# An Improvement of Global Complex Roots and Poles Finding Algorithm for Propagation and Radiation Problems

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***Abstract*—An improvement of the recently developed global roots finding algorithm has been proposed. The modification allows to shorten the computational time by reducing the number of function calls. Moreover, both versions of the algorithms (standard and modified) have been tested for numerically defined functions obtained from spectral domain approach and field matching method. The tests have been performed for three simple microwave structures (open waveguides and conformal antenna resonator). The results have been verified and the increase of efficiency for the improved version of the algorithm has been confirmed.**

***Index Terms*—radiation, propagation, root finding, conformal antennas, dielectric waveguides.**

## I. Introduction

The complex function analysis is an important contribution to solving the propagation and radiation problems. The propagation coefficient of a shielded or unshielded waveguide is represented by a complex number (guided, leaky or complex modes), which usually represents losses or radiation. Similarly, a complex domain is required when finding resonant frequencies of radiators or resonators. In many cases, evaluation of these parameters boils down to finding the roots of more or less complicated function. Unfortunately, hardly ever those roots can be found analytically (even for simple methods, such as mode matching technique [1]). For more sophisticated methods, for instance field matching method [2] or spectral domain approach [3], [4] the problem is deepening (the function cannot be expressed in analytical form). Although many algorithms have been developed for the last few decades, root finding is still an open problem. The algorithms can be applied only for a certain class of functions or for restricted search regions. For local algorithms such as Muller's [5] or Newton's [6], the knowledge of an initial root value is required. Global root finding algorithms can be very efficient for polynomial functions [7], [8] and this fact caused the rise of many different techniques based on polynomial and rational approximation (for functions containing singularities). However, in some cases such approximation can bear little or no relation to the original function [9]. Moreover, the accuracy of the root obtained by these methods cannot be easily determined and local algorithms are usually used to its improvement. However, in local algorithms some of the roots might be omitted, due to the uncertain convergence (if the initial localization of the root is not sufficiently accurate). Another problem, arising during the rational approximation, is a random occurrence of spurious solutions [10].

Recently, a new global complex roots and poles finding method (GRPF) has been published [11]. It is based on the function phase analysis and does not require its derivative. It can be applied for a very wide class of problems defined on an arbitrary domain (also for functions containing branch cuts and singularities). Moreover, the efficiency of this algorithm is very high in comparison with the other established techniques, which has been confirmed in [11]. The algorithm consists of two main stages. In the first one the function is sampled in the nodes of triangular mesh evenly covering the assumed domain. Then the candidate edges (edges along which the function phase changes of more than two quadrants) are found and the triangles attached to these edges are combined into the candidate regions. Next, at the boundary of these regions discretized Cauchy's Argument Principle (CAP) is applied, in order to confirm the existence of roots or poles. In the second stage the accuracy of the evaluated roots/poles is improved by self-adaptive mesh refinement.

This paper is a continuation and extension of the publication [11]; a simple modification of the second stage of the GRPF algorithm is presented. In the original GRPF method, new nodes are added in the center of all the edges in the candidate regions. In the new approach, extra nodes are added only to the candidate edges. It is shown that the reduction of the nodes added in subsequent iterations can notably improve the convergence of that process. Such an improvement leads to the smaller number of the function evaluations, which results in much shorter processing time (especially if the function evaluation is time-consuming). The numerical tests are performed on analytically and numerically defined functions (obtained from spectral domain approach [3], [4] and field matching method [2]) for both original and modified GRPF algorithms. For a multi-valued function a new single-valued function obtained as a pointwise product of the all Riemann sheets can be used [12]. In the considered examples the convergence of the modified GRPF is up to two times faster than for the original one.

## II. Modified GRPF algorithm

The aim of the GRPF algorithm [11] is to find all the roots/poles of the considered function $f(z)$ in a fixed region $\Omega \subset \mathbb{C}$. The method is based on the function phase analysis and do not involve derivative of the function. The algorithm consists of two main stages and the modification concerns the second stage only. The first stage remains unchanged and begins with covering the domain with triangular mesh using Delaunay triangulation (with the initial mesh step $\Delta r$). Next, the function value at each node $z_n \in \Omega$ is calculated. Parameter $Q_n$ describes the quadrant in which the function value lies:

$$Q_n = \begin{cases} 1, & 0 \leq \arg f(z_n) < \pi/2 \\ 2, & \pi/2 \leq \arg f(z_n) < \pi \\ 3, & \pi \leq \arg f(z_n) < 3\pi/2 \\ 4, & 3\pi/2 \leq \arg f(z_n) < 2\pi \end{cases} . \quad (1)$$

The root or pole is located at the point, where regions of four different quadrants meet, as it is presented for the exemplary function in Fig. 1.

