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A Nonlinear Model of a Mesh Shell

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Abstract—For a certain class of elastic lattice shells experiencing finite deformations, a continual model using the equations of the so-called six-parameter shell theory has been proposed. Within this model, the kinematics of the shell is described using six kinematically independent scalar degrees of freedom — the field of displacements and turns, as in the case of the Cosserat continuum, which gives reason to call the model under consideration as the theory of micropolar shells. Nonlinear equations of state for the surface energy density of the shell deformation are derived. The obtained relations of the continuum model are a special case of the general defining relations of elastic micropolar shells for finite deformations.

Keywords: *mesh shells, framed curve, micropolar shells, nonlinear elasticity, equations of state.*

INTRODUCTION

Starting with the works of V.G. Shukhov lattice and mesh shells are one of the common classes of thin-walled structures widely used in construction, see [1, 2]. Such shells are also actively used in modern machine, aircraft and rocket production (works by V. V. Vasiliev and other authors [3–6]). Thin-walled mesh elements also find other applications, for example, for the production of biocompatible implants [7]. It should be noted that the equations of state of discrete shells may differ significantly from the classical models of continuous shells [8–10].

In this paper, we consider a special class of mesh shells formed by two families of flexible fibers orthogonal to each other. In other words, mesh structures resembling a fishing net with square cells are considered. It is assumed that the nodes in which the articulation of the fibers takes place are rather rigid, so that the fibers retain their orthogonality after deformation. In the work, a transition was made from the discrete system of equations describing the deformation of each fiber to the averaged continuous model of the nonlinear theory of shells. Equations fitted with a curve [11–13] are used as a model of nonlinear deformation of fibers, within which the deformations are described by kinematically independent fields of displacements and rotations. The continuum model is described in terms of the nonlinear theory of shells presented in [13–16]. Within the framework of this variant of the theory of shells, displacements and turns are also considered as independent kinematic quantities. In addition to the bending and torsional moment, the drilling moment is also taken into account. On the edge of the shell, six boundary conditions are specified. In the case of static boundary conditions, the values of three forces and three moments are specified, and in the case of kinematic conditions, three movements and three rotations are given. Thus, the kinematics of the shell is characterized by six parameters, and the model is often called six-parameter. In fact, here the description of the deformation coincides with the kinematics of the two-dimensional Cosserat continuum (micropolar medium). Various applications of this theory are presented, for example, in [15, 17–19]. An important aspect of the nonlinear theory of shells is the formulation of the equations of state, that is, the dependence of the deformation energy, forces and moments on the measures of the deformations of the shell. For the six-parameter theory of shells, the equation of state with finite deformations was discussed in [14, 15, 20–22].

