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Compressed Projection Bases for Model-Order Reduction of Multiport Microwave Components Using FEM

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***Abstract*—This paper presents a projection basis compression technique for generating compact reduced-order models (ROM) in the FE analysis of microwave devices. In this approach redundancy is removed from the projection basis by means of the proper orthogonal decomposition technique applied to the projected system of linear equations. Compression allows for keeping the size of a reduced-order model as small as possible without compromising ROM's accuracy. Effectiveness of the basis compression technique, including memory and time consumption as well as the size of the resultant ROM, are discussed for both global and local model-order reduction schemes.**

***Index Terms*—finite-element method, model-order reduction**

I. Introduction

Model-order reduction (MOR) is one of the most commonly used techniques to reduce the simulation time of the electromagnetic problems, analyzed by means of discrete methods, such as Finite Difference (FD) or Finite Element (FE). The main idea of MOR is to approximate the original complex model with a much simpler (reduced) one which preserves the original input-output behavior of the analyzed structure. The reduction approaches can be applied both to the global system of equations (so called: global reduction) [1]–[5] and to the selected subregions of the computational domain (local reduction, macromodeling) [6], [7].

In order to obtain reliable reduced-order models (ROMs) in the wide-band frequency simulations, multi-point MOR methods have to be applied [1]–[3]. They assume that the projection basis is generated at many expansion (frequency) points in which the moments of the reduced and the original transfer functions are matched up to the specified order. Although such an approach guarantees very high accuracy in the whole frequency band, the projection basis and consequently ROM in this case may eventually become very large, especially when the multiport structures are considered. In effect, the efficiency of the overall reduction process can be significantly deteriorated. Very compact ROMs are obtained with the Reduced Basis Method (RBM) [8]. Here the projection basis is found from snapshots of the solutions. Even though ROM produced by RBM is small, its construction requires more matrix factorizations than what is needed in moment-matching techniques.

A problem of a large projection basis occurs when the goal of the simulation is to obtain a parametrized ROM which is valid in the specified geometry and/or material parameter space $\mathcal{P}$. In this case the projection basis Q is composed of several projection subbases Q_i, where $i = \{1, 2, \ldots, m\}$. Each of them is evaluated in one of the properly selected m points from the space $\mathcal{P}$. However, if the high accuracy of the parametrized ROM is required, the number of vectors in the basis can be unnecessarily large.

To ensure that the size of the projection basis (and consequently the ROM) is kept small without compromising the accuracy of the ROM we propose a basis compression technique. The basis compression is an additional projection step that finds the best linear combination of the original projection vectors for a given problem. In this paper we explain the compression technique and provide a deeper insight into the details of this operation in both global and local model-order reduction approaches. We consider memory and time consumption, as well as the accuracy of the reduction and compression process.

II. Theory

The N-dimensional Finite Element discretization of a time-harmonic Maxwell's equations for a dielectric-loaded, lossy structure Ω excited through P ports with M_i modes at the i-th port leads to the following second-order input–output system of equations (the details of the formulation are provided in [9]):

$$\begin{aligned}(\Gamma + sG + s^2C)E(s) &= sBI,\\ U &= B^T E(s),\end{aligned} \tag{1}$$

where $\Gamma, G, C \in \mathbb{C}^{n\times n}$ are the FEM system matrices, $B \in \mathbb{C}^{n\times m}$ denotes a normalized port selection matrix, $E(s) \in \mathbb{C}^{n\times m}$ is a matrix of unknown FEM coefficients, I, and U are the vectors of amplitude of the normalized currents and voltages, respectively, $s = j\omega/c$ is the complex variable, c is a speed of light, and m is the total number of excitation modes.

