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ACCURACY ASSESSMENT OF MOBILE SATELLITE MEASUREMENTS IN RELATION TO THE GEOMETRICAL LAYOUT OF RAIL TRACKS

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Abstract

The paper presents the campaigns of mobile satellite measurements, carried out in 2009–2015 on the railway and tram lines. The accuracy of the measurement method has been analysed on the basis of the results obtained in both horizontal and vertical planes. The track axis deviation from the defined geometric shape has been analysed in the areas clearly defined in terms of geometry, *i.e.* on the straight sections and sections with constant longitudinal inclination. The values of measurement errors have been estimated on the basis of signals subjected to appropriate processes of filtration. The paper attempts to evaluate the changing possibilities of using the GNSS techniques to determine the shape of the railway track axis from 2009 to 2015. The determined average value of the measurement error now equals a few millimetres. This achievement is very promising for the prospects of mobile satellite measurements in railway engineering.

Keywords: positioning data processing, track geometric layout, GNSS satellite surveying, horizontal and vertical accuracy evaluation.

1. Introduction

The problem of examining track deformation is widely discussed in literature. The measurement methods based on tachymetry, GNSS positioning and INS surveying [1–5] are still examined to ensure a better reconstruction of the real shape of a railway track [6–8].

In the early 2009, an interdisciplinary research team from Gdansk University of Technology and the Naval Academy in Gdynia started the research project of using the GNSS/INS surveying in assessment of railway track geometry. The main purpose of the research is to improve techniques and methods for designing and assessing the track axis geometry. The mobile GNSS measurement technology developed by the research team turned out to be very useful in the mentioned problems by collecting track positions with high density and accuracy. In such conditions the geometric layouts of rail routes can be identified, improved and assessed [9–13].

The idea of mobile satellite measurements is based on carrying out geodetic measurements during the movement of a measurement platform with the installed GNSS multisystem geodetic satellite receivers (GPS – USA, GLONASS – Russia and Beidou – China). In order to increase

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the accuracy of position determination, the measurement campaigns were performed in real time with a frequency of 20 Hz using differential corrections acquired from the networks of GNSS reference stations – *Real-Time Networks* (RTN): ASG-EUPOS [14], LeicaSmartNet [15], taking into account the availability of satellite constellation [17]. Registration of fixed 3D coordinates in Word Geodetic System – WGS84 enables to record the course of the travelled route, which determines the geometrical layout in both horizontal and vertical planes.

Initially, the research was focused on the practical use of the measured coordinate of track axis in the horizontal plane. Due to the fact that the accuracy obtained in this plane proved to be more than sufficient for the purposes of the design process, the computation algorithms and computer-aided design software were developed [11]. The vertical coordinates were outside the scope of interest, since they were regarded as an element without a direct relation to the design of the route. Alternatively, it could prove to be useful in the diagnosis of track geometry, but this issue still requires clarification. Due to the required high precision of measurements, its accuracy should be verified at the current stage of development of the described measurement technique [13]. The paper aims to clarify this issue, with respect to the measured coordinates in both horizontal and vertical planes.

The first measurement campaign was carried out on a section of the railway line Koscierzyna – Kartuzy in northern Poland. The next measurement campaigns were performed in 2010 in the section Gdańsk Main Station – Gdańsk Zaspa (Freight Station) and on the line Gdańsk Osowa – Somonino. In 2012, 2013 and 2014 satellite measurements were carried out on tram lines in Gdansk and the historic narrow-gauge railway line in Koszalin [18–20]. The most recent research took place in June 2015, on the entire length of a newly constructed suburban railway line called the Pomeranian Metropolitan Railway. The coordinates of the railroad track axis were determined using various configurations of geodetic GNSS receivers. Their number, location on the measurement platform and configuration were subjected to numerous modifications. The terrain configurations were different in each of the measurement campaigns. The least preferred conditions in a sense of terrain obstacles was found in Koszalin (2010) – the line was going through a forest. On the other hand, the Pomeranian Metropolitan Railway was characterized by the most advantageous terrain conditions. The measurements in the tram network have been also made in various terrain configurations.