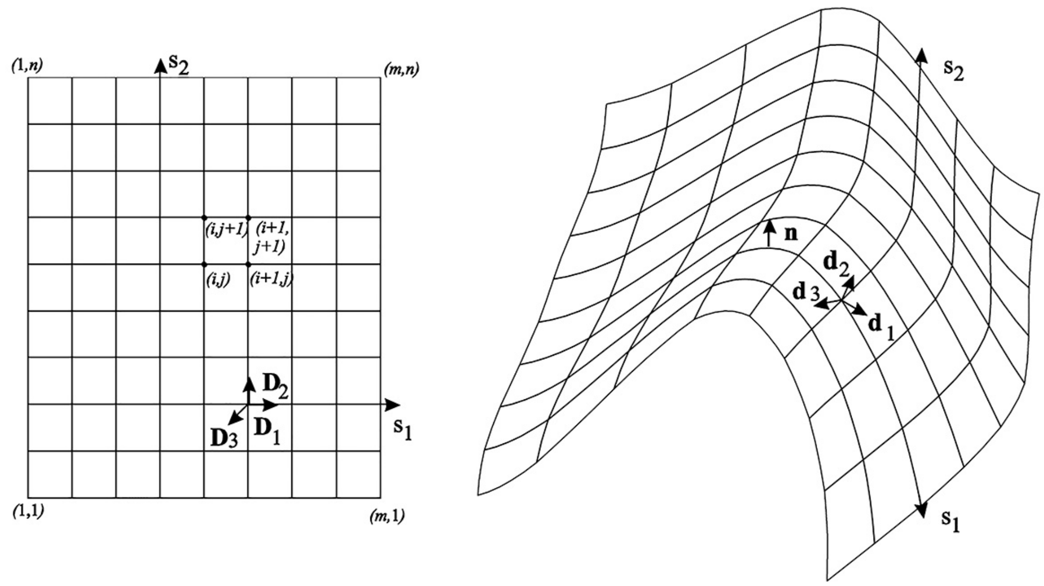

Fig. 1.

1. DEFORMATIONS OF A DISCRETE MESH SHELL

Let us consider the deformation of a mesh shell formed by two orthogonal families of flexible elastic fibers. The initial and deformed state of the shell are depicted in the figure. Here we confine ourselves to such a variant of the mesh structure, in which the connection of the cladding fibers is rigid, so that we can assume that the fibers retain their orthogonality even after deformation. As a theory describing the final deformations of the fiber, we use the theory of a rigged curve, also known as the Cosserat curve [11—13]. The kinematics of a rigged curve is described by two vector fields:

$$\mathbf{r} = \mathbf{r}(s), \quad \mathbf{d}_k = \mathbf{d}_k(s), \quad k = 1,\, 2,\, 3, \tag{1.1}$$

where $\mathbf{r}$ is the radius vector of the curve in the deformed state, $\mathbf{d}_k$ is the orthonormal vectors, directors defining the orientation (rigging) of the curve, s is the length of the arc of the curve in the undeformed state. Instead of directors $\mathbf{d}_k$, it is more convenient to use the orthogonal tensor $\mathbf{P} = \mathbf{d}_k \otimes \mathbf{D}_k$ itself, where the directors $\mathbf{D}_k$, defined on the undeformed curve C_0, are introduced, $\otimes$ denotes the tensor product. Hereinafter, direct tensor calculus is used [23, 24]. Without loss of generality, we assume that the vector $\mathbf{D}_1$ is tangent to C_0.

Equilibrium equations in the metric of the undeformed state have the form [12]:

$$\mathbf{t}'(s) + \mathbf{f} = \mathbf{0}, \quad \mathbf{m}' + \mathbf{r}'(s) \times \mathbf{t}(s) + \mathbf{c} = \mathbf{0}, \tag{1.2}$$

where $\mathbf{t}$ and $\mathbf{m}$ are the internal forces and moments acting in the fiber, $\mathbf{f}$ and $\mathbf{c}$ are the external forces and moments specified on the C_0 curve, $\times$ denotes the vector product, and prime indicates the derivative with respect to s: $(\ldots)' = \partial(\ldots)/\partial s$.

Restricting ourselves to considering hyperelastic materials, we introduce the strain energy in the form

$$U = U(\mathbf{e}, \mathbf{k}), \quad \mathbf{e} = \mathbf{P}^T \cdot \mathbf{r}' - \mathbf{D}_1, \quad k = -\frac{1}{2}\left(\mathbf{P}^T \cdot \mathbf{P}'\right)_\times, \tag{1.3}$$

where $\mathbf{e}$ and $\mathbf{k}$ are vector measures of deformations, $\mathbf{T}_\times$ is a vector invariant of a tensor of rank two $\mathbf{T}$, defined by the formula [24]

$$\mathbf{T}_\times = (T^{mn}\mathbf{i}_m \otimes \mathbf{i}_n)_\times = T^{mn}\mathbf{i}_m \times \mathbf{i}_n$$

for an arbitrary vector basis $\mathbf{i}_m$. The vectors of internal forces and moments are related to the strain energy by the formulas

$$\mathbf{t} = \frac{\partial \mathbf{U}}{\partial \mathbf{e}} \cdot \mathbf{P}^T, \quad \mathbf{m} = \frac{\partial \mathbf{U}}{\partial \mathbf{k}} \cdot \mathbf{P}^T. \tag{1.4}$$