Due to the model-order reduction process applied to the original FEM system of equations, one obtains the so-called reduced-order model (ROM) with much smaller number of variables, comparing to the original model ($r \ll n$):

$$\begin{aligned}(\Gamma_R + sG_R + s^2 C_R)E_R(s) &= sB_R I,\\ U &= B_R^T E_R(s),\end{aligned} \quad (2)$$

where the reduced matrices: $\Gamma_R, G_R, C_R \in \mathbb{C}^{r\times r}$ and $B_R \in \mathbb{C}^{r\times m}$ are computed by the projection of the original FEM matrices on a subspace spanned by the vectors of the reduced basis Q:

$$\begin{aligned}\Gamma_R &= Q^T \Gamma Q,\\ G_R &= Q^T G Q,\\ C_R &= Q^T C Q,\\ B_R &= Q^T B\end{aligned} \quad (3)$$

and the original solution matrix is approximated by the one obtained by means of the ROM, as follows:

$$E(s) \approx QE_R(s). \quad (4)$$

Note that the basis Q can be obtained by means of one of the (possibly multipoint) moment-matching reduction approaches, applied both to the global FEM system of equations [1]–[5] or to the selected subregions of the computational domain [6], [7]. What is more, Q can be used to obtain parametrized ROM, valid in the specified geometry and/or material parameter space $\mathcal{P}$. To this end, Q has to be composed of several projection subbases Q_i (where $i = \{1, 2, \ldots, m\}$), evaluated in selected points of the parameter space $\mathcal{P}$.

The reduction approaches produce projection basis which span the solutions in all frequencies in the band of interest. However, in all above cases the number of vectors in the projection basis may become very large and may contain redundant vectors that are not needed to span the solution, especially when the wide frequency band is considered. The redundancy in the projection basis can significantly influence the efficiency of the reduction approach. In order to keep the size of the projection basis as well as the size of the ROM as small as possible, we propose to utilize the operation called *basis compression*.

A. Basis compression

The goal of the *basis compression* operation is to find an optimal subspace of $span\{Q\}$, which is sufficient to represent the solution vectors for all frequencies in the band of interest, in such a way that the redundant vectors are removed from $span\{Q\}$. To this end, we perform the proper orthogonal decomposition (POD [10]) on eq. (2), which means that the reduced system of equations is solved for all considered D frequency points $s_1, s_2, ..., s_D$. All snapshots are stored in the matrix W_R:

$$W_R = [E_R(s_1), E_R(s_2), \ldots, E_R(s_D)]\,. \quad (5)$$

Since the number of unknowns in (2) is much smaller comparing to the original system of equations, matrix W_R is generated extremely fast. Next, we remove the redundancy from W_R by means of the singular value decomposition (SVD) [11] and in effect we obtain matrix $\widetilde{W}_R \in \mathbb{C}^{r\times w}$, with $w \le r$:

$$\widetilde{W}_R = \mathrm{SVD}(W_R), \quad (6)$$

which spans the solution space. Finally, the original projection basis Q is projected onto a subspace spanned by the vectors of $\widetilde{W}_R$:

$$\widetilde{Q} = Q\widetilde{W}_R. \quad (7)$$

with $\widetilde{Q} \in \mathbb{C}^{n\times w}$. It is seen that $\widetilde{Q}$ is a linear combination of the projection vectors generated by a MOR scheme. This new basis is used to project the original FEM matrices. This yields:

$$\begin{aligned}\widetilde{\Gamma}_R &= \widetilde{Q}^T \Gamma \widetilde{Q},\\ \widetilde{G}_R &= \widetilde{Q}^T G \widetilde{Q},\\ \widetilde{C}_R &= \widetilde{Q}^T C \widetilde{Q},\\ \widetilde{B}_R &= \widetilde{Q}^T B,\end{aligned} \quad (8)$$

where matrices $\widetilde{\Gamma}_R$, $\widetilde{G}_R$, $\widetilde{C}_R \in \mathbb{C}^{w\times w}$ and $\widetilde{B}_R \in \mathbb{C}^{w\times m}$ with $w \le r$.

