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Use of a Least Squares with Conditional Equations Method in Positioning a Tramway Track in the Gdansk Agglomeration

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ABSTRACT: Satellite measurement techniques have been used for many years in different types of human activity, including work related to staking out and making use of rail infrastructure. First and foremost, satellite techniques are applied to determine the tramway track course and to analyse the changes of its position during its operation.
This paper proposes using the least squares with conditional equations method, known in geodesy (LSce). When applied, this method will allow for improvement of the final determination accuracy. This paper presents a simplified solution to the LSce alignment problem. The simplification involves replacement of the parameter binding equations with equivalent observational equations with properly selected weights. The results obtained with such a solution were demonstrated with a randomly selected section of a tramway track in Gdańsk.
The article presents the theoretical foundations of the test method, the experiment organisation and the results obtained with MathCad Prime 3.0 software. It also presents the outcome of a study associated with the execution of the project No POIR.04.01.01-00-0017/17 entitled "Developing an innovative method of precision determination of a rail vehicle trajectory" executed by a consortium of the Gdańsk University of Technology and Gdynia Maritime University.

1 INTRODUCTION

The satellite system era started in the mid-1980s, when the American NAVSTAR GPS system became fully operative. Currently, there are 11 satellite systems in different parts of the world, with more under construction, which have had an increasing impact on the global economy [Czaplewski 2015]. Satellite systems have become a widely recognised and used tool in many types of navigation. Maritime navigation mainly employs systems for determination of vessel coordinates, but also for ensuring sailing safety at every stage of a sea journey [Urbanski et al. 2008]. As in sailing, the main job that satellite systems do in air navigation is to position an aircraft, especially during its take-off and landing [Bialy et al. 2011]. Satellite systems are used in land navigation with popular SatNav receivers, which are so popular that it is not often realised that navigation signals come from satellite systems. Rail vehicles are another area in which land navigation is used. Here, satellite systems are used in stock-taking, diagnostics and design work in railways. The greatest advantage of mobile satellite measurements is their ability to perform measurements in a unified and coherent system of spatial coordinates [Specht, Koc 2016; Specht et al. 2019]. Satellite systems are also used in other areas of research and practical applications, e.g. in geodesy. Apart from conventional methods of determining coordinates for a point on the globe, they are also used - combined with mathematical alignment methods – to increase the accuracy of final determinations, e.g. [Bakula, Kazmierczak 2017, Czaplewski, Waz, 2017, Swierczynski, Czaplewski 2015].

For several years, co-authors of this paper have been presenting opportunities of using satellite techniques in measurements of railway infrastructure [Koc 2012, Specht et al. 2011, Specht et al. 2014]. Research in this area is also conducted in other parts of the world, e.g. [Gikas et al. 2008, Chen et al. 2015, 2018]. The European Commission has financed several projects aimed at applying satellite solutions for positioning multiple units and the results were published, among others, in [Filip et al., 2001, Urech et al., 2002, Mertens P., Franckart J.-P., 2003]. There is also the Positive Train Control (PTC) system in the USA, which has been used on most American railways since 2015. It employs the NAVSTAR GPS system in both its basic form and in the differential version [Betts et al. 2014]. Currently, the Automated Train Management System (ATMS) is under construction in Australia; starting in 2020, it will enable train positioning with high reliability and accuracy [ACIL 2013]. However, each solution based only on satellite systems faces the problem of positioning accuracy because of GNSS system accessibility, disrupted not only for natural reasons [Czaplewski 2018]. Therefore, the authors attempted to adapt methods of result alignment, which are well-known in geodesy. This paper proposes the use of the least squares with conditional equations method to increase the accuracy of mobile measurements, which can be supported by additional precision positioning of GNSS receiver antennas on measurement platforms. The proposed adaptation of the alignment method was verified using a measurement campaign carried out in Gdansk in autumn 2018.