The system of equations (1.1)–(1.4), supplemented by the corresponding boundary conditions, is a boundary problem describing the final deformations of an elastic beam, taking into account its tensile compression, shear deformations, bending and torsion. Accordingly, these equations can be used for beam systems, including the mesh shell. For simplicity, we neglect the transverse shear deformations, then U will depend only on the tangential deformation $\varepsilon = \mathbf{e} \cdot \mathbf{D}_1$: $U = U(\varepsilon, \mathbf{k})$, see [11].

Consider the deformation of the mesh shell, formed by two orthogonal fiber families, similar to that shown in the figure. Assume that the director vectors $\mathbf{D}_1$ and $\mathbf{D}_2$ are tangent to the 1st and 2nd families, respectively. Denoting the values assigned to the first and second families, respectively, using indices 1 and 2, we write the Lagrangian equilibrium equations for the discrete mesh shell

$$\mathbf{t}'_{1,1}(s_1) + \mathbf{f}_1 = \mathbf{0}, \tag{1.5}$$

$$\mathbf{m}'_{1,1} + \mathbf{r}'_{1,1}(s_1) \times \mathbf{t}_1(s_1) + \mathbf{c}_1 = \mathbf{0}, \tag{1.6}$$

$$\mathbf{t}'_{2,2}(s_2) + \mathbf{f}_2 = \mathbf{0}, \tag{1.7}$$

$$\mathbf{m}'_{2,2} + \mathbf{r}'_{2,2}(s_2) \times \mathbf{t}_2(s_2) + \mathbf{c}_2 = \mathbf{0}. \tag{1.8}$$

Here $\mathbf{t}_\alpha$ and $\mathbf{m}_\alpha$ are the vectors of internal forces and moments acting in the α-family, $\mathbf{f}_\alpha$ and $\mathbf{c}_\alpha$ are the corresponding external forces and moments, and differentiation is introduced along each curve: $(\ldots)'_{,\alpha} = \partial(\ldots)/\partial s_\alpha$, α=1, 2.

Enumerating the nodes of the shell, as shown in the figure, we can write the total energy of the mesh shell as the sum

$$E = \sum_{j=1}^{n} \sum_{i=1}^{m-1} \int_{\mathrm{s}_1^{(\mathrm{i})}}^{\mathrm{s}_1^{(\mathrm{i+1})}} \mathrm{U(s_1)ds_1} + \sum_{\mathrm{i=1}}^{\mathrm{m}} \sum_{\mathrm{j=1}}^{\mathrm{n-1}} \int_{\mathrm{s}_2^{(\mathrm{j})}}^{\mathrm{s}_2^{(\mathrm{j+1})}} \mathrm{U(s_2)ds_2}, \tag{1.9}$$

where m and n are the numbers of fibers in two directions, and the strain energy densities are introduced as

$$U(s_\alpha) = U(\varepsilon_\alpha, \mathbf{k}_\alpha), \quad \varepsilon_1 = \mathbf{r}'_{,1} \cdot \mathbf{P} \cdot \mathbf{D}_1 - 1, \quad \varepsilon_2 = \mathbf{r}'_{,2} \cdot \mathbf{P} \cdot \mathbf{D}_2 - 1, \quad \mathbf{k}_\alpha = -\frac{1}{2}\left(\mathbf{P}^T \cdot \mathbf{P}'_{,\alpha}\right)_\times. \tag{1.10}$$

For simplicity, we assume that the considered fibers are the same and are described by the same equations of state.

