Modelling of abdominal wall under uncertainty of material properties

Katarzyna Szepietowska[1], Izabela Lubowiecka[1], Benoit Magnain[2], and Eric Florentin[2]

[1] Faculty of Civil and Environmental Engineering, Gdansk University of Technology
Narutowicza 11/12, 80-233 Gdańsk, Poland,
`katszepi@pg.edu.pl`,
[2] INSA Centre Val de Loire, Univ. Orléans, Univ. Tours, LaMé, EA 7494, F-18022, Bourges, France

Abstract. The paper concerns abdominal wall modelling. The accurate prediction and simulation of abdominal wall mechanics are important in the context of optimization of ventral hernia repair. The shell Finite Element model is considered, as the one which can be used in patient-specific approach due to relatively easy geometry generation. However, there are uncertainties in this issue, e.g. related to mechanical properties since the properties may vary naturally or as an effect of identification accuracy etc. The aim of the study is to include uncertainties in the modelling and investigate their influence on the model response. The parameters of Gasser-Ogden-Holzapfel hyperelastic material model including fibre orientation are treated here as random variables. The uncertainties are propagated with the use of regression based polynomial chaos expansion method. Sobol' indices are used as the measures of global sensitivity analysis and they provide information about the influence of input uncertainties on the uncertainty of the model output. Uncertainty of parameter affecting stiffness of ground substance (C_{10}) has the highest contribution to the variation of the displacement of chosen point in the center of the abdominal wall.

Keywords: uncertainty quantification, global sensitivity analysis, hyperelasticity, Gasser-Ogden-Holzapfel material model, polynomial chaos, Sobol' indices

1   Introduction

The study addresses the issues of the modelling of abdominal wall. Understanding mechanical behaviour of abdominal wall is particularly interesting in the context of ventral hernia repair. In order to improve the efficiency of hernia repair, some mechanical approaches have been employed. Various surgical meshes were investigated in the literature, e.g. [1] and models of implants were developed [2]. However, it was also acknowledged that the mechanics of abdominal wall pays a crucial role in designing implants that would be mechanically compatible with human tissue [3]. Thus the mechanical properties of abdominal wall should be known.

The extensive review on mechanics of both abdominal wall and implant was written by Deeken and Lake in [4]. It can be noticed in their compilation of existing experimental studies on tissues of abdominal wall, that the properties of the same tissue reported in the literature varies between articles, e.g. due to different testing protocols. Existing numerical models are mainly based on *ex vivo* properties of animals [5] or human [6] samples. Some properties were also identified *in vivo*: Young's modulus [7] or parameters of isotropic hyperlastic material model on animals [8]. It can be seen that a lot of uncertainties appear in the modelling of abdominal wall, e.g. due to challenges in accurate identification of properties, natural variability of properties and so on. The aim of this study is to include those uncertainties in the modelling and study their influence on the model output.

The models mentioned above [5, 6] are detailed and include various components of abdominal wall with geometry based on medical images (MRI or CT scans). Lubowiecka et al. [9] proposed simpler membrane model of abdominal wall with geometry corresponding to the external surface of abdominal wall. It was created in the perspective of patient-specific approach and *in vivo* identification of material properties by inverse methods using measurements of displacement caused by known changes of pressure during peritoneal dialysis. The behaviour of the model was compared with experiment. In [10] we propagated uncertainties related to the value of intraabdominal pressure, the parameters of linear elastic orthotropic model and the direction of orthotropy. We studied the influence of these uncertainties on the uncertainty of the output. However, soft tissues exhibit nonlinear elastic behaviour. Borzeszkowski et al. [11] applied Gasser-Ogden-Holzapfel (GOH) [12] hyperelastic material model to abdominal wall shell model and performed a parametric analysis to study the influence of various parameters. The aim of the present study is to perform uncertainty quantification and global sensitivity analysis of abdominal wall shell model including uncertainty of GOH material and structure parameters related to fibres alignment. The general purpose is to find most important and negligible variables in the context of the further identification of abdominal wall properties and optimisation under uncertainty of hernia repair parameters with the use of abdominal wall model. Polynomial chaos expansion method is used to propagate the uncertainties and to calculate Sobol' indices [13], which are global sensitivity measure of the influence of investigated uncertainties to the uncertainty of abdominal wall response. The global sensitivity of GOH was already performed in other applications [14], but since the sensitivity analysis outcomes depends on the problem and the studied quantity of interest so new results are expected.

