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Basic Sensitivity Analysis of a Telecommunication Tower Complementing Standard Reinforcement Design Process

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Abstract. This paper presents straightforward sensitivity assessment of a telecommunication tower. The analysis is set to identify the elements of the tower which may be reinforced with the greatest structural advantage. As current expert opinions on structural redesign of similar structures due to a planned addition of extra loads are mainly based on deterministic computations or engineering intuition, the probabilistic-based approach is proposed to verify and supplement the reinforcement design projects, in contrary to the existing action course. In the paper, the analysis is performed on an example of a standard, spatial telecommunication truss tower subjected to a standard set of loads, special attention is paid to wind and icing load. The cross-sectional areas of curbs and braces of each tower segment are adopted as random variables of the problem, in the form of sets of percentage increments of the initial cross-sectional area values. Joint impact is observed of individual variable change and the change imposed by groups of key variables with a similar impact on the structural load capacity. Finally, engineering-oriented conclusions are presented, of probabilistic origin. A beneficial solution of the reinforcement is proposed, maintaining the reliability of the initial tower.

INTRODUCTION

Nowadays, data transmission technologies are one of the most rapidly developing branches of the technical industry, a fact observed both in terms of new technologies, and in the growing number of network users (and therefore the increment in the number of local devices the network must handle). The latter factor introduces a continuous need to expand the network coverage, triggering the necessity to install many new devices.

As the telecommunication network efficiency clearly requires a proper elevation of dedicated transmitters and receivers, high--rise supporting transmitter structures are inseparable elements of the teletechnical network. In particular, towers and masts of high slenderness are often used for this purpose in different areas. In many cases, adaptation or modification of existing structures is the only solution for the expanding the network, either due to the cost criterion or the impossibility of construction of a new structure in the dense residential urban areas.

This, in turn, means that a need exists to install new ballast loads (related with new data transfer devices) on the structures designed in the past, originally designed accounting for different load schemes, combinations and limit states. Moreover, alongside the dead load of the installations, large faces of antennas and receivers produce new non-negligible secondary loads induced by wind, snow and icing. According to the Authors' professional experience, the majority of existing telecommunication towers are currently operating within 90-95% of their nominal load-bearing capacity, so an increase the sum of loads acting on these structures may imply a necessity to perform a technical expertise of their current load capacity state before the load addition or even their re-design.

There is a growing number of technical expertise papers addressing the need for reinforcement of existing towers and their planning. However, current expert opinions are mainly based on deterministic computations or engineering

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intuition only [1]. In order to enrich the standard reinforcement design, application of simple probabilistic methods, easy to implement and interpret even by engineers unfamiliar with random methodology are proposed.

A preliminary sensitivity analysis is performed, based on the simulation approach. It is meant to present a more in-depth information on the optimal reinforcement quantity and location, given a set objective function. For this purpose, the current load capacity margin of the structure is determined – the reinforced structure is planned to detect a similar margin as the original tower.

In the paper a twofold analysis was made. First, the influence of reinforcing individual structural elements (curbs and cross-braces) on the total load resistance of the object was examined. Based on the information from this analysis, several elements bringing the most desired effects while providing a relatively low cost of reinforcement were chosen. Their overall contribution to the increase in the total resistance of the structure is also inspected.

In the second part of the analysis, the paper deals with simulation of a step-wise increment of the number of additional antennas installed on the tower and its influence on the total load resistance of the structure.

The paper aims to present the superposition of both analytical cases – to determine the outlook of beneficially strengthened tower in the situation of increasing the equipment loads.

COMPUTATIONAL MODEL OF THE TOWER

The calculations were carried out on a real-life, typical telecommunication tower located in Wielanowo, near Szczecin, in the West Pomeranian Voivodship, Poland.

The general design plot of the tower was shared by the PROTEL Radio and Television Design Office Company, Augustowska Street 22, 02-981 Warsaw, Poland. The tower design outline is presented in Fig. 1a. The plot aided the Authors' re-creation of the tower structural model, performed in Autodesk Robot Structural Analysis Professional 2016 commercial software environment [2]. The visualization of the model is presented in Fig. 1b. The model has been cross-checked with the original data on the tower key response, concerning the fulfillment of both Ultimate Limit State (ULS) and Serviceability Limit State (SLS) requirements.

The tower is elevated 84 m above the sea level, it is located in the second wind zone. The structure is placed in a rural area, on a second category site terrain (low vegetation, single obstacles spaced at a distance not lower than 20 percent of the tower height) [3].

The tower has a total height of 50 m and it is divided into five segments of equal height. It is a pyramid frustum of a constant convergence of 3%. The base side of the pyramid varies from 4.4 to 1.4 m.