Using trapezoid formulas

$$\int_{s_1^{(i)}}^{s_1^{(i+1)}} U(s_1) ds_1 = \frac{h}{2}\Big[U(s_1^{(i)}) + U(s_1^{(i+1)})\Big], \qquad \int_{s_2^{(j)}}^{s_2^{(j+1)}} U(s_2) ds_2 = \frac{h}{2}\Big[U(s_2^{(j)}) + U(s_2^{(j+1)})\Big],$$

we can reduce the dependence (1.9) to a completely discrete form

$$E = \frac{h}{2}\sum_{j=1}^{n}\Big[U(s_1^{(1)}) + U(s_1^{(m)}) + 2\sum_{i=2}^{m-1} U(s_1^{(i)})\Big] + \frac{h}{2}\sum_{i=1}^{m}\Big[U(s_2^{(1)}) + U(s_2^{(n)}) + 2\sum_{j=2}^{n-1} U(s_2^{(j)})\Big].$$

Thus, the total energy of the mesh grid takes the form of a weighted sum of energies specified only at the grid nodes, that is, at the points $(s_1^{(i)}, s_2^{(j)})$:

$$E = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\Big[U\big(s_1^{(i)}\big) + U\big(s_2^{(j)}\big)\Big], \tag{1.11}$$

Here c_{ij} is some weighting factors obtained by summing the terms in the above formula. Taking into account (1.10), equation (1.11) can also be written as follows, explicitly indicating the dependence on strain measures

$$E = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\Big[U\big(\varepsilon_1^{(i,j)}, \mathbf{k}_1^{(i,j)}\big) + U\big(\varepsilon_2^{(i,j)}, \mathbf{k}_2^{(i,j)}\big)\Big]. \tag{1.12}$$

It should be noted that the vector $\mathbf{r}$ and the tensor $\mathbf{P}$ are defined only on the grid lines. In other words, depending on the choice of the grid line there are dependencies of $\mathbf{r} = \mathbf{r}(s_1)$, $\mathbf{P} = \mathbf{P}(s_1)$ or $\mathbf{r} = \mathbf{r}(s_2)$, $\mathbf{P} = \mathbf{P}(s_2)$. Instead of functions of one variable, one can introduce surface fields $\mathbf{r} = \mathbf{r}(s_1, s_2)$, $\mathbf{P} = \mathbf{P}(s_1, s_2)$ so that they coincide with these functions when narrowing to the grid region ω. Replacing the double sum with the surface integral, we obtain the continual model of the mesh grid

$$E = \iint_{\omega} \Big[U(\varepsilon_1, \mathbf{k}_1) + U(\varepsilon_2, \mathbf{k}_2)\Big] d\omega. \tag{1.13}$$

It can be shown that discretization of the functional (1.13) with some accuracy leads to an expression of the form (1.12). Since the discretization of the energy functional of the discrete mesh shell (1.9) and its continual analog (1.13) leads to the same formulas (with a certain accuracy), we call these models equivalent. To characterize the continual model obtained in more detail, we consider the equations of the statics of the six-parameter theory of shells.

2. EQUATIONS OF THE CONTINUAL MESH SHELL

Following [13, 16], we consider the boundary value problem of the six-parameter theory of shells (micropolar shells). The kinematics of the shell is specified using the radius vector of the position of the base surface of the shell $\mathbf{x} = \mathbf{x}(s_1, s_2)$ and the orthogonal tensor $\mathbf{Q} = \mathbf{Q}(s_1, s_2)$. Lagrange equilibrium equations and possible boundary conditions are given by the formulas

$$\nabla \cdot \mathbf{T} + \mathbf{f} = \mathbf{0}, \quad \nabla \cdot \mathbf{M} + [\mathbf{F}^T \cdot \mathbf{T}]_\times + \mathbf{c} = \mathbf{0}, \tag{2.1}$$

$$\ell_1: \quad \mathbf{x} = \mathbf{x}_0(s), \quad \ell_2: \quad \boldsymbol{\nu} \cdot \mathbf{T} = \boldsymbol{\tau}(s), \tag{2.2}$$