2 LEAST SQUARES WITH CONDITIONAL EQUATIONS METHOD WITH A SIMPLIFIED SOLUTION

There are alignment problems in geodetic measurements in which the parameters of an observational equations system must meet extra conditions (Wisniewski 1985, 2013, 2016). These conditions in a geodetic network can apply to coordinates of some points being part of the given network. This is a situation dealt with in the task of monitoring the position of a tramway track. It is possible when the distance between measurement points (antennas of GNSS receivers) are measured with accuracy sufficient to treat the quantities as relatively error-free (compared to other observations in the network). Coordinates of the antennas on a measurement platform, contained in vector $\mathbf{X}=[X_1,\ldots,X_r]^T$, must be a solution of the following system of conditional equations

$$\left.\begin{array}{l}\Psi_1(X_1,X_2,\ldots,X_r)=0\\ \Psi_2(X_1,X_2,\ldots,X_r)=0\\ \vdots\\ \Psi_w(X_1,X_2,\ldots,X_r)=0\end{array}\right\}\Leftrightarrow \Psi(\mathbf{X})=\mathbf{0} \tag{1}$$

On the other hand, according to the general principles of determination of geodetic network coordinates, vector $\mathbf{X}$ determination is based on satellite observations which enable formulating the following system of observational equations

$$\left.\begin{array}{l}y_1+v_1=F_1(X_1,X_2,\ldots,X_r)\\ y_2+v_2=F_2(X_1,X_2,\ldots,X_r)\\ \vdots\\ y_n+v_n=F_n(X_1,X_2,\ldots,X_r)\end{array}\right\}\Leftrightarrow \mathbf{y}+\mathbf{v}=\mathbf{F}(\mathbf{X}) \tag{2}$$

where $\mathbf{y}=[y_1,\cdots,y_n]^T$ is a vector of observation. $\mathbf{v}=[v_1,\cdots,v_n]^T$ denotes a vector of random observation errors with the covariance matrix $\mathbf{C_v}=\sigma_0^2\mathbf{P}^{-1}$ ($\mathbf{P}$ – matrix of weights, σ_0^2 – coefficient of variance). With known approximate coordinates $\mathbf{X}^0=[X_1^0,\ldots,X_r^0]^T$ of receiver antennas, the equation $\mathbf{y}+\mathbf{v}=\mathbf{F}(\mathbf{X})$ can then be replaced with a linear observational equation of the following form:

$$\begin{aligned}\mathbf{v}&=\mathbf{F}(\mathbf{X})-\mathbf{y}=\mathbf{F}(\mathbf{X}^0+d\mathbf{X})-\mathbf{y}=\\&=\left.\frac{\partial\mathbf{F}(\mathbf{X})}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}^0}d\mathbf{X}+\mathbf{F}(\mathbf{X}^0)-\mathbf{y}=\mathbf{A}d\mathbf{X}+\mathbf{l}\end{aligned} \tag{3}$$

The quantity $d\mathbf{X}$ is a vector of unknown increments such that $\mathbf{X}=\mathbf{X}^0+d\mathbf{X}$. The matrix

$$\mathbf{A}=\left.\frac{\partial\mathbf{F}(\mathbf{X})}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}^0}=\begin{bmatrix}\frac{\partial F_1(\mathbf{X})}{\partial X_1}&\frac{\partial F_1(\mathbf{X})}{\partial X_2}&\cdots&\frac{\partial F_1(\mathbf{X})}{\partial X_r}\\ \frac{\partial F_2(\mathbf{X})}{\partial X_1}&\frac{\partial F_2(\mathbf{X})}{\partial X_2}&\cdots&\frac{\partial F_2(\mathbf{X})}{\partial X_r}\\ \cdots&\cdots&\cdots&\\ \frac{\partial F_n(\mathbf{X})}{\partial X_1}&\frac{\partial F_n(\mathbf{X})}{\partial X_2}&\cdots&\frac{\partial F_n(\mathbf{X})}{\partial X_r}\end{bmatrix}_{\mathbf{X}=\mathbf{X}^0}$$

is a known matrix of coefficients ($rank(\mathbf{A})=r$), whereas

$$\begin{aligned}\mathbf{l}&=\mathbf{F}(\mathbf{X}^0)-\mathbf{y}=\\&=\begin{bmatrix}F_1(\mathbf{X}^0)-y_1 & F_2(\mathbf{X}^0)-y_2 & \cdots & F_n(\mathbf{X}^0)-y_n\end{bmatrix}^T\end{aligned}$$

is a vector of absolute terms. Considering that $\mathbf{X}=\mathbf{X}^0+d\mathbf{X}$, a system of observational equations can likewise be reduced to a linear form (1). Developing function $\Psi(\mathbf{X})$ to the following form