The major tower material is S235 structural steel. The material data is based on in-situ surveys, the list of steel parameters is given in Table 1. The segments of the tower are interconnected using bolts from the same steel.

The load-carrying structural elements (curbs) are made of hot-rolled equal angle (L) sections, the internal bracing elements (braces) are cold formed channel (C) sections. A detailed list of the applied profiles is given in Table 2.

The diagonal braces form a K-truss pattern. The cross-braces are joined with the tower curbs by means of gusset plates and bolts. It should however be noted, that the numerical model does not employ the gusset plates, it assumes bar elements joined directly at nodes, in the form of three-dimensional rigid connections. The columns are pin-supported (fixed accordingly in all three dimensions) on a solid ground foundation.

In this tower, three levels of possible assembly of telecommunication antennas are designed. However, only the highest one is employed in the paper in order to locate the incremental ballast load.

A standard set of tower loads is applied, several basic load cases are considered. The cases are: dead load, equipment load, wind action on two main perpendicular directions and icing. Special attention is paid to the correlation between the wind and icing loads and the equipment load. It was assumed that the load sum of these cases changes accordingly to the number of telecommunication receivers, the arrangement of these receivers (antennas) was intentionally set axisymmetrical.

TABLE 1. Mechanical parameters of structural steel forming the tower elements.

Mechanical properties	Symbol	Value	Unit
Young's elasticity modulus	E_s	210.0	GPa
characteristic yield strength	R_k	235.0	MPa
design yield strength	R_d	210.0	MPa
tensile strength	R_m	355.0	MPa

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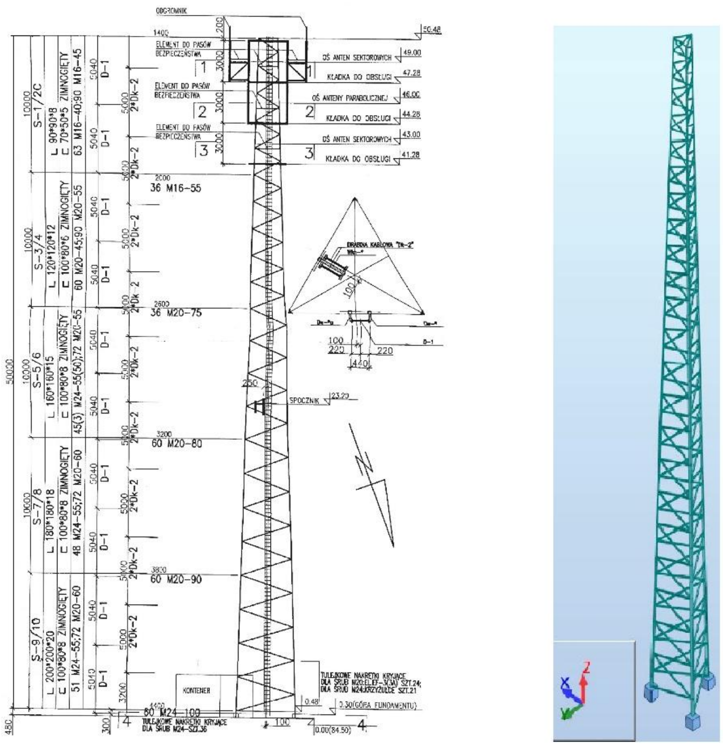
TABLE 2. List of profiles of structural elements of the analyzed tower, alongside with the extreme Von Mises stress and the current load capacity usage in respective structural elements.

Segment	Element	Designation	Sectional area value [cm^2]	Total length [m]	Total weight [kg]	Extreme stress value [MPa]	Current load capacity usage [%]
1	curbs	L 90×90×8	13.9	30.00	327.35	**146.48**	**69.75**
(top)	braces	C 70×70×5	8.5	59.22	395.15	66.66	31.74
2	curbs	L 120×120×12	27.5	30.00	647.63	114.47	54.51
	braces	C 100×80×6	15.6	75.24	921.39	34.92	16.63
3	curbs	L 160×160×15	46.1	30.00	1085.66	101.18	48.18
	braces	C 100×80×8	20.8	75.78	1237.34	26.08	12.42
4	curbs	L 180×180×18	61.9	30.00	1457.75	94.48	44.99
	braces	C 100×80×8	20.8	89.19	1456.29	21.22	10.10
5	curbs	L 200×200×20	76.3	30.00	1796.87	65.05	30.98
(bottom)	braces	C 100×80×8	20.8	102.87	1679.66	69.92	33.30



(a) (b)

FIGURE 1. Telecommunication tower in Wielanowo, in West Pomeranian Voivodship, Poland:
(a) general design plot of the tower shared by the PROTEL Radio and Television Design Office Company,
(b) visualization of the tower model developed in the Robot Structural Analysis 2016 Professional commercial software [2].

