PRACE NAUKOWE UNIWERSYTETU EKONOMICZNEGO WE WROCŁAWIU
RESEARCH PAPERS OF WROCŁAW UNIVERSITY OF ECONOMICS 2019, vol. 63, nr 6

ISSN 1899-3192
e-ISSN 2392-0041

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MARKOWITZ'S PORTFOLIO THEORY – OPTIMAL LENGTH OF ESTIMATION WINDOW FOR GOLD AND THE BIGGEST COMPANIES ON THE WARSAW STOCK EXCHANGE

TEORIA PORTFELOWA MARKOWITZA – OPTYMALNA DŁUGOŚĆ OKNA ESTYMACJI DLA INWESTYCJI W ZŁOTO ORAZ NAJWIĘKSZYCH SPÓŁEK NOTOWANYCH NA GIEŁDZIE PAPIERÓW WARTOŚCIOWYCH W WARSZAWIE

DOI: 10.15611/pn.2019.6.07
JEL Classification: G11, G17

Summary: The following article is dedicated to the construction of an investment portfolio consisting of 3 investments from the Polish capital market found in the WIG20 index and from investment in gold. The purpose of the study was to determine the optimal length of the estimation window for building a portfolio with minimal risk and maximum efficiency. The length of the estimation window was also assessed in terms of the rate of return and the maximum cumulative loss. Data from 2017 was used to build the portfolio, and the weightings determined for the portfolio based on these data were evaluated using data from 2018 (from January to October). Based on the research, it was found that the optimal length of the estimation window ranges from 144 to 160 daily observations from the past. However, depending on the investment objective (risk minimization or maximization of efficiency) and the characteristics describing the portfolio, other lengths of the estimation window may also be appropriate.

Keywords: portfolio theory, Polish capital market, estimation window, minimal risk, maximum efficiency.

Streszczenie: Artykuł poświęcono konstrukcji portfela inwestycyjnego składającego się z trzech inwestycji z polskiego rynku kapitałowego (inwestycji wchodzących w skład indeksu WIG20) oraz z inwestycji w złoto. Celem poniższego opracowania jest wyznaczenie optymalnej długości okna estymacji dla budowy portfela o minimalnym ryzyku oraz maksymalnej efektywności. Długość okna estymacji została także oceniona pod względem stopy zwrotu oraz maksymalnej skumulowanej straty. Do budowy portfela wykorzystano dane z roku 2017,

a wyznaczone w oparciu o te dane wagi dla portfela poddano ocenie dla danych z roku 2018 (od stycznia do października). Na podstawie badań stwierdzono, że długość optymalnego okna estymacji zawiera się w przedziale od 144 do 160 dziennych obserwacji z przeszłości. W zależności od przyjętego celu inwestycyjnego (minimalizacja ryzyka lub maksymalizacja efektywności) oraz charakterystyki opisującej portfel można mówić o szerszych wartościach powyższego przedziału okna estymacji.

Słowa kluczowe: teoria portfelowa, polski rynek kapitałowy, okno estymacji, minimalne ryzyko, maksymalna efektywność.

1. Introduction

Markowitz's portfolio theory was published in 1952 (Markowitz, 1952). Since then, the issue of portfolio optimization has received a lot of attention and been the subject of numerous scientific studies. The phrase "modern portfolio theory" returns over 105,000 scientific papers in the EBSCO multi-search. The following article is devoted to the construction of an investment portfolio consisting of three investments from the Polish capital market in the WIG20 index and from investment in gold. The purpose of the study is to determine the optimal length of the estimation window for building a portfolio consist of blue-chip stocks and gold with minimal risk and maximum efficiency. The author of this paper formulates the following hypothesis: there exists one common range for an estimation window for a portfolio with minimal risk and a portfolio with maximum efficiency which is optimal for investors and using it gives better results than a naive strategy. The length of estimation window is important because it shows that if its value belongs to a wider range, then it can be said that the capital market is developed and rather less risky. If the range for the length of the estimation window is narrow or does not exist, then it can be assumed that the capital market is developing and is also more risky. For companies called blue-chip (which are analysed in this paper) which are rather less risky and have a stable rate of return, finding the range of the optimal length of estimation window should not be very difficult. However, when so distant investments' purposes as minimal risk and maximum efficiency have to be connected, then that task can be difficult. It can be also said that when an investor constructs his/her portfolio using only a few observations from the past (a short length of estimation window) then his/her investment vehicle should be more risky than for an investor who uses a greater number of observations (a wider length of estimation window) from the past to calculate portfolio weights. These are the main factors why the length of the estimation window is so important to investors. Data from 2017 was used to build the portfolio, and the weightings determined for the portfolio based on these data were evaluated using data from 2018 (from January to October).