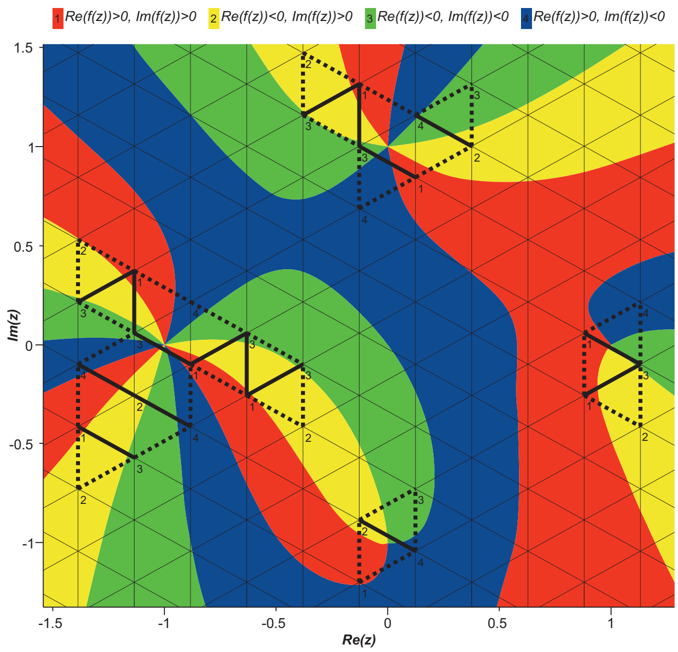


Fig. 1. The phase portrait of example function. The numbers (colors): 1 (red), 2 (yellow), 3 (green) and 4 (blue) represent the quadrants in which the function values lie. The candidate edges are marked by thick black lines. The black dotted lines represent the boundaries of the candidate regions [11].

An extra parameter $\Delta Q_p$ can be introduced to represent the difference in quadrants of the function values along the edge

$$\Delta Q_p = Q_{n_{p2}} - Q_{n_{p1}}, \qquad \Delta Q_p \in \{-2, -1, 0, 1, 2\}, \quad (2)$$

where $n_{p1}$ and $n_{p2}$ are nodes attached to edge $p$. Then, if $|\Delta Q_p| = 2$, the edge is marked as a "candidate edge" (close localization of the root/pole is expected). Next, all triangles attached to the candidate edges are combined into candidate regions. Then, the set $C$ of unique edges attached to candidate regions is created. The set $C$ can be divided into $k$ subsets $C^{(k)}$. All the edges from $C^{(k)}$ form a closed contour around the $k$-th candidate region. Then, the existence of roots or poles can be confirmed by discretized Cauchy's Argument Principle:

$$q = \frac{1}{2\pi} \sum_{p=1}^{P} \arg \frac{f(z_{p+1})}{f(z_p)}, \quad (3)$$

where $q$ represents difference between the sum of all roots counted with their multiplicities and the sum of all poles counted with their multiplicities. Since the $\Delta Q_p$ parameters are computed, (3) can be replaced by the following formula:

$$q = \frac{1}{4} \sum_{p=1}^{P} \Delta Q_p \quad (4)$$

and the localizations of the roots/poles are determined (with the accuracy lower than $\Delta r$).

In order to refine the accuracy of the solutions the second stage of the algorithm must be applied. In the original GRPF algorithm the new mesh points are added in the center of each edge from set $C$ (each edge in the candidate regions). In order to preserve well-conditioned mesh geometry (elimination of "skinny triangles") an extra second zone surrounding the candidate regions should be introduced. If the triangle in the second zone is ill-conditioned (the maximal to minimal triangle sides ratio is greater than three), the new point is added at the center of this triangle. Then, the triangulation involving the new nodes is performed again. As a result, a new self-adaptive mesh is obtained and once again the candidate regions (smaller) can be determined. The whole procedure is repeated until the assumed size of the region (accuracy $\delta$) is reached.