ULS AND SLS CALCULATIONS

In the Robot Structural Analysis 2016 Professional commercial software environment, the basic limit states were generated automatically. Automatic load case combinations (ponderations) are the designer-friendly means to define and compute essential summed-up basic loads effects presented in the previous section (all theoretically acceptable combinations of load cases). The calculations were set to define the requirements of the ULS and SLS criteria.

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In the ULS criterion, the extreme design values of Von Mises stresses induced by the respective extreme load combinations were checked and compared to the design value of yield strength of structural steel. A list of extreme Von Mises stress values in key elements of the tower and the current load capacity usage in respective structural elements is also given in Table 2. It can be seen, that mostly due to the wind load, the load capacity utilization of curbs rises with the increment in height above the foundation level. The maximum utilization is generated in the curbs of the top segment. However, the current value of 69.75% clearly shows, that an unnecessarily big safety margin is preserved, the structure is slightly over-dimensioned. On the other hand, no extreme load cases (gale wind, extreme icing, earthquake) are considered here (due to long load return periods they were intentionally neglected).

The SLS criterion is intended to indicate if the value of the maximum vertical deflection (displacement) caused by a relevant characteristic load combination in the most unfavorable case is smaller than the limit value given for a set type of structure. In the analysed model, the vertical displacement of the highest situated node was equal to $f = 0.16$ m. The allowable displacement value for telecommunication towers is given as $f_{lim} = H/150$, where H denotes the total height of the tower. Given $H = 50$ m, the computed limit deflection is $f_{lim} = 0.33$ m, meaning that the SLS criterion is fulfilled. It is worth noting, that for every structure analysed further, the SLS criterion is fulfilled. It is mostly because the stiffening effect was introduced by respective reinforcement variants. Due to that fact, the SLS check is omitted in the upcoming analysis.

SENSITIVITY ANALYSIS

Sensitivity analysis is the study of the impact of the change in certain structural model input parameters on the variability of its response results [3]. The input parameters (design variables) change may be associated with the occurrence of geometric or material imperfections, load variability, randomness of the support conditions, among others [4, 5]. The variability, either deliberate or random, may occur on every stage of the construction process. It may be analysed in the initial design, as well as it may appear on the stage of structural operation [6].

The types of mechanical response of the structure to be altered by initial variability are e.g. internal and support forces, limit state loads, extreme load bearing capacity, displacements. They are often denoted as state variables [6].

The first task of the sensitivity analysis is to establish the basic knowledge on the way uncertainty of the structural response can be attributed to different variability sources included in the input data [4]. Due to a complicated relation of variabilities, the concept of sensitivity analysis is often confused with the uncertainty analysis. The latter is very often implemented as an introduction to the sensitivity analysis (first the characteristics of the output variable are determined, next the sources of its uncertainty with appropriate weights are estimated) [7].

Sensitivity analysis, performed in the structural design process is aimed at eliminating the parameters that do not significantly affect the calculated result (in terms of computational time and extension), determining the impact of correlation between particular parameters, checking if the created mathematical model is reliable (by investigating the fluctuations in the result due to the input variabilities or verifying whether the change introduced by presumably irrelevant parameters is rationally low). Thus, the indirect aim of the sensitivity analysis is to recognize the structural mechanical response and a possible simplification of the mathematical model [3, 7, 8].

Structural sensitivity may be assessed using both deterministic and probabilistic methods. Currently, deterministic calculations are commonly conducted. Mostly they are carried out in the form of parametric analysis, however variance methods or a simple (e.g. graphical) assessment of the effect of individual parameters on output variables are also applied. It should be noted, that these approaches does not provide full information on the impact of geometric and material imperfections or the load uncertainty on the response of the structure [9, 10].

In the paper, deterministic techniques were applied, as the analysis is aimed to help the designers and it should not incorporate advanced probabilistic techniques. Similar work was performed by the leading Author in [11, 12].

An optimization problem was stated, which involves the need to strengthen the initial structural variant of a telecommunication tower. The reinforcement is aimed at the lowest cost possible (minimization of the used steel quantity), while introducing a demanded load capacity increment to allow for an additional load, assuming the general structural reliability remains unchanged. Thus, the presented sensitivity analysis is conducted – mainly to gather the information on key elements (with maximum impact on structural response) to be reinforced.

In should be noted that the presented analysis is purely demonstrative. It cannot be identified with the guidelines for the final modification of the tower, as the structural FE model is highly simplified – the wind load does not take dynamic action into account, load combinations do not include any accidental situations and the additional antenna support installations are nodal loads only. Due to these simplifications, the numerical results obtained from the presented model are not identifying optimal values of design variables, but indicate their possible beneficial values.