$$\Psi(\mathbf{X}) = \Psi(\mathbf{X}^0 + d\mathbf{X}) =$$
$$= \Psi(\mathbf{X}^0) + \left.\frac{\partial\Psi(\mathbf{X})}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}^0} d\mathbf{X} = \Psi(\mathbf{X}^0) + \mathbf{B}d\mathbf{X} \tag{4}$$

gives the following linear conditional equation

$$\mathbf{B}d\mathbf{X} + \Delta = \mathbf{0} \tag{5}$$

where:

$$\mathbf{B} = \left.\frac{\partial\Psi(\mathbf{X})}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}^0} = \begin{bmatrix} \frac{\partial\Psi_1(\mathbf{X})}{\partial X_1} & \frac{\partial\Psi_1(\mathbf{X})}{\partial X_2} & \cdots & \frac{\partial\Psi_1(\mathbf{X})}{\partial X_r} \\ \frac{\partial\Psi_2(\mathbf{X})}{\partial X_1} & \frac{\partial\Psi_2(\mathbf{X})}{\partial X_2} & \cdots & \frac{\partial\Psi_2(\mathbf{X})}{\partial X_r} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial\Psi_w(\mathbf{X})}{\partial X_1} & \frac{\partial\Psi_w(\mathbf{X})}{\partial X_2} & \cdots & \frac{\partial\Psi_w(\mathbf{X})}{\partial X_r} \end{bmatrix}_{\mathbf{X}=\mathbf{X}^0}$$

$$\Delta = \Psi(\mathbf{X}^0) = \begin{bmatrix} \Psi_1(\mathbf{X}^0) & \Psi_2(\mathbf{X}^0) & \cdots & \Psi_w(\mathbf{X}^0) \end{bmatrix}^T$$

Taking into account the conditions binding the parameters being determined and application of the goal function of the least square method leads to an alignment problem of the following form (Wiśniewski 2016):

$$\left.\begin{aligned} &\mathbf{A}d\mathbf{X} + \mathbf{l} = \mathbf{v} \\ &\mathbf{B}d\mathbf{X} + \Delta = \mathbf{0} \\ &\mathbf{C}_{\mathbf{v}} = \sigma_0^2\mathbf{P}^{-1} \\ &\varphi(d\mathbf{X}) = \mathbf{v}^T\mathbf{P}\mathbf{v} = \min \end{aligned}\right\} \tag{6}$$

To obtain a strict solution of the problem, it is necessary to replace the primary goal function $\varphi(d\mathbf{X}) = \mathbf{v}^T\mathbf{P}\mathbf{v}$ with the following Lagrange function

$$\varphi(d\mathbf{X}) = \mathbf{v}^T\mathbf{P}\mathbf{v} - 2\mathbf{k}^T(\mathbf{B}d\mathbf{X} + \Delta) \tag{7}$$

where $\mathbf{k}$ is a vector of unknown Lagrange multipliers.

The problem (6) can be solved practically in an different, numerically simpler way. To this end, the conditional equation $\mathbf{B}d\mathbf{X} + \Delta = \mathbf{0}$ must be replaced with an equivalent observational equation

$$\mathbf{B}d\mathbf{X} + \Delta = \mathbf{v}_* \tag{8}$$

Since it is required that $\mathbf{v}_* = \mathbf{0}$, then the vector of fictitious observational errors $\mathbf{v}_*$ must be assigned with such a covariance matrix $\mathbf{C}_{\mathbf{v}_*} = \sigma_0^2\mathbf{P}_*^{-1}$, that fictitious residuals $\hat{\mathbf{v}}_*$ should meet the condition $\hat{\mathbf{v}}_* = \mathbf{0}$ (within the calculation precision limits). For mutually independent observations, this results in the adoption of sufficiently large, diagonal elements of the matrix of weights $\mathbf{P}_*$ (theoretically, these should be infinitely large quantities). In this manner, the problem (6) is replaced with a conventional alignment problem with the following form