The investments included in the WIG20 index were chosen because this study was conducted from the perspective of a Polish investor, and only the biggest companies

were analysed. The choice of gold as an element of the portfolio was due to the perception of this investment as a safe haven. Gold as an investment is what is known as an alternative investment as it allows the effective diversification of the investment portfolio. Some state that all investments in raw materials, including gold, are a valuable source of diversification of the investment portfolio (Belousova and Dorfleitner, 2012).

The main contribution of this paper is to examine if two distant investment purposes can be realized based on the same length of estimation window. Four measures are tested. First, the standard deviation of the rate of return was applied to the risk assessment. Second, the Sharpe ratio was used to measure the risk. Finally, the length of the estimation window was assessed in terms of the obtained rate of return and the maximum possible cumulative loss. The main findings can be applied to Polish blue-chip companies in the adopted research period. The results of the constructed portfolio are compared to a naive strategy.

The study is laid out as follows. Section 2 contains the literature review. The third section presents the data used for the analysis and the characteristics of the research method used. The fourth section presents the most important conclusions from empirical research, and the summary is included in the final, fifth section.

2. Literature review

It has been over 75 years since Harry Markowitz's ground-breaking article was published. Since then, research on portfolio theory has involved many aspects. Markowitz's portfolio theory is applicable in many areas related to not only capital investments, but also various goods and resources (Alvarez, Larkin, and Ropicki, 2017; Garcia, González, Contreras, and Custodio, 2017; Hua, Liang, Zeng, Xu, and Zhang, 2015). The application of this theory can be noted in seemingly very remote areas, such as the energy sector (Favre-Perrod, Kienzle, and Andersson, 2010) and environmental protection (Runting, Beyer, Dujardin, Lovelock, Bryan, and Rhodes, 2018). Despite the literature research, the author could find only one work directly devoted to the issue of the optimal length of the estimation window for portfolio theory. One similar work, alas not connected with stock of big companies and gold but dedicated to automated trading strategies (Raudys, Raudys, and Pabarskaite, 2018), finds that the estimation window length should vary from two to 24 months. The authors also emphasized that the optimal length of the estimation window depends on the investment task and that to the best of their knowledge, this problem has not been considered in the literature. On the other hand, there exist many papers dedicated to the construction of an optimal portfolio in general, and the relevant conclusions from such research are presented below.

The basic advantages of using a portfolio theory are presented in a study on six companies listed on the Indian stock market (Biswas, 2015). The study looked at the period from 1st January 2009 to 30th June 2014 and used daily simple rates of return calculations in order to analyse diversified portfolios against results that would have

been obtained if all the funds had been invested in only one type of asset. It found that diversification positively affects both the risk of the investment portfolio (which reduced) and the rate of return (which increased).

Further advantages of using Markowitz's theory are presented in a study analysing twelve different investment portfolio management strategies, of which ten were based on portfolios with minimal risk, and comparing them against a naive strategy that was determined in two ways (Behr, Guettler, and Miebs, 2013). The study was conducted for six different databases. The authors observed that their portfolio strategy achieved higher Sharpe ratios than the 1/N strategy, amounting to an average Sharpe ratio increase of 32.5% across their six empirical datasets. However, statistically significant results related to the higher Sharpe value compared to the naive strategies were observed only for one of the ten analysed strategies with minimal risk.

A study focusing on determining the minimum risk portfolio from two different ways of constructing the investment portfolio found that investment portfolios reviewed every three days are characterized by low risk (Chu-Xin, Wan-Yi, and Shu-Jing, 2018). The above conclusion was derived on the basis of research on the Shenzhen stock market. In another article (Geambasu, Sova, Jianu, and Geambasu, 2016), it was proved that the classical approach to risk measurement gives worse results than the use of risk measures within post-modern portfolio theory (PMPT). The authors state explicitly that PMPT offers a better measure of risk and is more flexible and adapted to the investment process reality.

On the other hand, research on the twelve major sectors of ETF's operating in the United States confirmed that the use of a diversity booster (DB) allows for better results than using classic portfolio theory or a naive strategy (Schmidt, 2018). The research was conducted for the period 2009-2015, for a three-year estimation window with monthly rebalancing. In addition, the author stated that the use of DB allows sudden changes in the values of investment portfolio weights to be avoided.

The classic theory of Markowitz and the behavioural version of the construction of investment portfolios were compared in another study (Pfiffelmann, Roger, and Bourachnikova, 2016). The authors concluded that these methods cannot be used interchangeably. In almost 70% of cases, the classic Markowitz theory is, in fact, less effective than the other method. Yet the study also found that behavioural portfolio theory is characterized by a higher risk than classical theory.