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Variables of the Task

The structural reinforcement of telecommunication towers is most often performed by adding appropriate reinforcing pads by welding or bolting. This approach was considered in the paper, yet indirectly. In new, reinforced models, steel profiles different from the original ones were chosen from the steel product catalogues, with greater cross-sectional area values. These new elements account for the old ones, subjected to reinforcing.

As stated earlier, the considered truss tower consists of five independent segments, each one consists of a curb and a diagonal brace of a corresponding type with specified cross-sectional area values. This leads to adoption of $5 \times 2 = 10$ input design variables (one per each key element cross-sectional area) to be considered in the paper. The Authors consider it the most natural choice. The basic variables are collected and presented in Table 3.

It was assumed that the change of selected random variables is positive only – if the tower needs strengthening, the allowed impact of degradation of these parameters would be pointless. Moreover, it is assumed that a given input parameter changes twofold only – the cross-sectional areas may be elevated to approximately 125% and 150% of the initial value. These two increments provide a satisfactory knowledge of the structural response behavior. Therefore, all ten variables are considered discrete, in the form of given equally probable percentage increments of the initial cross-sectional areas. If one of the input variables changes (a reinforcement of a given element is provided), the remaining ones are assumed set at their initial values, consistently with the OAT (One-At-A-Time) technique [10].

The initial, nearly 125% and nearly 150% values of the cross-sectional areas of all tower elements in respective segments are also given in Table 3, along with the associated profiles.

TABLE 3. List of structural elements (variables) before and after reinforcements.

Var.	**Element**	**Initial profile**	**~125% of the initial profile**	**~150% of the initial profile**
1	segm. 1 curbs	L90×90×8 = 13.90 cm^2	L90×90×10 = 17.10 cm^2	L100×100×11 = 20.90 cm^2
2	segm. 1 braces	C70×70×5 = 8.50 cm^2	C90×60×5 = 10.50 cm^2	C100×60×6 = 13.20 cm^2
3	segm. 2 curbs	L120×120×12 = 27.50 cm^2	L150×150×12 = 34.80 cm^2	L150×150×14 = 40.30 cm^2
4	segm. 2 braces	C100×80×6 = 15.60 cm^2	C120×80×7 = 19.60 cm^2	C130×10×7 = 23.10 cm^2
5	segm. 3 curbs	L160×160×15 = 46.10 cm^2	L180×180×16 = 55.40 cm^2	L200×200×18 = 69.10 cm^2
6	segm. 3 braces	C100×80×8 = 20.80 cm^2	C125×80×9 = 25.65 cm^2	C135×90×9 = 31.50 cm^2
7	segm. 4 curbs	L180×180×18 = 61.90 cm^2	L200×200×20 = 76.30 cm^2	L200×200×24 = 90.60 cm^2
8	segm. 4 braces	C100×80×8 = 20.80 cm^2	C125×80×9 = 25.65 cm^2	C135×90×9 = 31.50 cm^2
9	segm. 5 curbs	L200×200×20 = 76.30 cm^2	L200×200×25 = 94.10 cm^2	L250×250×24 = 114.20 cm^2
10	segm. 5 braces	C100×80×8 = 20.80 cm^2	C125×80×9 = 25.65 cm^2	C135×90×9 = 31.50 cm^2

It was decided that the analyzed state variable (the structural response) will be proposed in the form of an dimensionless remaining safety margin m. The margin is based on the numerical calculations of the allowable load multiplier and it is defined on a basis of a dimensionless increment of the extreme Von Mises stresses in the element resulting in plastic failure of this element or the entire structure, in the most unfavorable load case. The safety margin m can be described by a ratio of structural steel design yield strength R_y and the extreme design values of Von Mises stresses induced by the respective extreme load combinations σ_{extr}.

Sensitivity of the Tower to The Strengthening of its Individual Elements – Numerical Case without the Equipment Loads (Output Sensitivity)

First of all, the analysis concerning the impact of modification of separate design variables on the structural load bearing capacity was performed for the tower free of any equipment loads. This analysis is presented to indicate the natural safety margins of the structure and the elements that are contributing to these margins firstly.

Fig. 2 shows the impact of the increment of the cross-sectional areas of individual elements on the change in structural load capacity (expressed by the remaining safety margin for the tower is not loaded with equipment.

On the basis of the initial sensitivity analysis, presented in Fig. 2a, it can be deducted, that the most advantageous change is introduced by two variable parameters only, both associated with support segment 5 – the cross-sectional area of curbs and braces of the lowest segment. Thus, this segment is definitely crucial as it is possibly associated with support region rigidity positive change (the vertical deflection of the tower top is lowered). The presented change in the safety margin assessment is slightly non-linear. The remaining eight variables do not introduce any change in the response, the tower is insensitive to their variations if no equipment loads are present.