2. Possibilities of increasing accuracy of satellite measurements

In the railroad inventory surveys carried out in 2009÷2010, the differential corrections of GNSS signal were used both in real time and in post-processing mode. During measurements, the team used the governmental Polish Active Geodetic Network ASG-EUPOS which was launched in the middle of 2008. The network corrections were acquired from the RTN using only GPS, at an estimated accuracy: Distance Root Mean Squared error DRMS – less than 30 mm (horizontally) and Root Mean Squared error – RMS – 50 mm (vertically) [21].

The principle of the RTK network functionality is based on the cooperation of several reference stations, which are constantly sending satellite measurement results to the main server with specialized software. Calculated amendments are sent from the server to the user in a standard format (RTCM) at 1-second intervals. The smallest RTK network consists of the recommended minimum of five reference stations (there is no upper limit), with 50 km maximum distance from each other. The purpose of the RTK network is to minimize the effect of reduction in accuracy dependent on the distance between the base station and the receiver that affects the calculated position.

When the team planned the first measurement campaign, the limitations of receiving GNSS signals resulting from obstacles occurring in the urban area were took into account. Therefore, after a detailed analysis of the conditions of measurements carried out in 2009–2010 on the railway track, it was decided to thoroughly verify the methodology of the research. The measurements were taken with the use of real-time ASG-EUPOS corrections due to the existing (in the afternoon) network congestion associated with the transmission of GPS pseudo-range corrections. It was decided to carry out measurements in the post-processing mode, which enables to use signals from different reference stations in the calculation of results using dual-constellation GNSS receivers, thus using the signals of two satellite systems: GPS and GLONASS. This led to an increase of accuracy of coordinate determination, directly related to the available number of satellites. Due to the fact that by 2012 the ASG-EUPOS network did not have the capability to send corrections for dual-mode receivers, a local GPS/GLONASS reference station located in the area of measurement (up to 10 km) was also used. Since 2012, the *virtual reference stations'* (VRS) corrections were transmitted to the GPS satellite system (as before in ASG-EUPOS) as well as to the Russian GLONASS system. In this way, the number of available satellites is nearly doubled, which in an urbanized area has a major impact on the accuracy of calculations and significantly improves the availability of high-precision measurements. Currently, the VRS corrections are sent to at least three constellations (GPS, GLONASS, Galileo/Beidou). They enable to perform measurements with the use of two-frequency geodetic GNSS receivers with an accuracy of up to 8 mm (DRMS) horizontally and up to 15 mm (RMS) vertically [22].

In addition, to improve the quality of the measurement itself, the team has been working on finding a high quality measurement vehicle. A measurement trolley should have a proper construction to ensure tight fitting to the track and determine the correction of the track axis shift due to cant. By treating these assumptions as targets, at the first stages of research it was decided to employ existing capabilities and to use existing rail vehicles or their components (twin-axle bogies). As it turned out, even such temporary solutions have led to the confirmation of the effectiveness of the proposed measurement method. In the measurements carried out on railway lines in 2009–2010 a PWM-15 trailer and WM-15 platform were used. In the measurements of tram lines and Pomeranian Metropolitan Railroad the 300-series pre-war tram boogies and a track geometry measurement trolley were used. In Table 1 the measurement systems – their configurations – are presented [23].

Table 1. Specification of the measurement systems.