$$\left.\begin{aligned} &\mathbf{A}d\mathbf{X} + \mathbf{l} = \mathbf{v} \\ &\mathbf{B}d\mathbf{X} + \Delta = \mathbf{v}_* \\ &\mathbf{C}_{\mathbf{v}} = \sigma_0^2\mathbf{P}^{-1} \\ &\mathbf{C}_{\mathbf{v}_*} = \sigma_0^2\mathbf{P}_*^{-1} \\ &\varphi(d\mathbf{X}) = \mathbf{v}^T\mathbf{P}\mathbf{v} + \mathbf{v}_*^T\mathbf{P}_*\mathbf{v}_* = \min \end{aligned}\right\} \Leftrightarrow \begin{cases} \widehat{\mathbf{A}}d\mathbf{X} + \widehat{\mathbf{l}} = \widehat{\mathbf{v}} \\ \widehat{\mathbf{C}}_{\mathbf{v}} = \sigma_0^2\widehat{\mathbf{P}}^{-1} \\ \varphi(d\mathbf{X}) = \widehat{\mathbf{v}}^T\widehat{\mathbf{P}}\widehat{\mathbf{v}} = \min \end{cases} \tag{9}$$

where:

$$\widehat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix}, \widehat{\mathbf{l}} = \begin{bmatrix} \mathbf{l} \\ \Delta \end{bmatrix}, \widehat{\mathbf{v}} = \begin{bmatrix} \mathbf{v} \\ \mathbf{v}_* \end{bmatrix}$$

Vector $\widehat{\mathbf{v}}$ is a combined vector of observation errors with the covariance matrix

$$\widehat{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathbf{v}} = \sigma_0^2\mathbf{P}^{-1} & 0 \\ 0 & \mathbf{C}_{\mathbf{v}_*} = \sigma_0^2\mathbf{P}_*^{-1} \end{bmatrix} = \sigma_0^2\begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{P}_*^{-1} \end{bmatrix} = \sigma_0^2\widehat{\mathbf{P}}^{-1} \tag{10}$$

where

$$\widehat{\mathbf{P}} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_* \end{bmatrix}$$

is a combined matrix of weights (σ_0^2 – the coefficient of variance common to both elements of model (10)). Problem (9) has a solution as an estimator of increments $d\mathbf{X}$ with the following form

$$d\hat{\mathbf{X}} = -(\widehat{\mathbf{A}}^T\widehat{\mathbf{P}}\widehat{\mathbf{A}})^{-1}\widehat{\mathbf{A}}^T\widehat{\mathbf{P}}\widehat{\mathbf{l}} \tag{11}$$

where:

$$\widehat{\mathbf{A}}^T\widehat{\mathbf{P}}\widehat{\mathbf{A}} = \mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{B}^T\mathbf{P}_*\mathbf{B}$$

$$\widehat{\mathbf{A}}^T\widehat{\mathbf{P}}\widehat{\mathbf{l}} = \mathbf{A}^T\mathbf{P}\mathbf{l} + \mathbf{B}^T\mathbf{P}_*\Delta$$

Determination of the covariance matrix for the estimator gives

$$\mathbf{C}_{d\hat{\mathbf{X}}} = \hat{\sigma}_0^2(\widehat{\mathbf{A}}^T\widehat{\mathbf{P}}\widehat{\mathbf{A}})^{-1} \tag{12}$$

where:

$$\hat{\sigma}_0^2 = \frac{\hat{\widehat{\mathbf{v}}}^T\widehat{\mathbf{P}}\hat{\widehat{\mathbf{v}}}}{n + w - r} \tag{13}$$

Vector $\hat{\widehat{\mathbf{v}}} = [\hat{\mathbf{v}}^T \quad \hat{\mathbf{v}}_*^T]^T$ is a combined vector of residuals determined from the function

$$\hat{\widehat{\mathbf{v}}} = \widehat{\mathbf{A}}d\hat{\mathbf{X}} + \widehat{\mathbf{l}} \tag{14}$$

The following vector is a coordinate estimator for GNSS receiver antennas

$$\hat{\mathbf{X}} = \mathbf{X}^0 + d\hat{\mathbf{X}} \tag{15}$$

with the covariance matrix $\mathbf{C}_{\hat{\mathbf{X}}} = \mathbf{C}_{d\hat{\mathbf{X}}}$. Let us note that squares of mean errors of determined estimators are the diagonal elements of the matrix, i.e. $m^2_{\hat{X}_i} = [\mathbf{C}_{\hat{\mathbf{X}}}]_{ii}$

3 ORGANISATION OF THE EXPERIMENT

The measurement campaign was conducted during the night of 28/29 November 2018 between 11 p.m. and 4.00 a.m. in Gdansk with a Bombardier NGT6 tramway (Fig. 1).