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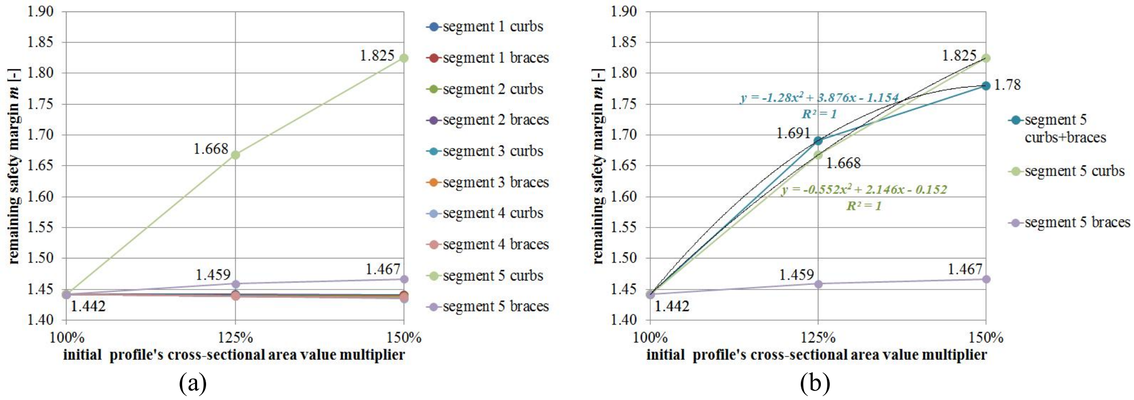

FIGURE 2. The increment of the cross-sectional areas of individual elements versus the remaining safety margin variation for the tower not loaded with equipment: (a) a separate analysis for each element, (b) a simultaneous increment in segment 5 elements cross sectional areas (appropriate approximation of the response provided).

Due to the results obtained in the preliminary assessment, in the second analytical step, the increase in the remaining safety margin of the structure for a simultaneous change of both most sensitive parameters is investigated.

Fig. 2b shows the impact of simultaneous increase in cross-sectional area of both segment 5 curbs and braces on the remaining safety margin. It can be observed that a one-step reinforcement of the entire segment is not recommended, as a greater remaining safety margin is obtained if only the segment 5 curbs are reinforced. This is presumably because a large difference in demonstrated in the sensitivity of both elements.

Sensitivity of the Tower to the Increase in the Tower Equipment Sum of Loads

The second part of the undertaken task considers the impact of increasing the applied equipment load (increase in the number of mounted antennas) on lowering the structural limit load capacity.

It is assumed, that the additional equipment load is induced by a specified number of antennas, mounted axisymmetrically on the tower in packets of three (a standard operation, due to technological reasons), until reaching a number of 15. The upper limit is dictated by functionality of the tower in general.

The loads induced by the presence of antennas and support rings are nodal point loads only. The appropriate resultant forces and moments (in the form of force couples) were determined and transferred to the tower main body. It should be emphasized that not only the equipment dead weight was transferred, but the wind load action on the antennas faces and icing gravity load too, in the form of resultant forces related to the number of antennas installed.

Subsequent calculations were made for the remaining safety margin in a case where the equipment loads are applied (alongside the resultants from additional atmospheric loads). In this case sensitivity is investigated with each of ten design parameters set to 100% of its initial cross-sectional area. The analytical results are presented in Fig. 3.

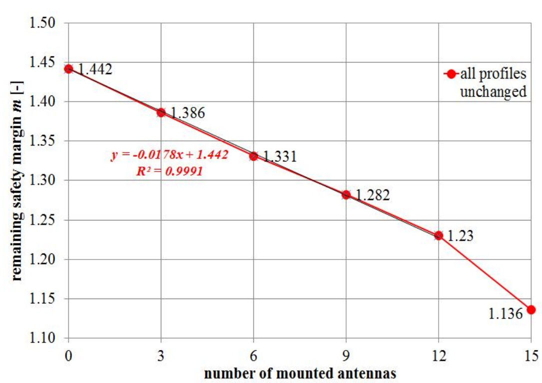

FIGURE 3. The number of mounted antennas versus the change in the remaining safety margin for the tower loaded with equipment and the resultants from additional atmospheric loads (appropriate approximation provided).

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Sensitivity of the Tower to the Strengthening of its Individual Elements – Numerical Case Concerning the Equipment Loads and Associated Wind and Icing Loads

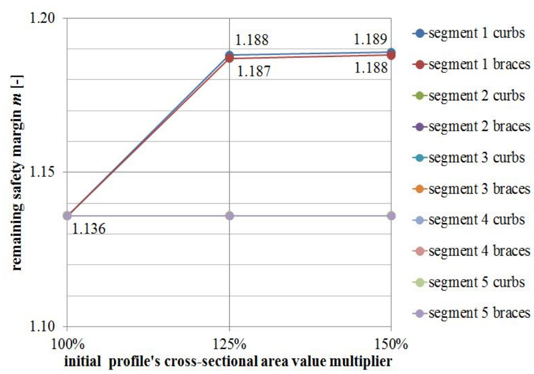
Furthermore, it was decided to perform an analysis of the impact of separate modification of design input variables on the structural response assuming a specified number of antennas and their associated atmospheric loads.