$$\ell_3: \quad \mathbf{Q} = \mathbf{H}(s), \quad \ell_4: \quad \boldsymbol{\nu} \cdot \mathbf{M} = \boldsymbol{\mu}(s), \tag{2.3}$$

where $\nabla = \mathbf{D}_\alpha \partial/\partial s_\alpha$ is the surface gradient operator defined in the initial state, s_α is the surface orthogonal coordinates, $\mathbf{F} = \nabla \mathbf{x} \equiv \mathbf{D}_\alpha \otimes \mathbf{x}'_{,\alpha}$ is the surface deformation gradient, $\mathbf{x}_0(s)$, $\mathbf{H}(s)$, $\boldsymbol{\tau}(s)$, $\boldsymbol{\mu}(s)$ are given on the corresponding parts of the shell contour, respectively, of the radius vector field, the rotation tensor, forces and moments . $\partial\omega = \ell_1 \bigcup \ell_2 \bigcup \ell_3 \bigcup \ell_4$ is the partition of the shell contour into parts on which kinematic and static boundary conditions are specified; $\boldsymbol{\nu}$ is the unit normal vector to the contour $\partial\omega$ orthogonal to the normal $\mathbf{n}$ to the shell: $\boldsymbol{\nu} \cdot \mathbf{n} = \mathbf{0}$, $\mathbf{T}$ and $\mathbf{M}$ are force and moment tensors of Piola type.

For an elastic shell, there is a strain energy W, which depends on the surface measures of strain $\mathbf{E}$ and $\mathbf{K}$ as follows:

$$W = W(\mathbf{E}, \mathbf{K}), \tag{2.4}$$

$$\mathbf{E} = \mathbf{F} \cdot \mathbf{Q}^T - \mathbf{A}, \quad \mathbf{K} = \frac{1}{2}\mathbf{D}_\alpha \otimes \left(\mathbf{Q}'_{,\alpha} \cdot \mathbf{Q}^T\right)_\times, \tag{2.5}$$

where $\mathbf{A} = \mathbf{I} - \mathbf{D}_3 \otimes \mathbf{D}_3$ and $\mathbf{I}$ are the unit tensor. In [25], a detailed analysis of the forms of the equations of state for various types of shell symmetry was carried out. In particular, considering the square-cell mesh shell, we can show that the material symmetry group of such a shell contains rotations around $\mathbf{D}_3$ at angles $\pm\pi/2$ and reflections $\mathbf{I} - \mathbf{D}_1 \otimes = \mathbf{D}_1$ and $\mathbf{I} - \mathbf{D}_2 \otimes \mathbf{D}_2$.

Taking into account the geometric meaning of the strain measures [26], it can be shown that the expressions $\mathbf{D}_1 \cdot \mathbf{E} \cdot \mathbf{D}_1$ and $\mathbf{D}_2 \cdot \mathbf{E} \cdot \mathbf{D}_2$ describe tensile-compression deformations in the $\mathbf{D}_\alpha$ directions. Comparing the expressions for the strain measures (1.3) and (2.5), one can notice that the relations

$$\varepsilon_1 = \mathbf{D}_1 \cdot \mathbf{E} \cdot \mathbf{D}_1, \quad \varepsilon_2 = \mathbf{D}_2 \cdot \mathbf{E} \cdot \mathbf{D}_2, \quad \mathbf{K} = \mathbf{D}_\alpha \otimes \mathbf{k}_\alpha$$

are satisfied if we assume that the position vectors $\mathbf{r}$ and $\mathbf{x}$, as well as the rotation tensors $\mathbf{P}$ and $\mathbf{Q}$ are the same: $\mathbf{r}$=$\mathbf{x}$, $\mathbf{P}$=$\mathbf{Q}$.