Date	***GNSS* systems**	**RTN**	**Processing**
Feb. 2009	*GPS – 4 antenas*	*VRS, FKP, MAC, ASG-EUPOS*	Real Time, Service *NAVGEO* Distance auto-recording 30 cm
Apr. 2010	*GPS – 3 antenas*	*VRS, FKP, MAC, ASG-EUPOS*	Real Time, Service *NAVGEO*, Distance auto-recording 30 cm, Leica Office software
Nov. 2010	*GPS – 3 antenas*	*VRS service: ASG-EUPOS*	Real Time, Service *NAVGEO*, Distance auto-recording 30 cm, Leica Office software
Feb. 2012	*GPS/GLONASS – 2 antenas*	*FKP* Gdansk University of Technology Reference Station	Post-processing, Auto-recording every 30 cm, Leica Office software
Oct. 2013	*GPS/GLONASS – 1 antena*	*VRS service:* Leica SmartNet *GNSS* network	Real Time, Auto-recording 20 Hz (Leica GS 15), Leica Office software
May. 2014	*GPS/GLONASS – 1 antena*	*VRS service:* Leica SmartNet GNSS network	Real Time, Auto-recording 20 Hz, Leica Office software
Jun. 2015	*GPS/GLONASS/BeiDou – 7 antenas*	*VRS* services: Trimble, Topcon and Leica networks	Real Time, Auto-recording 20 Hz, Trimble Business Center

3. Adopted methodology of analysis

A geometric layout of tracks consists of straight sections and curves of constant and variable curvature in the horizontal plane and sections of uniform inclination and circular arcs in the vertical plane. The straight sections as well as sections of uniform inclination are clearly defined in terms of geometry, which is why the assessment of their shapes, obtained from satellite measurements can provide a basis for determining the obtained accuracy.

Initially, the track axis coordinates measured in WGS84 system were transformed into PL-2000 national Cartesian coordinate system based on Gauss-Kruger projection. Next, in order to determine the course of the route on straight track sections in both horizontal and vertical planes, the measured signal of spatial coordinates XYZ was transformed into local coordinate systems. The relations between global and local coordinate systems are shown in Fig. 1.

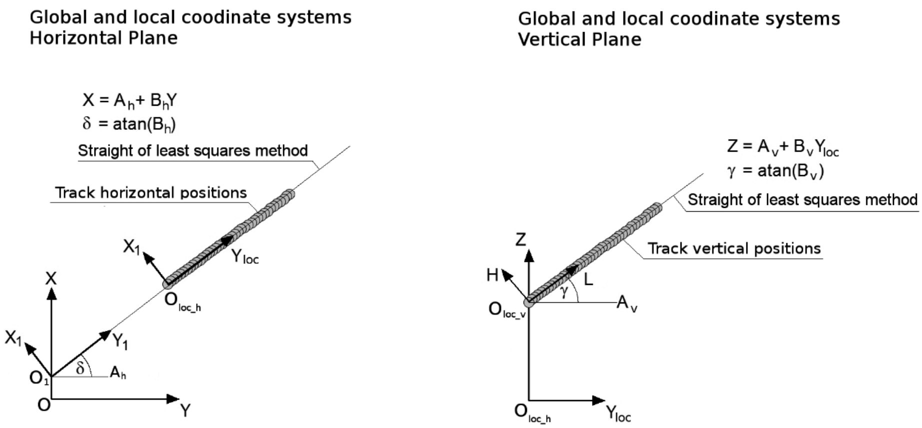

Fig. 1. The idea of transformation of the coordinate system in the horizontal and vertical planes.

For this purpose, selected points' clusters were approximated and described by the linear equations:

$$X = A_h + B_h Y, \tag{1}$$

$$Z = A_v + B_v Y_{loc}. \tag{2}$$

In the next step the coordinates were transformed to the local coordinate system $X_1(Y_1)$ and $H(L)$ using the following formulae [24]:

For $B_h > 0$, the angle $\delta = \mathrm{atan}(B_h)$:

$$Y_1 = Y\cos(\delta) + (X - A_h)\sin(\delta), \tag{3}$$

$$X_1 = -Y\sin(\delta) + (X - A_h)\cos(\delta). \tag{4}$$

For $B_h < 0$, the angle $\delta = \pi + \mathrm{atan}(B_h)$:

$$Y_1 = Y\cos(\delta) - (X - A_h)\sin(\delta), \tag{5}$$

$$X_1 = Y\sin(\delta) + (X - A_h)\cos(\delta). \tag{6}$$

For $B_v > 0$ the angle $\gamma = \mathrm{atan}(B_v)$:

$$L = Y_{loc}\cos(\gamma) + (Z - A_v)\sin(\gamma), \tag{7}$$

$$H = -Y_{loc}\sin(\gamma) + (Z - A_v)\cos(\gamma). \tag{8}$$

For $B_v < 0$ the angle $\gamma = \pi + \mathrm{atan}(B_v)$:

$$L = Y_{loc}\cos(\gamma) - (Z - A_v)\sin(\gamma), \tag{9}$$

$$H = Y_{loc}\sin(\gamma) + (Z - A_v)\cos(\gamma), \tag{10}$$

where: B_h, A_h and B_v, A_v – coefficients of the linear equations of track sections in the horizontal and vertical planes; δ, γ – angles of straight inclination in the horizontal and vertical planes.

After these transformations with the use of the presented algorithm, the obtained signal is located in the coordinate system in which the horizontal axis is the direction corresponding to the track axis in the straight section in both planes. In [25] the authors proved that identification of the linear parameters of straight track sections with the use of GNSS methods is possible with 0.01° accuracy. As a result, vertical non-zero values determine the 171 deviation of the GPS signal from the direction of the straight line.

If the measurement follows a perfectly straight track (without any horizontal and vertical irregularities), the recorded signal values would represent a measurement error, which would enable to determine the accuracy of the presented method. However, in practice, the shape of track was deformed as a result of operation and technological processes of maintenance. Therefore, it should be expected that the measured signal transformed into the local coordinates system will represent the horizontal deformation of the track axis (in relation to the design assumptions) as well as the measurement error.

3.1. Track deformation index – Cross Track Error

As it was mentioned in Introduction, the main goal of the measurement campaigns was to determine both the location and shape of a railway track. Therefore, the correctness of the adopted methodology of satellite measurements implies the accuracy of determining the shape of the track's axis. In order to assess the accuracy, the basic error measure of the track position adopted by the authors was the distance between the measured positions of the receivers and the established (theoretic) points along the track axis. This indicator – defined as *Cross Track Error* (XTE) – will serve as a unified parameter in the statistical evaluation of a large number of homogeneous sections of the examined track. This analysis brought an objective assessment of changes in the measurement capabilities over the years 2009÷2015. For the final assessment, the module of XTE was adopted. From the navigation viewpoint – the values of XTE are always non-negative, therefore the left/right deviations are no longer distinguished. The idea of XTE is shown in Fig. 2.

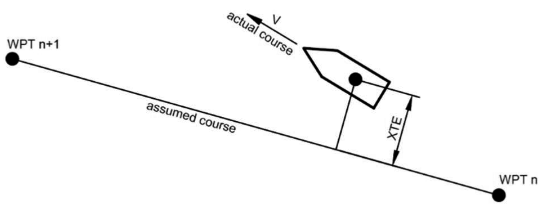

Fig. 2. A graphical interpretation of the XTE concept.

3.2. Signal analysis

In order to obtain the track alignment closer to the actual one – based on the position data – the authors analysed the position signal using the Fourier transformation [13]. Then, the density of the wave number obtained in this way was filtered. It was assumed that the superstructure rigidity in both horizontal and vertical planes does not allow for all deformation forms – particularly for the very short wavelengths. For this reason, after the analysis of the examined sections, thresholds for cutting off the high values of the deformation wave number in the horizontal and vertical plane were established. After this operation, by performing the *inverse Fourier transformation* (IFFT), a smoothed signal of track positions was obtained. This signal is interpreted as a supposed probable form of the deformed track axis. This waveform from that moment was a reference to the calculation of the XTE value. Thus, the actual XTE value defined a new indicator – ΔXTE – calculated in reference to the filtered signal of a track position.

In order to establish the filtering parameters (cut-off threshold) the authors measured a test track section using a common and very precise tachymetric method. It has been observed that the higher wave numbers usually occur in the form of evenly distributed signal noise. It should be emphasized that in order to objectively identify a local cut-off threshold it is necessary to refer to the reference measurement of the track irregularities (shape imperfections) in specific track conditions. This measurement can be made, for example, by self-registration draisines or trolleys [26, [27]. The chord – versine analysis enables to compare the results obtained by different methods [28, 29].