The 10 input design variables were assumed identical to previous calculations (see Table 3). Their change is assumed positive only, to approximately 125% and 150% of the initial values. The OAT technique is again used.

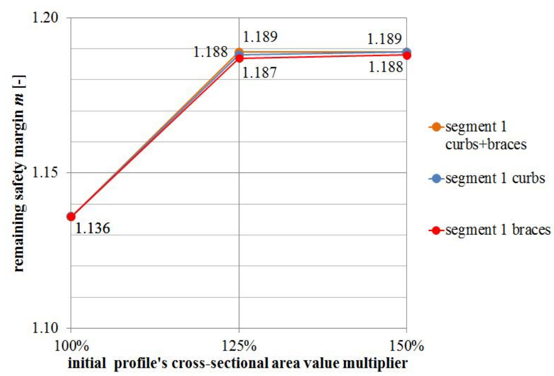
Although the calculations were made for every situation (for 3 – 15 antennas), only the latter, the analysis of the fully loaded tower is shown in the paper and presented in Fig. 4.

The ULS criterion was the decisive one again, the failure of the entire tower occurred because of single element plastic failure. However, there was a change in the failure location zone – the top segment curbs subjected to tensile forces failed. Due to this phenomenon, all design variables not associated with segment 1 are deemed insensitive.

This means that in the case of such a significant increase in the load of the tower in its upper segment, modification of the cross-sectional area of curbs and struts of this segment is a necessity, as the top segment 1 becomes crucial for the structure, as presented in Fig. 4a. Interestingly, the change in sensitivity is non-linear – after reaching a certain limit of the possible safety margin, further strengthening of the segment is pointless.



(a)



(b)

FIGURE 4. The increment of the cross-sectional areas of individual elements versus the remaining safety margin variation for the tower loaded with equipment and its associated wind and icing loads: (a) a separate analysis for each element, (b) a simultaneous increment in segment 1 elements cross sectional areas.

In the second step of the analysis it was decided, similarly to the case where no equipment load are taken into account, to find an increase in the safety margin in the case of a simultaneous change of both most sensitive parameters of the tower. Fig. 4b shows the impact of simultaneous increase in the cross-sectional area of both segment 1 curbs and braces on the remaining safety margin for the case of the full tower load.

Based on the results collected in Fig. 4b, it can be observed that a single-step reinforcement of the entire segment 1 is pointless. The same value of the remaining safety margin is obtained if the segment 1 curbs or struts are reinforced only – a desirable joint action of both elements reinforcement cannot be observed. In this case reinforcing the curbs will be a better option – although the elements have greater cross-sectional values than the braces, their total length (and mass) is slightly lower, so their reinforcement would be economically justified.

This happens because the full loading case triggers a different failure mode of the entire tower.

SENSITIVITY-BASED OPTIMIZATION – TOWER ELEMENTS REINFORCEMENT

Based on the initial results of sensitivity analysis in the case of no equipment load and assuming the equipment load is present, it was shown that two cases should be distinguished. If the number of antennas is relatively low (≤ 9 antennas) the cross-sectional areas of segment 5 curbs and braces hold the greatest impact on the increase in the safety margin of the tower. However, if the number of antennas is high (≥ 12 antennas), the failure mechanism changes and the cross-sectional areas of segment 1 curbs and braces tend to be the most important (the plastic failure of the tower is initiated in level 1 elements).

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For this reason, subsequent diagrams analyse the changes in the safety margin of the tower related to the number of antennas. The scenario concerning the increment in the cross-sectional area up to 125% of the initial value is presented separately from the 150% scenario. The relation between the change in the remaining safety margin and the number of antennas installed on the tower is given in Fig. 5.

The data in the figure finally confirms the observation that a two-interval approach should be applied – as a result, different optimization is performed for the interval detecting the number of antennas not greater than 9 and a different one in the scenario showing the number of antennas not lower than 12.