The diversification of the investment portfolio is not only related to the stock market. Another study (Grujić, 2016) presents the advantages of using the Markowitz theory for the domestic bond market. The research was conducted for selected bonds listed on the Banja Luka Stock Exchange. The criterion for selecting specific bonds was related to their liquidity, which was understood as the existence of a regular secondary market. Based on the conducted research, it was found that the use of Markowitz's theory is possible and beneficial for the bond market. It was emphasized, however, that the disadvantage of this approach is the relatively high exposure to non-diversified risk related to the insolvency of a single issuer.

3. Data and research methodology

The data used for the analysis was obtained from two websites. The first one (http://infostrefa.com/infostrefa/pl/index) gives the prices of listed companies from the main Warsaw Stock Exchange index (WIG20), while the second site (http://www.lbma.org.uk/) shows gold prices. The data were obtained for the period from 30/12/2016 to 29/10/2018. For each day the logarithmic rates of return were calculated (Dębski, 2007):

$$i = lnP_t - lnP_{t-1},$$

where: i – the daily logarithmic rate of return,
P_t – the investment price on day "t",
P_{t-1} – the investment price on day "$t - 1$".

On this basis a total of 459 rates of return were obtained for each of the 20 companies included in the index and for investment in gold. The rates of return were set for the research period from 02/01/2017 to 29/10/2018. Next the research period was divided into an estimation window (including data for 2017) and an evaluation window (data for 2018).

In the next step, based on all data from 2017 (250 observations), the structure of the optimal investment portfolio with minimal risk (measured by standard deviation of the rate of return) and maximum efficiency (understood as the ratio of the rate of return on the portfolio, minus the risk-free rate of return divided by standard deviation of the rate of return), were determined. The weights determined on the basis of data from 2017 were used to build the portfolio for 2018. On this basis the rate of return, risk, effectiveness, and the maximum loss that was achieved for the investment portfolio in 2018 were determined. In the next step the oldest observation

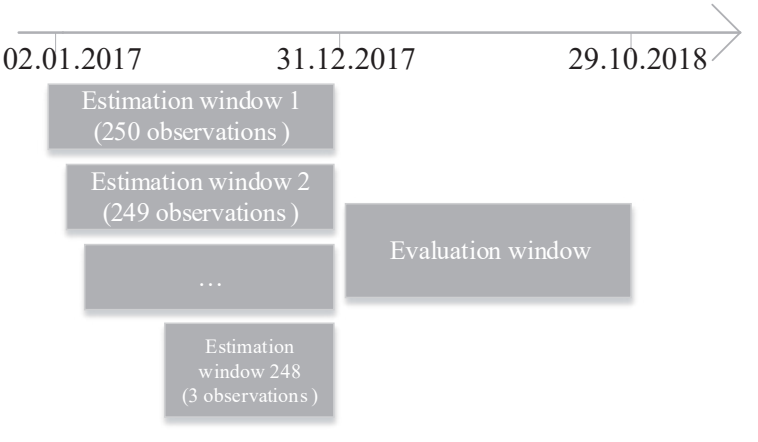

Fig. 1. Estimation and evaluation window

Source: own calculations.

was removed from the estimation window and the entire optimization and evaluation procedure was repeated assuming that the optimization was performed on the basis of 249 observations from 2017. The estimation window was shortened until the structure of the optimal portfolio was determined on the basis of the last three quotation dates of the year 2017 as shown in the figure below.

Three of the 20 investments in shares of Polish capital companies included in the WIG20 index and investment in gold were used to build investment portfolios. For each estimation window 1140 four-element investment portfolios were built, both with minimal risk and maximum efficiency. Taking into account that the evaluations were made for 248 different cases of total window length, a total of 282 720 portfolios were determined with a minimum risk and the same number for maximum efficiency portfolios, which gave in total more than half a million optimizations.

The rate of return, the risk of the portfolio, and efficiency were calculated from the following formulas (Jajuga and Jajuga, 2006):

$$i_p = \sum_{k=1}^{4} i_k w_k,$$

$$\sigma_p = \sqrt{\sum_{k=1}^{4} w_k^2 \sigma_k^2 + 2 \sum_{k=1}^{3} \sum_{m=k+1}^{4} w_k w_m \, \sigma_k \sigma_m p_{km}},$$

$$e_p = \frac{i_p - i_f}{\sigma_p}.$$

The calculations were made assuming that

$$\sum_{k=1}^{4} w_k = 1 \wedge \; w_k \geq 0,$$

where: i_p – portfolio rate of return,
i_k – rate of return from investment "k",
w_k – wage of investment "k" in the portfolio,
σ_p – portfolio risk (standard deviation of the rate of return),
σ_k – risk of investment "k" (standard deviation of the rate of return),
p_{km} – coefficient of correlation of return rates on investment "k" and investment "m",
e_p – effectiveness measure (Sharpe ratio),
i_f – free-risk rate of return.

In turn, the maximum cumulative loss (MDD) was defined as the maximum loss an investor can suffer in the investment by buying at the highest point and selling at the lowest point (Bacon, 2004).