The modification of the GRPF algorithm concerns only its second stage (the refinement) and reduces the number of nodes added in the subsequent iterations. In the improved algorithm, instead of adding new nodes in the centers of all edges from the candidate region, only a single node is added in the center of candidate edge. Also, in this approach extra points must be added in the centers of ill-conditioned triangles (in both zones), but their number is not high. Moreover, an extra node must be added in the center of the edge from candidate region, if it is located at the boundary of $\Omega$. Despite the similarities of the both approaches (regular and the modified) the difference in the number of nodes can be quite high and, as it is shown in the numerical results, can reach 50%. An example of the modified second stage of the algorithm performance is presented in Fig. 2.

The proposed modification can notably reduce (even two times) the total number of function calls. For more sophisticated functions, its evaluation process can be the most time-consuming part of the algorithm, so the presented improvement can significantly reduce the total computational time.

## III. Numerical results

In order to verify the efficiency of the proposed method, the results of some numerical tests performed for three different

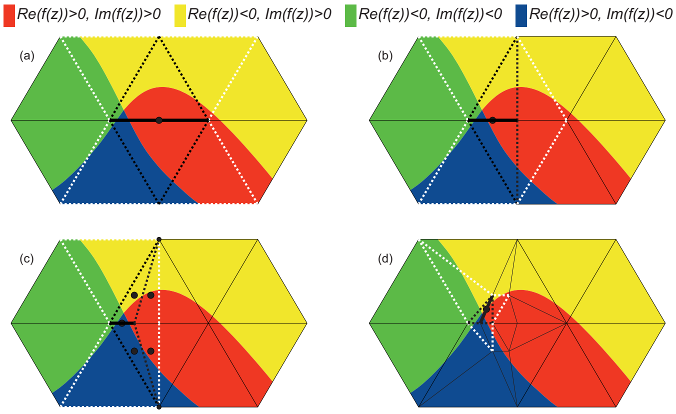


Fig. 2. Four subsequent iterations of the modified algorithm: a new mesh node is added in the half of the candidate edge (a), the same step is performed in the next iteration (b), except the point in the candidate edge ceter, four nodes are added in centers of "skinny triangles" (c), another node is added in the candidate edge (d)

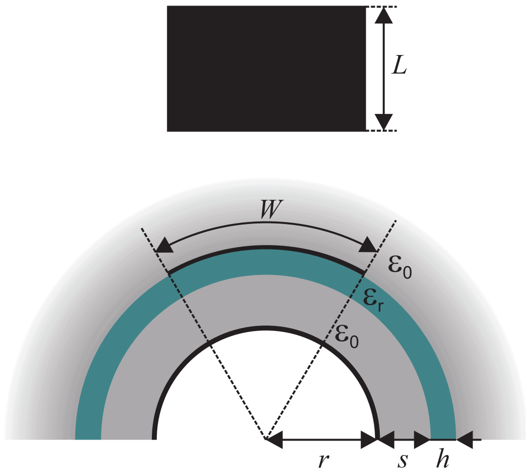


Fig. 3. Cylindrical-rectangular microstrip structure [3], where ground cylinder radius $r = 200$ mm, substrate permittivity $\varepsilon_r = 2.32$, and thickness $h = 2.4$ mm, patch dimensions $L = 80$ mm and $W = 168$ mm, air gap of thickness $s = 5$ mm

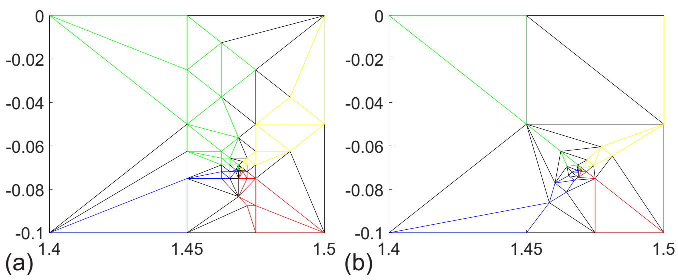


Fig. 4. Comparison of the self-adaptive meshes obtained for the structure presented in Fig. 3. The standard (a), and modified version of GRPF (b).

electromagnetic problems are presented. The computational time, the number of function evaluations and the accuracy of the obtained results are analyzed. The algorithm is implemented in MATLAB environment, and all the tests are performed using an Intel(R) Core i5-2500 CPU 3.30 GHz, 16-GB RAM computer.