Figure 1. Bombardier NGT6 tramway - source: www.gait.pl

Two mobile measurement platforms with the measuring instruments - GNSS receivers – were pulled by the railway vehicle (Fig. 2) The instruments were supplied by two leading producers of geodetic instruments.

a/

b/

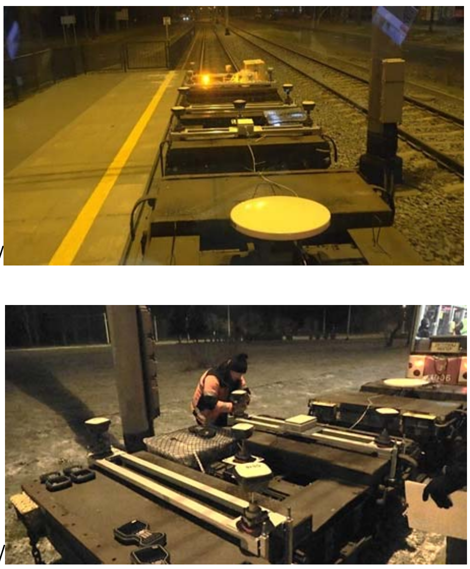

Figure 2 a - Mobile measurement platform, b - GNSS receiver tracks

Moreover, an inclinometer, an accelerator and a compass were installed to conduct other measurements not described in this paper. The measurements were performed on a 3-kilometre route in Gdansk (Fig. 3). They involved repeated passage of the measurement unit along a section with various numbers of buildings along it, which affected the accessibility of the satellite systems (Fig. 3). The mean speed of the unit at which the measurements were performed was 10 km/h.

Figure 3. Plan of the measurement unit passage - source: maps.google.pl

The diagram of the measurement platforms constructed for the tests is shown in Fig. 4. The receivers were deployed in such a way that four were situated at the vertices of a square, and the fifth was situated at the intersection of its diagonals. The construction of the mobile measurement platform enabled designing a geometric measurement structure in the shape of a square with the sides of 155 cm to 170 cm. The precision placement of the GNSS receivers in the track axis and above the rail was performed in the local system with the use of an electronic total station and a prismatic mirror placed on a dedicated tripod with an accuracy of ca. 1 mm (rms).

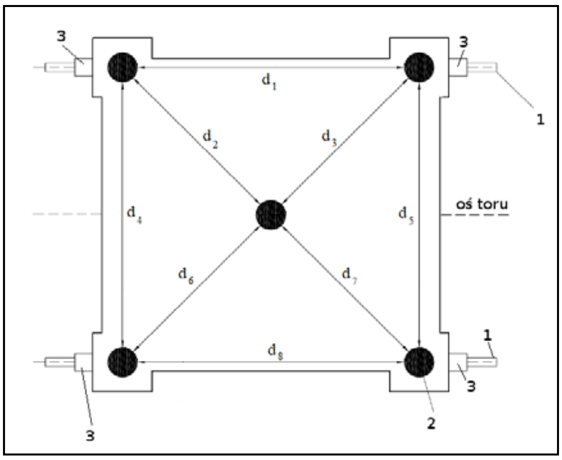

Figure 4 A diagram of the mobile measurement platform (*1* – rails, 2 – points of forced centring of a GNSS receiver, *3* – platform wheels, d_i – distances between the GNSS centring points) – source: prepared by the authors

The position data were recorded in real time with the 1 Hz frequency during the measurement campaign for different GNSS receiver configurations:

– positioning with the use of GPS correction data,
– positioning with the use of GPS+GLONASS correction data,
– positioning with the use of GPS+GALILEO correction data.

4 THE METHOD APPLICATION FOR ANTENNA POSITION ALIGNMENT ON A MEASUREMENT PLATFORM.

The method described in section 2 was applied to align the satellite measurement results, also taking into account previous tachymetric measurements. The tachymetric measurements allowed for precision positioning of the GNSS receiver antennas on the measurement platforms relative to one another and dimensioning the whole measurement system on the measurement platform. The observation results are shown in Fig. 5.