While performing the calculations, the portfolio risk was minimized (the parameters of these portfolios were marked with an additional "mr" subscript) or efficiency was maximized (the parameters of these portfolios were marked with an additional subscript "me"). It was also assumed that there was no possibility for short sales (the weights could not be negative). All weights were rounded to two decimal places.

In addition, the rates of return, risk, efficiency, and maximum cumulative loss for 1140 portfolios were calculated based on the data from the assessment period, assuming a naive strategy where equal capital was allocated to each investment (the parameters of these portfolios were marked with an additional subscript "n"). The results obtained using the naive strategy were the basis for the comparisons made in this study.

4. Research results

The table below presents the value of basic descriptive statistics for the longest estimation window, which was 250 observations from 2017. The statistics are presented for 20 companies included in the WIG20 index and for gold.

Table 1. Value of descriptive statistics for the longest estimation window $n = 250$

Investment	Descriptive statistic							
	Mean	Median	Standard deviation	Kurtosis	Skewness	Range	Minimum	Maximum
1	2	3	4	5	6	7	8	9
ALIOR	0.15%	0.09%	0.018	1.68	0.47	0.13	–5.38%	7.46%
CCC	0.13%	0.09%	0.019	1.61	0.14	0.15	–6.82%	8.07%
CDPROJEKT	0.25%	0.00%	0.022	1.95	–0.01	0.18	–8.48%	9.31%
CYFRPLSAT	0.00%	0.02%	0.015	1.11	–0.27	0.11	–6.03%	4.67%
ENERGA	0.13%	0.11%	0.020	2.63	0.39	0.17	–7.78%	8.81%
EUROCASH	–0.16%	–0.15%	0.020	11.32	–1.42	0.22	–14.76%	7.19%
JSW	0.15%	0.10%	0.025	0.24	0.17	0.15	–5.95%	8.83%
KGHM	0.07%	–0.04%	0.020	0.42	0.35	0.13	–6.01%	6.76%
LOTOS	0.16%	0.04%	0.022	1.06	0.15	0.16	–7.21%	8.43%
LPP	0.18%	0.11%	0.019	1.17	–0.02	0.13	–6.88%	6.19%
MBANK	0.13%	0.11%	0.021	0.08	0.15	0.13	–5.49%	7.10%
ORANGEPL	0.02%	–0.08%	0.019	13.24	–0.09	0.24	–12.04%	11.80%
PEKAO	0.01%	–0.08%	0.014	5.10	–0.32	0.13	–7.61%	5.68%
PGE	0.06%	0.00%	0.019	1.97	–0.02	0.14	–8.50%	5.92%
PGNIG	0.04%	0.00%	0.017	1.74	–0.02	0.13	–6.44%	6.33%
PKNORLEN	0.09%	0.15%	0.021	1.35	–0.56	0.14	–8.11%	6.20%
PKOBP	0.18%	0.08%	0.017	0.50	0.42	0.10	–3.64%	6.18%

1	2	3	4	5	6	7	8	9
PZU	0.10%	0.15%	0.015	0.63	0.05	0.09	–4.55%	4.55%
SANPL	0.09%	0.00%	0.018	1.16	0.52	0.11	–4.64%	6.74%
TAURONPE	0.03%	0.00%	0.018	0.92	0.49	0.11	–4.50%	6.94%
GOLD	0.06%	0.03%	0.007	1.26	0.34	0.04	–1.81%	2.56%

Source: own calculations.

Table 2. Value of descriptive statistics for the evaluation window

Investment	Descriptive statistic							
	Mean	Median	Standard deviation	Kurtosis	Skewness	Range	Minimum	Maximum
ALIOR	–0.15%	–0.21%	0.021	2.68	–0.12	0.17	–9.36%	7.39%
CCC	–0.25%	–0.19%	0.027	3.88	–0.45	0.24	–13.41%	10.58%
CDPROJEKT	0.20%	0.16%	0.030	0.93	–0.30	0.18	–11.13%	7.31%
CYFRPLSAT	–0.05%	0.00%	0.019	6.88	–0.73	0.19	–11.03%	8.26%
ENERGA	–0.23%	–0.27%	0.025	7.53	–0.25	0.28	–14.60%	13.02%
EUROCASH	–0.11%	0.00%	0.021	4.16	–0.37	0.19	–10.95%	8.38%
JSW	–0.17%	–0.13%	0.024	1.33	–0.03	0.17	–7.99%	8.91%
KGHM	–0.13%	–0.12%	0.022	3.69	–0.20	0.20	–10.90%	9.27%
LOTOS	0.08%	0.07%	0.021	0.37	0.31	0.12	–4.88%	7.03%
LPP	–0.05%	–0.05%	0.019	2.02	0.43	0.14	–5.18%	8.73%
MBANK	–0.08%	–0.05%	0.022	11.15	0.57	0.26	–11.31%	14.49%
ORANGEPL	–0.13%	0.00%	0.019	2.55	0.01	0.15	–6.57%	8.44%
PEKAO	–0.12%	–0.24%	0.026	37.84	0.67	0.41	–19.62%	21.15%
PGE	–0.09%	0.11%	0.022	0.27	–0.15	0.13	–6.84%	5.88%
PGNIG	–0.01%	0.17%	0.018	1.66	–0.02	0.14	–7.59%	6.31%
PKNORLEN	–0.09%	–0.23%	0.022	–0.30	–0.01	0.11	–5.81%	5.09%
PKOBP	–0.06%	–0.07%	0.019	2.46	–0.47	0.16	–9.17%	6.95%
PZU	–0.05%	–0.13%	0.018	1.25	0.01	0.13	–6.56%	6.52%
SANPL	–0.07%	–0.05%	0.020	0.90	0.29	0.12	–4.88%	7.35%
TAURONPE	–0.25%	–0.42%	0.024	0.53	0.19	0.14	–6.99%	6.73%
GOLD	–0.02%	–0.07%	0.006	0.50	0.26	0.03	–1.90%	1.51%