2   Materials and methods

2.1   Finite Element model of abdominal wall

The model of abdominal wall was created in commercial Finite Element (FE) software MSC.Marc. The geometry of the model is based on measurements of human external surface of abdominal wall [15]. The model is composed of 1872

shell 4-node quadrilateral elements. Nodes on the boundary of abdominal wall (Fig. 1) have fixed translations. Thickness is assumed to be 3 cm. The model is subjected to pressure equal to 981 Pa corresponding to intraabdominal pressure caused by liquid intruded during peritoneal dialysis. The loading corresponds to experiment described in [9] when the same model (but with orthotropic linear elastic material law) was validated with experiments performed on the patient undergoing dialysis. The analysis is geometrically and physically nonlinear.
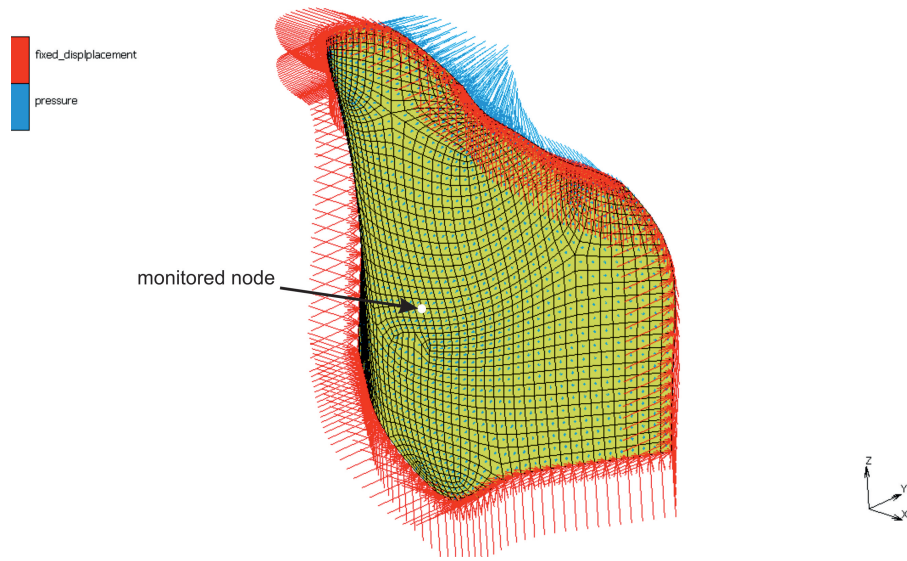

Fig. 1. FE model

2.2   Constitutive modelling

In this work the assumed material model of abdominal wall is Gasser-Ogden-Holzapfel (GOH) model [12], which is hyperelastic anisotropic model. Although this constitutive law was developed to model arterial layers, it is also used in modelling of other soft tissues, e.g. abdominal wall tissues [16], tendons [17]. In the model a strain-energy function Ψ is assumed to be in decoupled form

$$\Psi = \Psi_{vol} + \bar{\Psi}, \tag{1}$$

where Ψ_{vol} is purely volumetric contribution Ψ_{vol} and $\bar{\Psi}$ is isochoric part. It is assumed that $\bar{\Psi}$ is superposition isotropic contribution corresponding to ground-matrix $\bar{\Psi}_g$ and contribution corresponding to embedded fibres $\bar{\Psi}_f$

$$\bar{\Psi} = \bar{\Psi}_g + \bar{\Psi}_f, \tag{2}$$

Neo-Hookean model is chosen for Ψ_g:

$$\bar{\Psi}_g = C_{10}(\bar{I}_1 - 3), \tag{3}$$

where C_{10} is the stress-like material parameter, $\bar{I}_1$ is the first invariant of the modified right Cauchy-Green deformation tensor. Only one family of fiber is assumed, then:

$$\bar{\Psi}_f = \frac{k_1}{2k_2} \left[\exp\{k_2[\kappa(\bar{I}_1 - 3) + (1 - 3\kappa)(\bar{I}_4 - 1)]^2\} - 1 \right], \tag{4}$$

where k_1 is the stress-like material parameter, k_2 is the dimensionless material parameter. $\bar{I}_4$ is the invariant equal to the square of the stretch in the direction of the mean orientation α_f of the family of fibres. κ describes level of fibre dispersion and can be in the range $0 \leq \kappa \leq 1/3$, where $\kappa = 0$ corresponds to perfect alignment of fibers (transverse isotropy) and $\kappa = 1/3$ correspond to isotropy.

Parameters of the GOH model and fiber orientation are assumed to be uncertain.

2.3   Uncertainty quantification and sensitivity analysis

Polynomial chaos expansion Uncertainties can be included in the modelling by probabilistic approach. Since a commercial software is applied, the non-intrusive uncertainty propagation method is needed. Such methods are based on some number of deterministic calculations and do not require modification of the FE code of the model. Polynomial chaos (PC) method exists in non-intrusive variants (Non-Intrusive Spectral Projection Method [18] or regression based approach [19]) and enable performing uncertainty quantification and global sensitivity analysis method with relatively small computational costs when compared to widely-used Monte Carlo method. In the PC method the model output Y, Quantity of Interest (QoI), is expanded as follows:

$$Y \approx \sum_{\boldsymbol{\alpha} \in \mathcal{A}} a_{\boldsymbol{\alpha}} \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\xi}), \tag{5}$$

where $\boldsymbol{\xi}$ is an input random vector, $a_{\boldsymbol{\alpha}}$ are coefficients, $\mathcal{A}$ is a truncation set of $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}$ are M-uplets $(\alpha_1, \ldots, \alpha_M) \in \mathbb{N}^M$, and $\Phi_{\boldsymbol{\alpha}}(\boldsymbol{\xi})$ is a multivariate polynomial basis constructed by multiplying polynomials ϕ_{α_i} of order α_i

$$\Phi_{\boldsymbol{\alpha}}(\boldsymbol{\xi}) = \prod_i^M \phi_{\alpha_i}^i(\xi_i). \tag{6}$$

Polynomials have to be orthonormal with respect to a given distribution. In this case Legendre polynomials are employed because the random variables follow uniform distribution. Classic truncation has been performed, such that $\mathcal{A} = \{\boldsymbol{\alpha} \in \mathbb{N}^M : \sum_{i=1}^M \alpha_i \leq p\}$, where p is PC degree. Regression-based approach [19]

is used to find the coefficients. The model is performed on N regression points in order to obtain the vector of exact solutions. Then the coefficients are calculated by solving least square problems. The drawback of non-intrusive methods like this one, is that the accuracy depends on the number and choice of regression points. Different strategies were compared on the hernia-related models in [20]. Based on the experience presented the points from Sobol sequence are used. $N = (M - 1)P$, where P is size of PC basis (cardinality of truncation set $\mathcal{A}$). Different techniques have been developed to control error [21]. Here, Leave-One-Out error estimate was calculated in order to evaluate PC meta-model performance because of simplicity.

Global sensitivity analysis Global sensitivity analysis is a study of sensitivity of the output to variations of the input. It enables varying all variables at the same time with variations over whole domain. Sobol indices [22] are one of the global-sensitivity measures. The calculation is based on ANalysis Of VAriance (ANOVA) decomposition. Estimation of Sobol indices by MC is very expensive computationaly. It has been shown by Sudret [13] and Crestaux et al. [23] that thanks to the orthonormality of the PC basis, estimation of Sobol indices can be performed with use of the PC coefficients without additional computational cost. Sobol index $S_{i_1,\ldots,i_s}$ shows how much of the total output variance is due to the uncertainty of variables $\xi_{i_1}, \ldots, \xi_{i_s}$. To compute it by PC method a set of $\boldsymbol{\alpha}$-tuples corresponding to polynomials depending only on all input variables $\xi_{i_1}, \ldots, \xi_{i_s}$ must be found:

$$\mathcal{A}_{i_1,\ldots,i_s} = \{\boldsymbol{\alpha} \in \mathcal{A} : \alpha_k \neq 0 \Leftrightarrow k \in \{i_1, \ldots, i_s\}\}. \tag{7}$$