For a statistical analysis, other than the standard evaluation of position uncertainty (using standard deviation of mean), a Weibull distribution for ΔXTE analysis was used. The authors deliberately departed from the symmetry of deviations at the ΔXTE (analysis without considering the left/right direction of track's deflection). In this analysis the expected value (estimated by mean) is always positive and characterizes the average distance between the measured position and the determined track axis line. This analysis illustrates both the measurement uncertainty and the track deformation level.

For each measurement series, the position of the track axis was determined by filtering out high wave numbers using FFT. The cut values of the transformations were classified as measurement errors. Then, after applying the inverse Fourier transform, the position of the measuring points was obtained. Fig. 3 shows the results of FFT transformation for one of the sections. A low-pass filter with a cut-off frequency threshold was adopted. This value was selected by qualitatively evaluating the result of reverse transformation versus the raw signal.

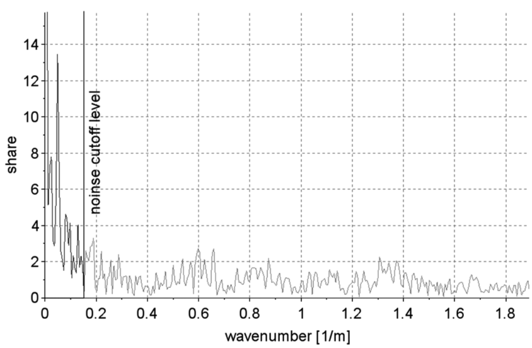

Fig. 3. The Fourier transform of a (measured) track section in the horizontal plane.
The vertical line indicates the signal filtering limit.

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4. Analysis of measurement data

4.1. Selection of sections for analysis

In order to analyse the accuracy of determining coordinates during satellite measurements, all data obtained over the years 2009–2015 were collected (apart from those obtained in the measurements made in 2014 in Koszalin which were considered unrepresentative). A total of 39 measurement files were loaded, with a total of 871,008 coordinates. The shares of the numbers of coordinates specified for each year are shown in Fig. 4. A significantly smaller proportion of the number of coordinates obtained in 2012 results from the adopted measurement step, and in 2014 – from the very short segment of track which was tested. Notwithstanding, the results are presented because the number of assessed sections was still of the same order of magnitude as in other years. However, it should be emphasized that the years 2012 and 2014 have a relatively smaller impact in the analysis.

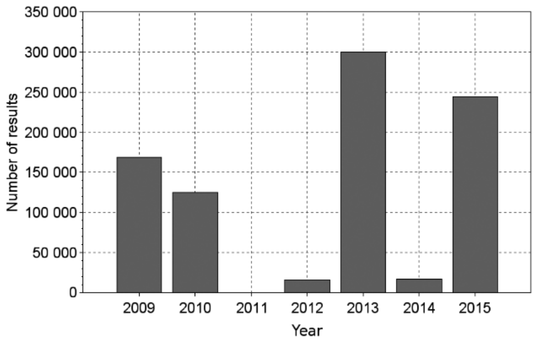

Fig. 4. Distribution of the numbers of coordinates obtained in each year.

The first step in the analysis was to determine the accuracy of measurements recorded by the devices in each year. For each measurement, the receiver records a determined uncertainty radius of the estimated coordinates (based on the geometric arrangement of the satellites which were visible on the horizon). In 2014, the determination accuracy specified by the receiver was not recorded. The mean value for all years was 603.9 mm and the median was 20 mm. From the disparity between the mean and the median, a significant influence of individual coordinates, measured with a very high uncertainty was clearly visible. These coordinates were specified in the code-phase mode and were removed.