### A. Cylindrical-Rectangular Microstrip Resonant Structure

The first example is a cylindrical-rectangular microstrip resonator, which geometry is presented in Fig. 3. Such a simple structure can be applied in conformal antennas (see [3], [13]–[15]). The problem is solved numerically using the spectral domain approach. In the last step of this approach a numerically defined determinant function is composed. The roots of this function represent the searched resonant frequencies. The analysis of that function in the $\Omega = \{z \in \mathbb{C} : 1.4 < \mathrm{Re}(z) < 1.5 \wedge -0.1 < \mathrm{Im}(z) < 0\}$ with $\Delta r = 0.1$, using the regular and modified GRPF algorithm, results in a single resonant frequency $f_r = 1.469084230386305 - 0.071068929056597i$ GHz. The final results are exactly the same for both approaches, however the efficiency significantly differs - see Table I. The reduction in the CPU time is a result of the smaller number of nodes in the presented modified algorithm.

TABLE I
ANALYSIS OF THE STRUCTURE PRESENTED IN FIG. 3 - COMPARISON OF THE RESULTS OBTAINED WITH THE USE OF BOTH ALGORITHMS (REGULAR GRPF PLACED IN BRACKETS)

| accuracy | CPU time [s] | no. of nodes | no. of iterations |
|---|---|---|---|
| $\delta = 1e-3$ | 40 (64) | 30 (50) | 11 (7) |
| $\delta = 1e-6$ | 86 (158) | 68 (125) | 27 (18) |
| $\delta = 1e-9$ | 131 (227) | 104 (180) | 43 (27) |
| $\delta = 1e-12$ | 173 (310) | 138 (246) | 60 (37) |

### B. Triangular Dielectric Fiber

As the next structure, a dielectric optical fibers is considered (see Fig. 5). The structure has a triangular cross section

TABLE II
ANALYSIS OF THE STRUCTURE PRESENTED IN FIG. (5) - COMPARISON OF THE RESULTS OBTAINED WITH THE USE OF BOTH ALGORITHMS (REGULAR GRPF PLACED IN BRACKETS)

| accuracy | CPU time [s] | no. of nodes | no. of iterations |
|---|---|---|---|
| $\delta = 1e-3$ | 16 (28) | 162 (281) | 17 (8) |
| $\delta = 1e-6$ | 25 (47) | 249 (467) | 38 (18) |
| $\delta = 1e-9$ | 34 (66) | 336 (658) | 58 (28) |
| $\delta = 1e-12$ | 44 (83) | 437 (829) | 82 (38) |

(equilateral) and can be investigated using a simple field matching technique [2]. The method is quasi-analytical and leads to the construction of determinant function composed of boundary conditions. The root of this function represents the searched propagation coefficient. The analysis of that function in the $\Omega = \{z \in \mathbb{C} : 1.25 < \mathrm{Re}(z) < 1.35 \wedge -0.4 < \mathrm{Im}(z) < -0.35\}$ with $\Delta r = 0.1$, using the regular and modified GRPF algorithm, results in finding a leaky mode characterized by normalized propagation coefficient $\gamma = 1.260551725649258 - 0.384838512540439i$. Again, the final results are exactly the same for both approaches, and again the efficiency significantly differs - see Table II. As in the previous example, the reduction in the CPU time is a result of the smaller number of nodes in the modified GRPF.

tween these two approaches for simple functions is irrelevant.

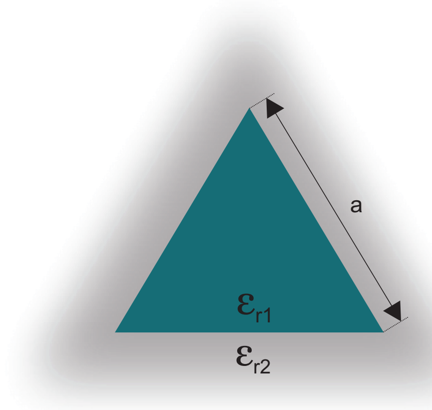


Fig. 5. Dielectric fiber of triangular cross section [2], where permittivity $\varepsilon_{r1} = 8.41$, $\varepsilon_{r2} = 2.4025$ and $a = 0.5$ $\mu$m

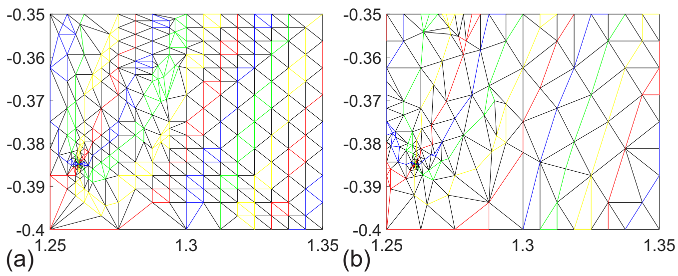


Fig. 6. Comparison of the self-adaptive meshes obtained for the structure presented in Fig. 5. The previous algorithm (a), and modified version (b).