Source: own calculations.

Based on the descriptive statistics determined for the longest estimation window, it can be stated that in 2017, out of the 21 analysed investments, 20 achieved a positive mean daily rate of return. Only Eurocash company included in the WIG20 index recorded a mean daily rate of return that was smaller than zero. A total of 13 of the

listed companies had a higher return than investment in gold. For risk measured by the standard deviation of the rate of return, all the companies had a higher risk than the investment in gold. For investments in gold, the standard deviation was equal to 0.007 percentage points and was two times lower than for the least risky stock company of PEKAO Bank, for which this value was 0.014. The same situation occurred for the range of observed values. Gold had the smallest value of this measure, which indicates the smallest empirical area of this investment's variability. The rates of return for gold were in the range of –1.81% to 2.56%. This is more than two times lower than for the least risky company assessed by this measure. Table 1 also presents the kurtosis and skewness values for the analysed investments.

The next table presents the value of basic descriptive statistics for the researched investments for the evaluation window. The evaluation period covered 209 quotations from 2018.

Contrary to the estimation window, almost all the investments recorded negative mean returns in the period adopted for assessment. Only for two companies were the mean daily rates of return greater than zero: LOTOS and CDPROJEKT, for which the values of this descriptive statistic were 0.08% and 0.20%, respectively. The risk of analysed investments was similar to the estimation window. The least risky investment turned out to be gold, for which the standard deviation of the rates of return and range was lower than the least risky listed company.

The value of the rate of return on the investment portfolios with minimal risk and maximum efficiency according to the length of the estimation window was as shown in Figure 2. All the findings presented below are the results of research based

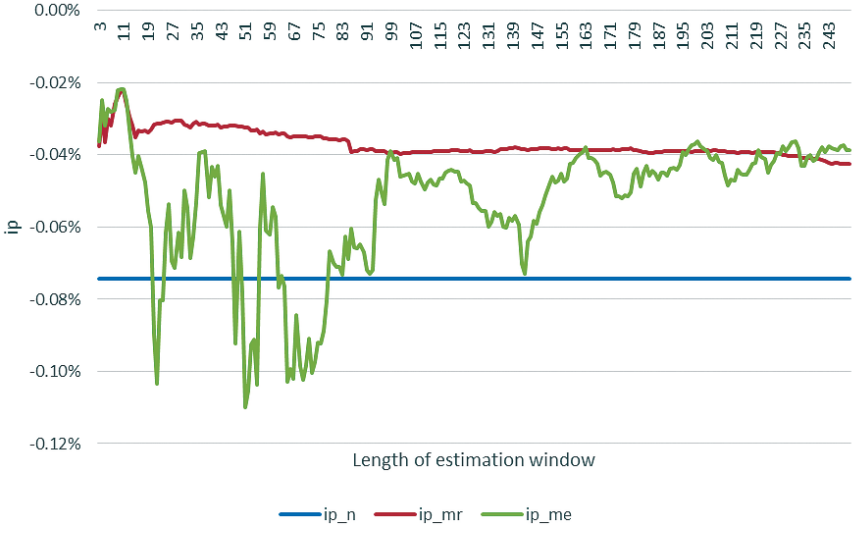

Fig. 2. Mean rate of return for designated portfolios according to the length of the estimation window

Source: own calculations.

on a given sample of companies (the blue-chip companies on the WSE) and for the analysed years. In this paper, other years and smaller companies were not analysed so for smaller companies the results can be different in the same research period and could be subject to further research.