After removing the measurements made with the use of code-phase, a greater correspondence between the mean value of 26.5 mm and the median of 17.8 mm for the radius of uncertainty of determination of coordinates (according to the receiver) can be noticed. Distributions of uncertainty in each year are shown in Figure 8. The average values are shown in dark bars and the medians in light ones. The carrier-phase measurements were made in comparable conditions except for the measurements made in 2010 (and 2014 – for which no data are available), what can be concluded from the comparable median of the uncertainty radii of coordinates' determination. There is also a smaller share of code-phase measurements in 2012–2015 compared with these in the years 2009–2010.

During the measurements some of the coordinates were derived with fixed integer ambiguities (164,485 samples), but only using the codes as opposed to using phase angles of the satellite signals (706,523 samples). In 2014 coordinates determined with the use of code-phase were not analysed because they were deleted together with the rest of the additional information. The shares of code-phase measurements (light bars) at the background of the carrier-phase measurements (dark bars) for each year are shown in Fig. 5.

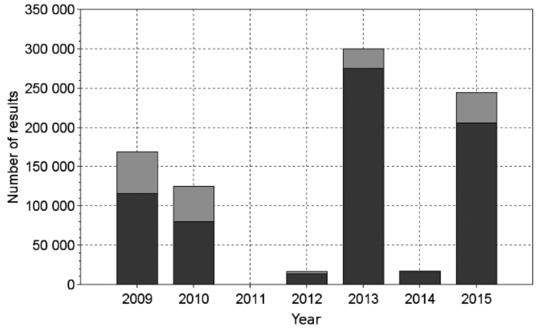

Fig. 5. Distribution of the numbers of coordinates obtained in code-phase (light bars) and carrier-phase (dark bars) in each year.

Then, the analysis of selected straight sections began and more than 100 signal ranges corresponding to the measured straight sections were extracted from the files containing the measurement data. Presently, each of the sections was analysed by a function that defines a linear equation. For the analysis only the sections with a good quality of signal were chosen. At this stage of analysis, it was determined that 95 of the measurement signal cut sections satisfy the conditions for further analysis. The sections with a significant influence of field obstacles resulting in the occurrence of constant shifts in the measured signal were rejected. An example of a segment with a high offset (between 170 and 240 m) is shown in Fig. 6. Although the entire length was measured in the carrier-phase mode, the uncertainty of determination of coordinates over a certain length is much greater than for the rest.

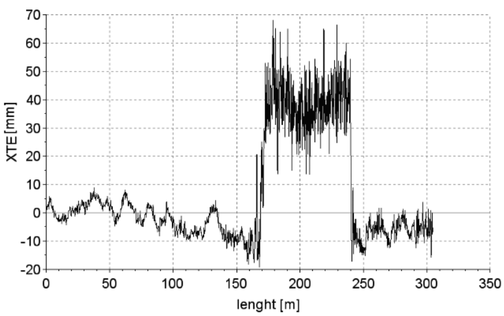

Fig. 6. An example of a raw XTE signal rejected from further analysis.

5. Assessment of parameters ΔXTE and ΔH

From the whole database of measurements, respectively 95 and 72 straight track sections have been analysed for horizontal and vertical assessment. The various numbers of test sections in each year – Table 2 – were determined by the characteristics of these sections as well as by the numbers of GNSS receivers in the measurement system.

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Table 2. The numbers of sections selected for analysis of the horizontal and vertical coordinates in each year of satellite measurements.

Date	**2009**	**2010**	**2012**	**2013**	**2014**	**2015**
Number of test sections – ΔXTE analysis	22	18	20	9	16	10
Number of test sections – ΔH analysis	14	27	14	9	–	8

As it was mentioned in Section 3.2, ΔXTE and ΔH were analysed for their absolute values. In Fig. 7 examples of histograms for 2009 and 2015 are presented. It should be emphasized that in 2015 the measured track was newly built and the elevation of the line was much higher in comparison with the railway line examined in 2009.