Similarly, the same procedure can be used to perform *basis compression* on the local projection basis in macromodeling technique. To this end, operations Eq. (5)-(7) are applied to a local system of equations. For the detailed description see [6].

In effect, the number of vectors in the projection basis can be considerably lowered, without devoting the accuracy of the ROM.

III. Numerical Results

In this section, the efficiency of the proposed basis compression technique is discussed. Two examples are considered: a microstrip branch coupler analyzed by means of Reliable Greedy Multipoint Model-Order Reduction technique (RGM-MOR) [2] and a dual-mode waveguide filter analyzed using local parametrized macromodeling technique [6]. As a FEM simulation we use InventSim [12].

A. Microstrip coupler

The geometry of the analysed coupler is depicted in Fig. 1 (all geometry dimensions all detailed in [13]). Scattering parameters computed at 476 frequency points in the frequency band 0.5-10 GHz are shown in Fig. 2. The FEM formulation resulted in the system of equation with ca. 1.15 million unknowns. The target accuracy of the reduced-order model has been set to 10^{-6}. A Reliable Greedy Multipoint Model-Order Reduction (RGM-MOR) technique was applied to compute the projection basis for fast frequency sweep. In RGM-MOR [2] moments of the transfer function are matched at many expansion (frequency) points selected in an automated way based on an error estimator. Subsequent block moments at each expansion point are added to the projection basis until

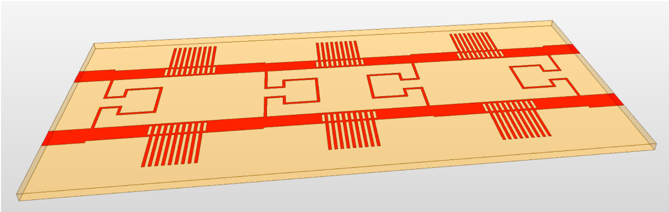


Fig. 1. Three-section branch-line microstrip coupler geometry.

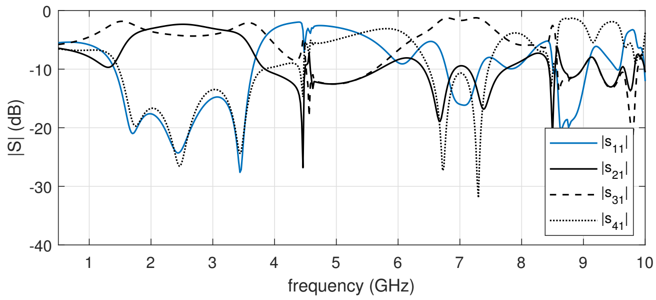


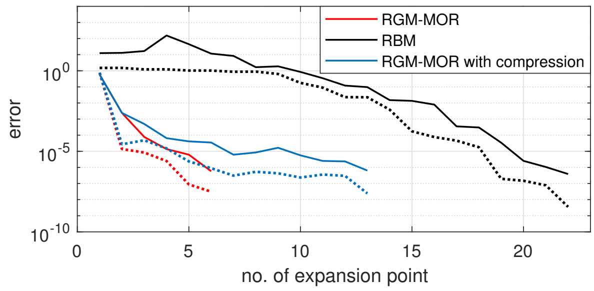
Fig. 2. Scattering parameters of the microstrip coupler.

the error in a chosen sub-band drops below assumed tolerance. Next, the projection basis is compressed and new expansion point is chosen.