Figure 2 shows how the mean rate of return on portfolios with minimal risk (ip_mr) and maximum efficiency (ip_me) changed in 2018, depending on how many observations from 2017 portfolio weights were determined. The presented values are mean values for all designated portfolios with a particular length of estimation window. In addition, the mean rate of return using naive diversification for 2018 data is presented using a horizontal line (ip_n). Regardless of how many observations from 2017 were taken into account when calculating the portfolio weights, it was not possible to choose the weights to achieve positive mean returns in 2018. This is due to the fact that almost all investments in 2018 recorded a loss, and the chart shows the mean rates of return for 1140 portfolios created for each estimation window. It should be emphasized, however, that for a portfolio with minimal risk, regardless of the length of the estimation window, the mean rates of return were each time higher than for the naive diversification. The highest rate of return was obtained if the length of the estimation window was 11 observations. From the shortest estimation window to the window with 85 observations, the mean return on the minimal risk portfolio was 0.04 percentage points higher than the return on the naive strategy. Greater variation in results was noted for the portfolio with maximum efficiency, as both lower and higher rates of return were achieved than for the naive strategy. Generally, it can be concluded that if the length of the estimation window was in the range of 3 to 12 observations, the rates of return were higher than for a naive strategy. In the next interval, when the length of the estimation window was from 12 to 90 observations, the rates of return were highly variable. When the length of the estimation window increased from 90 to 140 observations, the rates of return deteriorated, and then for the longest estimation window of over 140 observations, the rates of return increased.

Figure 3 shows how the length of the estimation window changed the mean risk value of the designated portfolios. For portfolios with minimal risk, as with the rate of return, the length of the estimation window did not significantly affect the risk value. For very short window lengths from 3 to 13 observations, higher risk values were obtained than for portfolios based on a range of data covering 13 to 85 observations. Then, the longer the estimation window, the slightly higher the risk.

In turn, portfolios with maximum efficiency had much greater variability depending on the assumed length of the estimation window. For very short estimation periods, as with the rate of return, lower risk values of maximum efficiency portfolios were noted. For estimation windows with a length of 12 to 95 observations, there was large variability in risk values. For some cases the obtained values were even three times higher than for the naive strategy, but in the same range, there were also some lengths of estimation window that achieved lower risk than the naive strategy. When weights

were determined based on more than 170 observations, then the portfolio risk value decreased as the length of the estimation window increased.

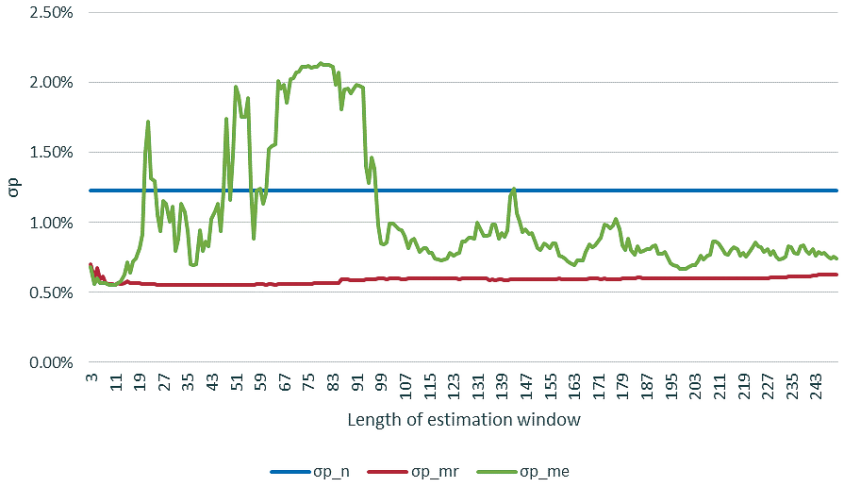

Fig. 3. Mean value of risk value for designated portfolios according to the length of the estimation window

Source: own calculations.

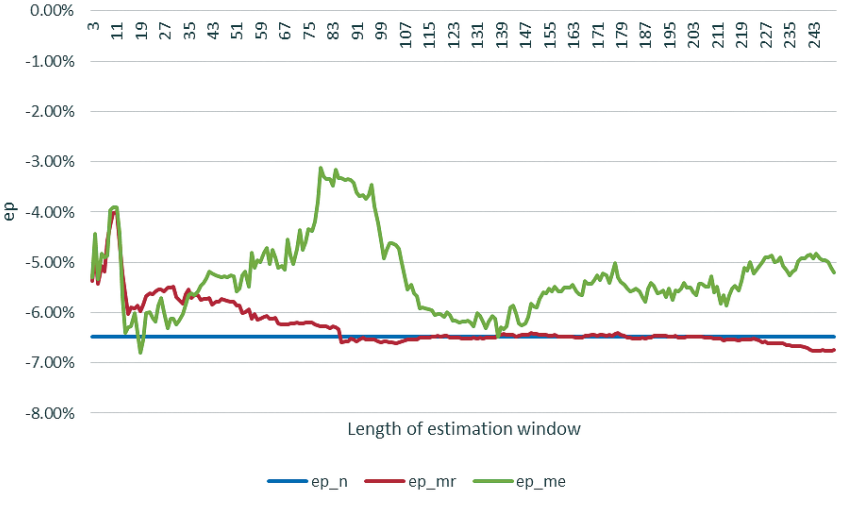

Fig. 4 Mean efficiency value for designated portfolios according to the length of the estimation window

Source: own calculations.