### C. Lossy Multilayered Waveguide

As the final example, a multilayered guiding structure presented in Fig. 7 is considered. This structures are widely used in microwave applications [16] and their analysis boils down to satisfying specific boundary conditions, which requires zero of the determinant function (see equation (9) in [11]). Again, the root of this determinant function represents the searched propagation coefficient. The analysis of the function in the $\Omega = \{z \in \mathbb{C} : 1.2 < \mathrm{Re}(z) < 1.8 \wedge -0.3 < \mathrm{Im}(z) < 0.3\}$ with $\Delta r = 0.1$, using the both algorithms, results in finding five guided modes characterized by normalized propagation coefficients:
$\gamma_1 = 1.353140429034829 - 0.000086139366031i$,
$\gamma_2 = 1.439795543998480 - 0.000052001476288i$,
$\gamma_3 = 1.504169866194328 - 0.000028029233217i$,
$\gamma_4 = 1.548692244042953 - 0.000012100934982i$,
$\gamma_5 = 1.574863045538465 - 0.000002974495292i$.
Obviously, the final results are exactly the same for both approaches. However, in opposition to the previous cases, for such a relatively simple function the efficiency is similar for both algorithms - see Table III. Since the evaluation of the function is not time-consuming, the difference in the CPU time is negligible (even if the number of function calls in the modified algorithm is much smaller). From the practical point of view (taking into account total CPU time) the choice between these two approaches for simple functions is irrelevant.

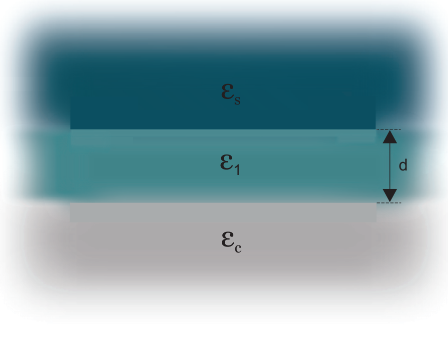


Fig. 7. Lossy multilayered waveguide, where layer permittivities are $\varepsilon_1 = 2.5075$, $\varepsilon_s = 4$, $\varepsilon_c = 15.99 + 0.52i$ and the layer thickness is $d = 1.8$ $\mu$m

TABLE III
ANALYSIS OF THE STRUCTURE PRESENTED IN FIG. 7 - COMPARISON OF THE RESULTS OBTAINED WITH THE USE OF BOTH ALGORITHMS (REGULAR GRPF PLACED IN BRACKETS)

| accuracy | CPU time [s] | no. of nodes | no. of iterations |
|---|---|---|---|
| $\delta = 1e-3$ | 0.06 (0.04) | 320 (461) | 24 (8) |
| $\delta = 1e-6$ | 0.11 (0.08) | 527 (770) | 40 (19) |
| $\delta = 1e-9$ | 0.16 (0.14) | 1062 (729) | 53 (29) |
| $\delta = 1e-12$ | 0.21 (0.20) | 1390 (927) | 69 (39) |

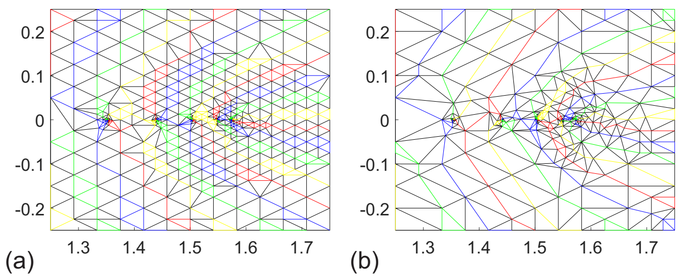


Fig. 8. Comparison of the self-adaptive meshes for the problem obtained for the structure presented in Fig. 7. The previous algorithm (a), and modified version (b).

## IV. Conclusion

An improvement to the recently published roots and poles finding method (GRPF) is presented. In the new approach the number of the function calls is smaller (due to the reduction in the number of the nodes) and results in a shorter processing time. The numerical tests were performed on both versions of the GRPF algorithm. The obtained results show that the new algorithm can be up to two times faster than the original one.

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