Thus, the deformation energy of the micropolar shell, corresponding to the continuous model of the mesh shell, has the form

$$W = U(\mathbf{D}_1 \cdot \mathbf{E} \cdot \mathbf{D}_1, \mathbf{D}_1 \cdot \mathbf{K}) + U(\mathbf{D}_2 \cdot \mathbf{E} \cdot \mathbf{D}_2, \mathbf{D}_2 \cdot \mathbf{K}) \tag{2.6}$$

that is, it represents a special case of the general relation (2.4). In other words, we can assume that the equations of state of a micropolar shell are completely determined by the defining relations of elastic fibers. Thus, to determine the static deformations of the mesh shells in the integral sense, it is possible to use finite element methods and special types of finite elements developed in the framework of the six-parameter theory of shells [15, 17–19]. Note also that the form of the equation of state (2.6) is similar to that used in the nonlinear theory of elasticity [27], where the potential deformation energy is represented as a sum of the form $W = f(\lambda_1) + f(\lambda_2) + f(\lambda_3)$, where λ_i is the main elongations, and f is a certain function. The case of inextensible fibers was previously considered in [28].

3. CONCLUSION

A continuum model of a mesh shell, formed by two families of flexible nonlinearly elastic fibers that retain their orthogonality in the process of deformation, is proposed. An expression for the deformation energy of the shell is obtained, which inherits the properties of fibers and is a special case of the general equations of state of the nonlinear theory of micropolar shells. This formulation allows the use of previously developed methods for solving boundary value problems for continual shells in the case of mesh shells. We note that here the assumption of preserving the orthogonality of the fibers was essentially used, its violation leads, generally speaking, to more complex models, see, for example, [29, 30]. A continuum model of a mesh shell, formed by two families of flexible nonlinearly elastic fibers that retain their orthogonality in the process of deformation, is proposed. An expression for the deformation energy of the shell is obtained, which inherits the properties of fibers and is a special case of the general equations of state of the nonlinear theory of micropolar shells. This formulation allows the use of previously developed methods for solving boundary value problems for continual shells in the case of mesh shells. We note that here the assumption of preserving the orthogonality of the fibers was essentially used, its violation leads, generally speaking, to more complex models, see, for example, [29, 30].

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REFERENCES

1. V. G. Shukhov, *Art of Construction* Ed. by R. Grefe, O. Perchi, F. Shukhov, et al. (MIR, Moscow, 1994) [in Russian].
2. M. Beckh, *Hyperbolic Structures: Shukhov's Lattice Towers — Forerunners of Modern Lightweight Construction* (John Wiley & Sons, Chichester, 2015).
3. V. V. Vasiliev, V. A. Barynin, and A. F. Razin, "Anisogrid Lattice Structures — Survey of Development and Application," Comp. Struct. **54** (2–3), 361–370 (2001).
4. V. V. Vasiliev, V. A. Barynin, and A. F. Razin, "Anisogrid Composite Lattice Structures — Development and Aerospace Application," Comp. Struct. **94** (3), 1117–1127 (2012).
5. V. V. Vasiliev and A. F. Razin "Prospects for the Use of Mesh Composite Structures in Civil Aviation," Polyot, No. 11–12, 3–12 (2016).
6. A. V. Azarov, "Continual and Discrete Models of Mesh Composite Cylindrical Shells," Mech. Comp. Mat. Struct. **18** (1), 121–130 (2012).
7. W. J. Li, C. T. Laurencin, E. J. Caterson, R. S. Tuan, et al., "Electrospun Nanofibrous Structure: a Novel Scaffold for Tissue Engineering," J. Biomed. Mater. Res. Part A. **60** (4), 613–621 (2002).
8. Cz. Woźniak, *Lattice Surface Structures* (PWN, Warsaw, 1970) [in Polish].
9. M. Kleiber and Cz. Woźniak, *Nonlinear Mechanics of Structures* (Polish Scientific Publishers and Dordrecht: Kluwer Acadcmic Publishers, Warszawa, 1991)
10. G. I. Pshenichnov, *A Theory of Latticed Plates and Shells* (World Scientific, Singapore, 1993).
11. V. V. Eliseev, *Mechanics of Elastic Bodies* (Izdat. SPBTU, St. Petersburg, 1999) [in Russian].
12. S. S. Antman, *Nonlinear Problems of Elasticity*, 2nd ed. (Springer, New York, 2005).
13. V. A. Eremeyev, L. P. Lebedev, and H. Altenbach, *Foundations of Micropolar Mechanics* (Springer, Heidelberg, 2013).
14. A. Libai and J. G. Simmonds, *The Nonlinear Theory of Elastic Shells* 2nd ed. (Cambridge University Press, Cambridge, 1998).
15. J. Chróścielewski , J. Makowski, and W.Pietraszkiewicz, *Statyka i Dynamika Powłok Wielopłatowych. Nieliniowa Teoria i Metoda Elementów Skończonych* (IPPT PAN, Warszawa, 2004) [in Polish].