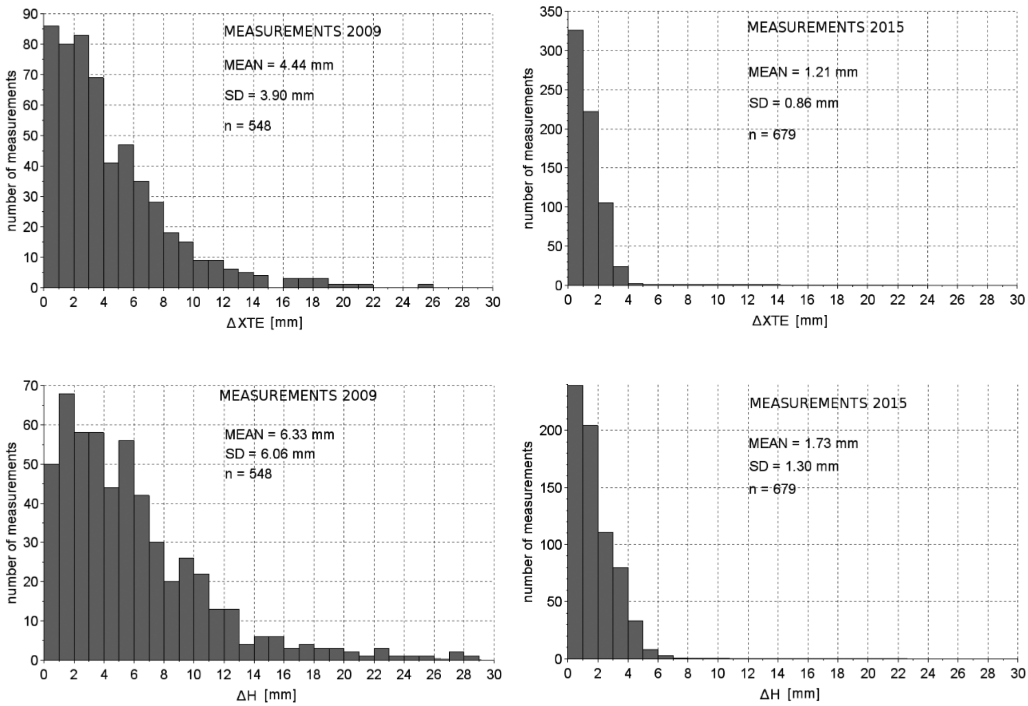

Fig. 7. Empirical distributions of ΔXTE and ΔH values for selected test sections from the 2009 and 2015 measurement campaigns.

Weibull probability density functions together with the histograms of ΔXTE and ΔH for all measurement data are presented in Fig. 8. The calculated expected value for the horizontal plane was 2.6 mm and for 90% of samples the values were between 0÷6.5 mm. For the vertical plane the expected value was 4.3 mm and for 95% samples their values were between 0÷10.5 mm.

For each measurement series, the altitude of the track axis was determined by filtering high wave numbers using Fast Fourier Transform. The higher wave numbers were cut with thresholds of 0.15 1/m for both planes. It should be noticed that, by accepting smaller values of filter thresholds, more of the wave signal residual components have been left, resulting in a gain of the calculated ΔXTE values.

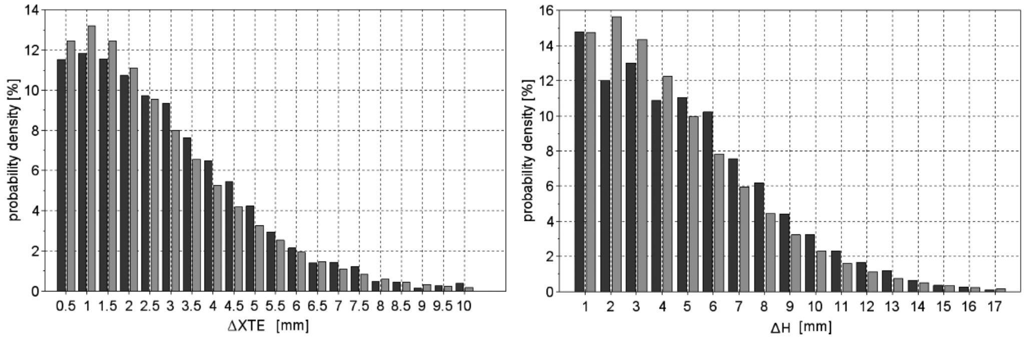

Fig. 8. The histograms of ΔXTE and ΔH (dark bars) and the probability density functions of Weibull distribution (light bars) with parameters respectively W (δ = 1.3, λ = 2.8) and W (δ = 1.4, λ = 4.7).