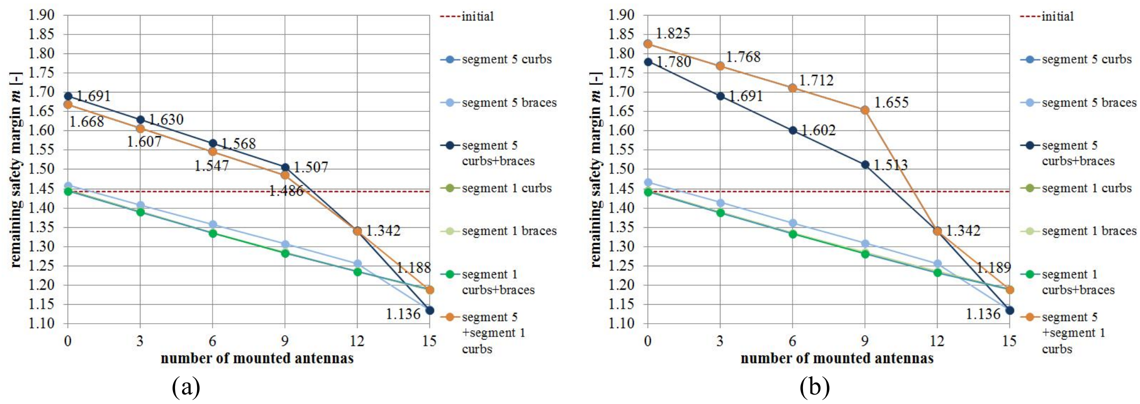

FIGURE 5. The number of mounted antennas versus the change in the remaining safety margin for the tower loaded with equipment, concerning an increment (a) to 125%, (b) to 150% of initial value of given cross sectional areas.

Optimization Case 1 – the Number of Antennas not Greater than 9

In the first optimization analysis case, two previous results were used – the approximation of the increase in the remaining safety margin due to the increment of the cross-sectional areas of segment 5 curbs alone and curbs and cross-braces simultaneously (presented in Fig. 2b) and the decrease in the margin caused by the increase in the number of mounted antennas (presented in Fig. 3).

If three antennas are installed, decrement of the safety margin induced by the equipment loads is

$$y = 1.442 - 0.0178 \cdot x \quad \rightarrow \quad decr_3 = \left| -0.0178 \cdot x \right| = 0.0178 \cdot 3 = 0.0534 \tag{1}$$

While the target of the optimization is to replenish the original safety margin of the structure, the value expressed by formula (1) has to be replaced with the increase of the cross-sectional areas of segment 5 elements with an identical value. It brings the need to achieve a safety margin of

$$reinf_3 = 1.442 + decr_3 = 1.442 + 0.0534 = 1.4954 \tag{2}$$

As observed in Fig. 2b, only the increase in the cross-sectional areas of curbs alone or the simultaneous increase in curbs and braces brings a satisfactory effect, the analysis of the cross-braces alone is neglected in the paper.

While the curbs are reinforced only, the determination of the optimal value is conducted by

$$reinf_C = -0.552 \cdot x^2 + 2.146 \cdot x - 0.152 \rightarrow x = \frac{-2.146 + \sqrt{2.146^2 - 4 \cdot (-0.552) \cdot (-0.152 - reinf_3)}}{2 \cdot (-0.552)} = 1.0527 \tag{3}$$

which means that a 105.27% of the initial value of the segment 5 curbs area is required to achieve the targeted goal.

If the initial total mass of segment 5 curbs is 1796.87 kg (see Table 2), then new curb mass is equal to 1891.57 kg. This in turn brings a mass increase equal to 94.73 kg. Assuming the price of structural steel (material only, no labor costs considered) as 0.567 €/kg, the cost of additional steel required for the reinforcement is 53.71 €.

While the curbs and braces are reinforced together, determination of the optimal value reads

$$reinf_{C+B} = -1.280 \cdot x^2 + 3.876 \cdot x - 1.154 \rightarrow x = \frac{-3.876 + \sqrt{3.876^2 - 4 \cdot (-1.280) \cdot (-1.154 - reinf_3)}}{2 \cdot (-1.280)} = 1.0423 \tag{4}$$

If the initial total masses of segment 5 curbs and braces are 1796.87 kg and 1679.66 kg respectively, the new total element mass is equal to 3623.59 kg, a mass increase is equal to 147.12 kg, and the cost is 83.42 € here.

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The calculations above are repeated for 6 and 9 antennas (additionally, for 12 and 15, for the purpose of the following analysis), they are presented collectively in Table 4.

TABLE 4. The results of the reinforcement of the tower when the number of antennas is not greater than 9.

