertheless it succeeded to prove that the provisions regarding shear strength could still be improved to give more accurate and reliable results. It should be noted that the aim of the article was not to decide which method is the most accurate, but to check if the available methods provide us with reliable results.

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