In Table 3 the final statistics are presented. The results prove that in the years 2009–2015, the accuracy obtained by the adopted methodology on the basis of a mobile satellite measurement method has evidently increased. This is undoubtedly due to the progress of technology used in newer GNSS receivers, as well as the development of GNSS space segment and ground reference networks. The adopted indicator $\overline{\Delta H}$ (mean value), representing the precision of GNSS measurements in the vertical plane, has been reduced several times over the measurement period, approaching the $\overline{\Delta XTE}$ indicator value, which determines the accuracy of measurements in the horizontal plane. At the beginning, the values of ΔXTE have been more favourable than the ΔH ones, but now there is only a minor difference between them. Table 3 clearly shows that the obtained accuracy of mobile satellite measurements is at present very high. The results obtained by the authors are presented separately for the horizontal and vertical planes. They correspond to the ones obtained for the spatial accuracy of determining the position in the research [27].

Table 3. Summary of mean values and standard deviations of indicators ΔXTE and ΔH.

Campaign	**Mean value**		**Standard deviation**	
	$\overline{\Delta XTE}$ **[mm]**	$\overline{\Delta H}$ **[mm]**	$\sigma_{\Delta XTE}$ **[mm]**	$\sigma_{\Delta H}$ **[mm]**
2009	3.63	10.59	3.92	4.84
2010	4.54	10.66	5.47	16.68
2012	3.38	8.00	3.17	12.39
2013	2.45	5.11	2.32	4.79
2014	3.11	–	2.61	–
2015	1.74	3.00	1.83	2.98

There is no doubt that in the measurements carried out, the main factor contributing to the increase in accuracy was the incorporation of a second (non-GPS) constellation of satellites - the Russian GLONASS system and the use of a dual-system active geodetic network. Considering the upcoming development of new systems: Chinese Beidou and European Galileo, the proposed measurement technique is very promising.

It is clear from the analysis that the achieved accuracy is sufficient for design purposes. As a result of an appropriate calculation procedure, the values of the differences in determination of track shape were found based on several measurement series. Assuming Weibull distribution for these differences, the expected values were: 2.6 mm for the horizontal plane and 4.3 mm for the vertical plane. These achievements are very promising from the point of view of the use of mobile satellite measurements in track diagnosis, if we take into account the improvised method of mounting the antennas used in the research, resulting from limited capabilities. Significant improvements in accuracy should be ensured by a proper construction of the rail vehicle dedicated to carry the receivers, ensuring strict reproduction of track shape and enabling to make corrections due to differences in railhead elevation.

6. Conclusions

A significant increase in measurement precision is made possible by the fact that the pseudo-range corrections from the virtual station are transmitted both to the GPS satellite system and to the GNSS GLONASS system. This greatly increases the number of available satellites, which has a major impact on the accuracy of determination and significantly increases the availability of high-precision tools (especially in the urban areas).

The GNSS campaigns carried out over the years 2009–2015 were also focused on checking the accuracy. Therefore, for a long time it was preferable to consider only the horizontal coordinates of the railway axis, but the accuracy of the vertical coordinates is also discussed in this paper. As a result of the described calculation procedure, it has been shown that, over the years 2009–2015, the accuracy of the measurement in the mobile satellite measurement method has clearly increased. This applies both to the horizontal plane and to the vertical plane. These findings are very promising from the perspective of the use of mobile satellite measurements also in diagnosing the conditions of railways. As expected, significant improvements in accuracy should be ensured by a proper construction of the rail vehicle dedicated to carry the receivers, ensuring strict reproduction of track shape and enabling to make corrections due to differences in railhead elevations.

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