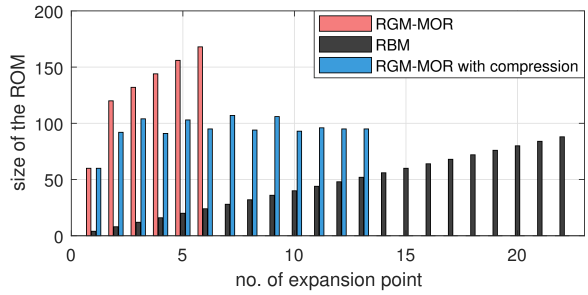
In the example considered above RGM-MOR technique with basis compression resulted in the reduced order model with 95 unknowns. In the course of the greedy algorithm, 13 expansion frequency points were chosen. The results of the RGM-MOR with compression were compared with the standard RGM-MOR approach (with no compression) and the Reduced Basis Method [8] (RBM). Fig. 3a shows the maximum value of the actual and estimated error as a function of the number of expansion points for three reduction schemes. It can be seen that the actual and estimated errors are well correlated in all three cases. The size of the reduced-order model as a function of number of expansion points is shown in Fig. 3b. It can be seen that the RGM-MOR with compression is a compromise between RBM and standard RGM-MOR in terms of the number of FE system matrix factorizations (equal to the number of expansion points) and the size of the projection basis. The reduced basis method (RBM) yields the smallest size of the basis (88 vectors), but requires as many as 22 sparse matrix factorizations (expansion points). RGM-MOR needs only 6 factorizations but the final size of the reduced order model is 168. Applying compression every time new expansion point is added results in the ROM size of 95 which is almost as small as RBM, while the number of factorizations is about half of that needed by RBM.

B. Dual mode filter

In this example, the basis compression technique was combined with parametrized local model-order reduction scheme [6] to speed up the analysis of dual mode waveguide filter. The structure was partitioned into 5 subdomains – the geometry



(a)



(b)

Fig. 3. Comparison of RBM, RGM-MOR and RGM-MOR with compression. a) Estimated error (solid line) and actual error (dotted line) as a function of number of expansion points. b) Size of the reduced-order model as a function of number of expansion points.

of the filter and its segmentation is shown in Fig. 4. Local parametric MOR was applied in three subregions containing tuning screws and a cross-shaped iris. The idea behind local parametrized MOR is that the projection basis Q_i for reduction in subdomain Ω_i is formed by concatenating several bases created for different geometries of the region Ω_i. In this case, three bases were used to create parametrized local ROMs in each of three subregions. The size of the original ROM obtained by a local MOR technique was 3876. Compression reduced this number to 1685. Since reduced FE matrices are composed of several macromodels, and each macromodel is a dense matrix, operations such as error estimation and solving become time and memory consuming. In terms of CPU time, achieving compression factor of two, results in 8-fold reduction of the matrix factorization time. Compression is also beneficial for CPU memory saving. In the example considered above, basis compression resulted in the reduction of number of nonzero elements in FE matrices from 4.3 million to 730 thousand and savings in the CPU memory needed to store them from 72 MB to 11.5 MB. What is important, basis compression preserves the accuracy of the ROM - in Fig. 5 the filter response obtained with basis compression and characteristics computed without compression are presented - very good agreement is observed.

IV. Conclusion

This paper has presented a basis compression technique and its application in different model order reduction schemes,

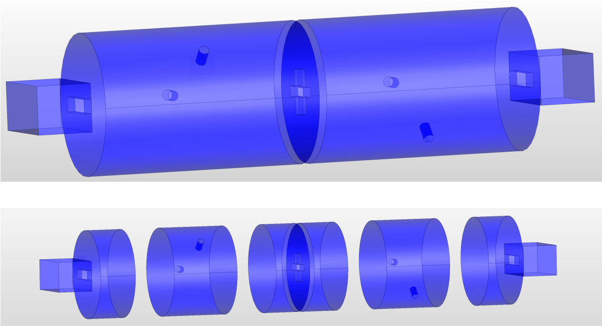


Fig. 4. Dual mode filter geometry and its segmentation. The dimensions can be found in [8].