Number of antennas	Decrement of *m – decr* [-]	Demanded new *m – reinf* [-]	Reinforced elements	The determined optimal quantity of c-s area [%]	Increase in the mass [kg]	Estimated cost [€]
3	0.0534	1.4954	curbs only	105.27	94.73	53.71
			curbs+braces	104.23	147.12	83.42
6	0.1068	1.5488	curbs only	110.88	195.43	110.81
			curbs+braces	108.88	308.82	175.10
9	0.1602	1.6022	curbs only	116.88	303.39	172.02
			curbs+braces	114.11	490.52	278.13
12	0.2136	1.6556	curbs only	123.40	420.46	238.40
			curbs+braces	120.20	702.25	398.17
15	0.2670	1.7090	curbs only	130.58	549.42	311.52
			curbs+braces	127.81	966.91	548.24

Optimization Case 2 – the Number of Antennas not Lower than 12

The second case is triggered by re-location of the failure zone. Here, the necessity rises to reinforce the elements of the top segment 1. The parallel necessary segment 5 reinforcement in this case is presented in Table 4.

As observed in Fig. 4, the reinforcement of any element of the top segment is appropriate. Such a conclusion arises from the observation of structural behavior – strengthening the braces alone redefines the stresses map, allowing for the transfer of a greater component of the load set to the braces and the increase in the safety margin of curbs (the cross-sectional difference between both elements show the lowest ratio in the entire structure).

In the economical approach, the optimal version is the choice of the element cheapest to strengthen. Thus, simple comparison of the total reinforcement cost is performed in this case. It should be pointed out, that retaining the original load-carrying capacity margin is impossible, as strengthening the elements of segment 1 beyond the limit of 125% of the initial value of the cross-sectional area does not further contribute to the global tower capacity. The calculations of reinforcement costs for 12 and 15 are presented collectively in Table 5.

TABLE 5. The results of the reinforcement of the tower when the number of antennas is not lesser than 12.

Number of antennas	Reinforced elements	The determined optimal quantity of c-s area [%]	Increase in the mass [kg]	Estimated cost [€]
12 or 15	curbs only	125.00	81.84	46.40
	braces only	125.00	98.79	56.01
	curbs+braces	125.00	180.63	102.41

CONCLUSIONS

The substantial conclusion from the performed sensitivity analysis with ten random variables is the determination of most beneficial to reinforce. The supporting and the top segments are indicated as key ones.

First of all the focus of structural reinforcement was set on changing the cross-sections of curbs of support segment 5. When these curbs are enlarged up to 125% of their initial cross-sectional area value, the load sum from nine antennas set generates structural response at the same level as the initial model with no equipment loads. When the curbs are set to 150% of their original area, a less satisfactory increase in the safety margin occurs. Enlarging the struts of the support segment brings small increase in the safety margin, their reinforcement is proved unnecessary. Moreover, if the support segment struts are reinforced alongside respective curbs, the safety margin decreases in comparison to curbs reinforcement only. This is caused by the occurrence of extreme stresses in the curbs.

However, it should be noted that regardless of the increase in the cross-sectional areas of the considered support segment curbs, the safety margin of the tower loaded with a maximum number of antennas is proven constant and it amounts to 1.136, a situation coming into focus when the tower is loaded with more antennas than nine. This is due to the fact that the extreme stresses are relocated to the top segment elements loaded with tensile forces.

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In this case only the reinforcement of the abovementioned elements brings an effect. Although the increase in the safety margin is also small (from 1.136 to 1.189), the issue of extreme tensile stresses occurrence and the need for their minimization becomes very important. The type of reinforcement of segment 1 elements is inert – the curbs, the braces, or both elements together may be reinforced with the same effect, so an economical criterion benefits here. The reinforcement of curbs only is suggested, similarly to segment 5. Performing the reinforcement of these two element groups at once creates an envelope of the maximum remaining safety margins, which is desirable.

On the basis of the undertaken calculations, a total cost of the additional steel required for the reinforcement of the tower against the action of loads resultant from mounting 15 antennas (assuming the price of structural steel only, no labor costs considered) is 357.92 € (311.52 € for the strengthening of segment 5 curbs and 46.40 € for the strengthening of segment 1 curbs).

The work does not take into account fatigue and degradation of structural steel before the tower reinforcement was commenced. It is strongly suggested that such an analysis should be performed before the sensitivity analysis, as the uncertainties related to the usage time may strongly reduce the expected safety margins.

Regarding the numerical model, it is interesting that a completely linear structure (loads increase proportionally, so do the cross-sectional areas) demonstrates a non-linear change in its limit load capacity. This fact is clearly related to the abovementioned re-location of the failure zone. This effect should be analyzed in a wider spectrum, an insightful analysis of the modified models should be made to control this phenomenon.

This work is primarily application-based. Deterministic sensitivity analysis of the telecommunication tower was presented to show a possible method of supporting the technical expertise of increasing load capacity of the tower with probabilistic methods. The shown analysis is simple and accessible, it does not require any specialist knowledge in the field of probabilistic engineering, thus it can be recommended for designers unfamiliar with random methodology. Sensitivity analysis correctly indicates the cross-sections relevant for modification and properly predicts the structural response of the tower after the reinforcement, with a small computational effort.

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