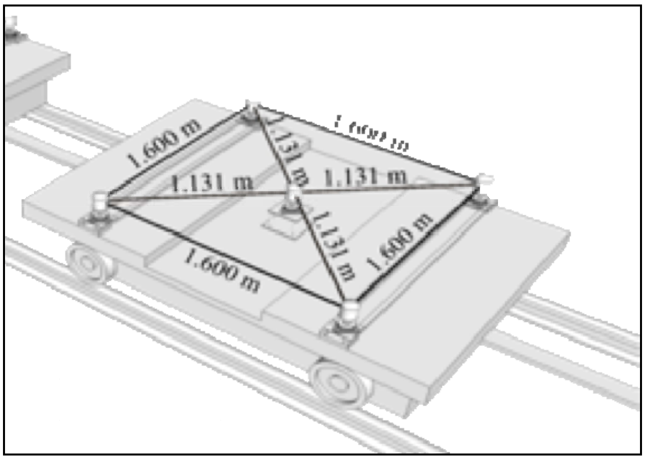

Figure 5. Distances between antennas on the measurement platform - source: prepared by the authors

The alignment process was performed for all kinds of observations. However, because of the large amount of the study material, only its random part is presented in this paper (Fig.3). 60 seconds of recording with one type of receivers using adjustment from GPS + GALILEO + GLONASS were used for the alignment. The least squares with conditional equations method was applied with the use of MathCad Prime 3.0 software, enabling calculations and export of the alignment results to a file in a format compatible with ArcGIS embedded in the GIS laboratory at the Department of Geodesy and Oceanography of the Maritime University in Gdynia.

No partial results are presented because of the large size of the measurement sample. However, the final results are shown in Fig. 6. The graphic presentation of the results was prepared in the ArcGIS program.

Figure 6. Aligned recording results for a selected section of the tramway track loop in Gdansk – source: prepared by the authors

The GNSS antenna position, aligned by the least squares with conditional equations method, is marked in yellow. The red colour denotes the antenna position determined by satellite techniques for one recording moment.

Figure 7 shows mean errors of position coordinate estimators obtained from the covariance matrix (12).

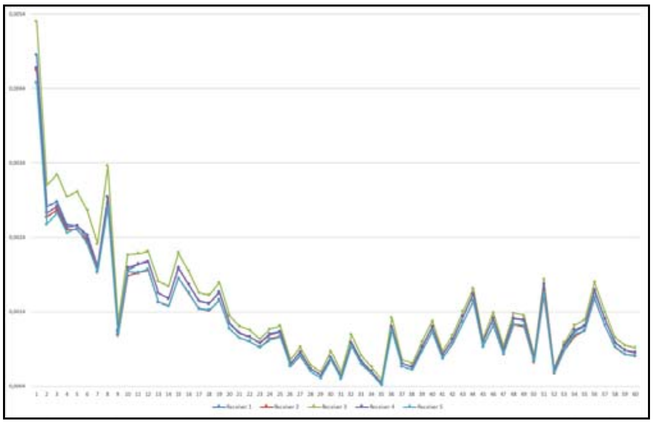

Figure 7. Errors of mean antenna position coordinate estimators – source: prepared by the authors

The errors lie within the interval $m_{\hat{x}} \in \langle 0.0004, 0.0053 \rangle (\mathrm{m})$.

5 CONCLUSIONS

This paper proposes the non-standard use of a method of aligning observation results which is well-known in conventional geodesy to calculate the results of satellite measurements in mobile measurement campaigns. The constant development of satellite techniques, aided by the data processing methods in geodesy, can bring improved quality to mobile measurements which require precision determination of point coordinates in various studies and practical implementations of technical assignments.

The justifiability of the theoretical assumptions was confirmed by recording data in the measurement campaign described in the paper. The use of the proposed alignment method should be confirmed with data from other measurement campaigns, which will be the object of studies in the next stages of the project execution. Moreover, the team will adapt the solutions derived from the solutions applied in this method and adapt other popular methods of aligning observation results. The work outcome will be presented in future publications showing the results of studies obtained as part of the project No POIR.04.01.01-00-0017/17.

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