The construction of a portfolio in accordance with Markowitz's theory also achieves higher efficiency than when using a naive strategy in a given set of companies and given

year (Figure 4). This is practically the case for all considered variants of estimation window length for portfolios with maximum efficiency, where the highest efficiency values were obtained for estimation windows with a length of 75 to 100 observations. A significant decrease in efficiency value was observed with the extension of the length of the estimation window from 100 to 130 observations, however, these values were still higher than for a naive strategy. For estimation windows with a length of more than 130 observations, there was a systematic increase in efficiency, but none of those efficiency values were higher than those obtained for estimation windows with a length from 75 to 100 observations. The situation was different for minimal risk portfolios. Here, the best efficiency values were obtained for short estimation windows – up to 50 observations. After 50 observations, values systematically decreased, and for cases containing the most observations in the estimation window, an even lower efficiency was achieved than for a naive strategy.

The last criterion that was evaluated for the designated portfolios was the value of the maximum possible loss that the investor is exposed to when constructing the portfolio based on portfolio theory. The results of this measurement are shown in Figure 5.

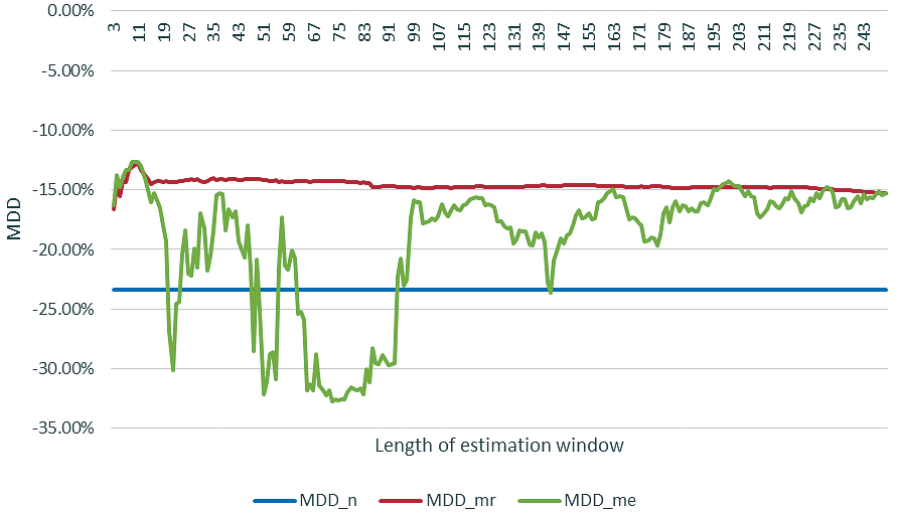

Fig. 5. Mean value of the maximum cumulative loss for designated portfolios according to the length of the estimation window

Source: own calculations

Portfolios with maximum efficiency, as with efficiency values, had more diverse results than portfolios with minimal risk in a given sample in the tested years. The maximum loss values for estimation windows up to 100 observations were highly variable, and often worse than for a naive strategy. It can be stated again that better results (less cumulated loss) can be achieved by determining portfolio weights based

on longer estimation windows. For portfolios with minimal risk, it can be concluded from the results obtained that the length of the estimation window has no major impact on the maximum loss that the investor achieves. Moreover, the value of this maximum loss is lower by nearly eight percentage points than in a naive strategy.

A summary of the results obtained is presented in the diagrams below, which illustrate the optimal length of the estimation window for the given criteria, combined with a naive strategy.