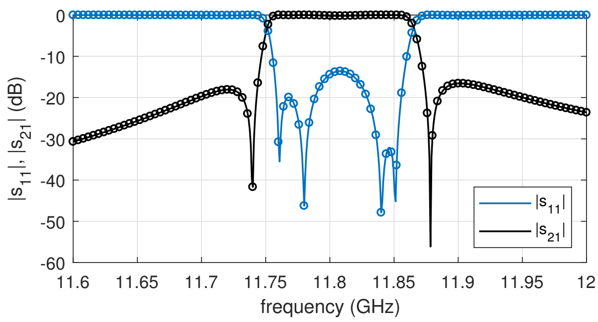


Fig. 5. Scattering parameters of dual-mode waveguide filter – local reduction without basis compression (solid line) and with compression (circles).

including global approach (RGM-MOR) for wideband fast-frequency sweep and local (macromodeling) for a parametric ROMs constructed with several concatenated bases. Such aspects as memory usage and time of analysis involving basis compression have been discussed. The presented results shows that basis compression allows for reducing the memory and time consumption without compromising the accuracy of the ROM.

Acknowledgment

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References

[1] R. D. Slone, J.-F. Lee, and J.-F. Lee, "Automating multipoint Galerkin AWE for a FEM fast frequency sweep," *IEEE Trans. Magn.*, vol. 38, no. 2, pp. 637–640, 2002.

[2] G. Fotyga, M. Czarniewska, A. Lamecki, and M. Mrozowski, "Reliable Greedy Multipoint Model-Order Reduction Techniques for Finite-Element Analysis," *IEEE Antennas and Wireless Propagation Letters*, vol. 17, no. 5, pp. 821–824, 2018.

[3] T. Narayanan and M. Swaminathan, "Preconditioned second-order multipoint passive model reduction for electromagnetic simulations," *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 11, pp. 2856–2866, 2010.

[4] V. de la Rubia, "Reliable reduced-order model for fast frequency sweep in microwave circuits," *Electromagnetics*, vol. 34, no. 3-4, pp. 161–170, 2014.

[5] R. Dyczij-Edlinger and O. Farle, "Finite element analysis of linear boundary value problems with geometrical parameters," *COMPEL-Int. J. Comput. Math. Elect. Electron. Eng.*, vol. 28, no. 4, pp. 779–794, 2009.

[6] M. Czarniewska, G. Fotyga, A. Lamecki, and M. Mrozowski, "Parametrized local reduced-order models with compressed projection basis for fast parameter-dependent finite-element analysis," *IEEE Trans. Microw. Theory Techn.*, vol. 66, no. 8, pp. 3656–3667, Aug. 2018.

[7] G. Fotyga, M. Rewienski, and M. Mrozowski, "Wideband macromodels in finite element method," *IEEE Microw. Compon. Lett.*, vol. 25, no. 12, pp. 766–768, Dec. 2015.

[8] V. de la Rubia, U. Razafison, and Y. Maday, "Reliable fast frequency sweep for microwave devices via the reduced-basis method," *IEEE Trans. Microw. Theory Techn.*, vol. 57, no. 12, pp. 2923–2937, Dec. 2009.

[9] M. Rewieński, A. Lamecki, and M. Mrozowski, "Greedy multipoint model-order reduction technique for fast computation of scattering parameters of electromagnetic systems," *IEEE Trans. Microw. Theory Techn.*, vol. 64, no. 6, pp. 1681–1693, June 2016.

[10] A. Chatterjee, "An introduction to the proper orthogonal decomposition," *Current science*, vol. 78, no. 7, pp. 808–817, Apr. 2000.

[11] L. N. Trefethen and D. Bau III, *Numerical linear algebra*. Siam, 1997, vol. 50.

[12] A. Lamecki, L. Balewski, and M. Mrozowski, "An efficient framework for fast computer aided design of microwave circuits based on the higher-order 3D finite-element method," *Radioengineering*, vol. 23, no. 4, pp. 970–978, 2014.

[13] K. Y. You, A.-A. Nadera, J. C. Chong, K. Y. Lee, E. M. Cheng, and Y. S. Lee, "Analytical modeling of conventional and miniaturization three-section branch-line couplers," *Journal of Electrical Engineering & Technology*, vol. 13, no. 2, pp. 858–867, 2018.