16. V. A. Eremeyev and L. M. Zubov, *Mechanics of Elastic Shells* (Nauka, Moscow, 2008) [in Russian].
17. S. Burzyński , J. Chróścielewski, K. Daszkiewicz, and W. Witkowski, "Geometrically Nonlinear FEM Analysis of FGM Shells Based on Neutral Physical Surface Approach in 6-Parameter Shell Theory," Compos. Part B: Eng. **107**, 203–213 (2016).
18. S. Burzyński, J. Chróścielewski, and W. Witkowski, "Geometrically Nonlinear FEM Analysis of 6-Parameter Resultant Shell Theory Based on 2-D Cosserat Constitutive Model," ZAMM **96** (2), 191–204 (2016).
19. J. Chróścielewski, A. Sabik, B. Sobczyk, and W. Witkowski, "Nonlinear FEM 2D Failure Onset Prediction of Composite Shells Based on 6-Parameter Shell Theory," Thin-Walled Struct. **105**, 207–219 (2016).
20. J. Chróścielewski and W. Witkowski, "On Some Constitutive Equations for Micropolar Plates," ZAMM **90** (1), 53–64 (2010).
21. J. Chróścielewski, W. Pietraszkiewicz, and W. Witkowski, "On Shear Correction Factors in the Non-Linear Theory of Elastic Shells," Int. J. Sol. Struct. **47** (25), 3537–3545 (2010).
22. W. Pietraszkiewicz, "The Resultant Linear Six-Field Theory of Elastic Shells: What it Brings to the Classical Linear Shell Models?" ZAMM **96** (8), 899–915 (2016).
23. A. I. Lurie, *Nonlinear Theory of Elasticity* (Nauka, Moscow, 1980) [in Russian].
24. L. P. Lebedev, M. J. Cloud, and V. A. Eremeyev, *Tensor Analysis with Applications in Mechanics* (World Scientific, New Jersey, 2010).
25. V. A. Eremeyev and W. Pietraszkiewicz, "Local Symmetry Group in the General Theory of Elastic Shells," J. Elast. **85** (2), 125–152 (2006).
26. W. Pietraszkiewicz and V. A. Eremeyev, "On Natural Strain Measures of the Non-Linear Micropolar Continuum," Int. J. Sol. Struct. **46** (3–4), 774–787 (2009).
27. K. C. Valanis and R. F. Landel, "The Strain–Energy Function of a Hyperelastic Material in Terms of the Extension Ratios," J. App. Phys. **38** (7), 2997–3002 (1967).
28. V. A. Eremeyev, "On Characterization of an Elastic Network within the Six–Parameter Shell Theory," in *Shell Structures: Theory and Applications*, Ed. by W. Pietraszkiewicz and W. Witkowski (CRC Press, Boca-Raton, 2018), pp. 81–84.
29. F. dell'Isola and D. Steigman, "A Two-Dimensional Gradient–Elasticity Theory for Woven Fabrics," J. Elast. **118** (1), 113–125 (2015).
30. L. Placidi, E. Barchiesi, E. Turco, and N. L. Rizzi, "A Review on 2D Models for the Description of Pantographic Fabrics," ZAMP **67**(5), 121–140 (2016).