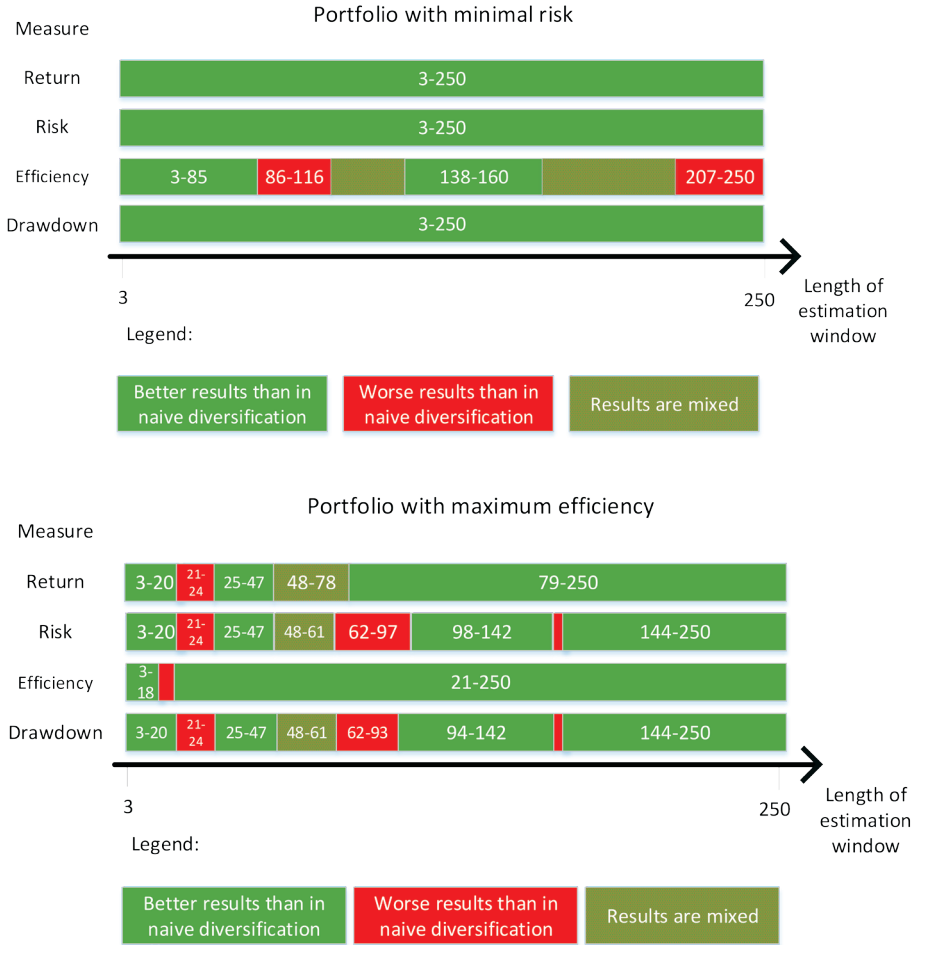

Fig. 6. Length of the estimation window and its impact on the basic measures of the investment portfolio compared to a naive strategy

Source: own calculations.

Based on the results of the research as illustrated in Figure 6, it can be concluded that the length of the estimation window for an investor investing in the largest Polish

companies by market capitalization and gold who wants to minimize investment risk (as measured by the standard deviation of the rate of return) should use an estimation window ranging in length from 3 to 85 or from 138 to 160 daily observations. The length of the estimation window will allow better results to be obtained than in a naive strategy for the portfolio characteristics of risk, rate of return, efficiency, and maximum cumulative loss. For a portfolio with minimal risk, it can also be seen that, apart from efficiency, using portfolio theory leads to better results than a naive strategy, and the length of the estimation window is not important.

In the case of an investor striving to maximize efficiency, better results were obtained than for a naive strategy when the number of observations from the past were in one of four ranges:

- <3, 20>,
- <25, 47>,
- <98, 142>,
- <144, 250>.

Hence, it can be concluded that regardless of the investment objective and the portfolio characteristics studied, the optimal length of the estimation window should be in the range of 144 to 160 observations. Therefore the research hypothesis adopted in the introduction of this paper cannot be rejected, and this is not the same finding as in [Raudys, Raudys, and Pabarskaite, 2018)] when stated that the length of the estimation window depends on the investment purpose. It should be noted that this conclusion was obtained on the basis of daily data and logarithmic rates of return and only for companies with the biggest capitalisation in the WSE and for investment in gold in a given year.

5. Conclusion

The study shows that an investor who seeks to minimize risk will achieve better results using portfolio theory than using a naive strategy regardless of the length of the estimation window. This is true for the rate of return, risk, and maximum accumulated loss. The effectiveness of such portfolios will be higher than for a naive strategy if the length of the estimation window is between 3 and 85 observations or between 138 and 160 observations. In turn, investors whose investment objective is to maximize efficiency should set the length of the estimation window to one of four ranges: from 3 to 20 observations, from 25 to 47 observations, from 98 to 142 observations, or from 144 to 250 observations. In addition, the analysis shows that portfolios constructed using Markowitz's portfolio theory, virtually regardless of the length of the estimation window, were more effective than portfolios constructed using a naive strategy. Therefore using a portfolio strategy for both types of investment portfolios gives better results than for naive diversification. If the investor does not define the purpose of building an investment portfolio, then the optimal length of the estimation window is in the range of 144 to 160 observations. So it can be said that for the biggest companies quoted on the WSE and for investment in gold there

can be calculated one common range for an optimal estimation window to realize so different investment purposes as minimal risk and maximum efficiency.

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