Then, sobol index $S_{i_1,\ldots,i_s}$ estimated by PC is:

$$S_{i_1,..,i_s}^{PC} = \frac{1}{D} \sum_{\boldsymbol{\alpha} \in \mathcal{A}_{i_1,\ldots,i_s}} a_{\boldsymbol{\alpha}}^2, \tag{8}$$

where D is total output variance, which can be estimated by PC coefficients $D = \sum_{\boldsymbol{\alpha} \in \mathcal{A} \backslash \mathbf{0}} a_{\boldsymbol{\alpha}}^2$. Total sobol index S_i^{Tot} is the sum of all indices corresponding to a given variable i, including mixed terms. It can also be estimated using PC coefficients:

$$S_i^{Tot,PC} = \frac{1}{D^{PC}} \sum_{\boldsymbol{\alpha} \in \mathcal{A}_i^{Tot}} a_{\boldsymbol{\alpha}}^2. \tag{9}$$

where $\mathcal{A}_i^{Tot} = \{\boldsymbol{\alpha} \in \mathcal{A} : \alpha_i \neq 0\}$.

Random variables and quantity of interest Due to the uncertainty of GOH model parameters and fibre orientation, five independent uniform random variables are assumed with limits presented in Table 1. First three: C_{10}, k_1, k_2 can be classified as material parameters and the last two: κ, α_f as a structure parameters [12].

Table 1. Limits of uniform distribution $\mathcal{U}(a,b)$ of each random variable

	C_{10} [kPa]	k_1 [kPa]	k_2	κ	α_f
a	2	1	200	0	0
b	35	10	2000	0.33	π

Range of C_{10} was chosen based on reported in [24] results of shear modulus (divided by 2) of living human abdominal wall. Upper bounds of k_1 and k_2 where taken from [16]. κ are α_f may vary in their possible range of values.

The quantity of interest here is the magnitude of displacement u in chosen nodes in the central area of abdominal wall (Fig. 1). During physical experiments [9] the displacement of external surface of abdominal wall can be measured and its value may be used in the *in vivo* identification of the material parameters of living abdominal wall [7, 8].

3   Results

Uncertainties have been propagated through the abdominal wall model with the use of PC method. The histogram of the QoI (the displacement of the chosen node in central area) is presented in Fig. 2. Mean equals 0.0067 m and standard deviation equals 0.0045 m.
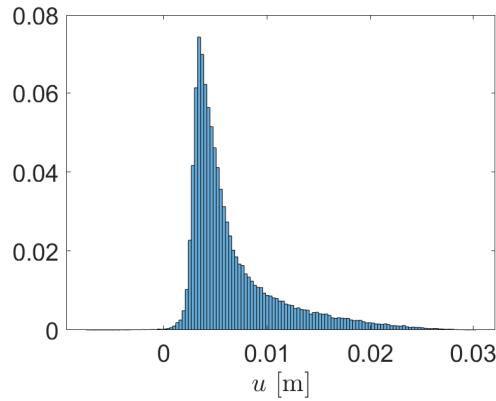

Fig. 2. Histogram of the QoI (displacement in the chosen node)

Table 2 shows the values of Sobol indices of first and second order and Table 3 shows total Sobol indices. The sensitivity indices values are also shown in Fig.

3. It can be shown that the uncertainty of C_{10} has a dominant contribution to the output variance: it has the highest first order Sobol' index $S_{C_{10}}$. Total index $S_{C_{10}}^{Tot}$ is higher mainly due to the interaction with κ and α_f (second order indices). The first order indices of variables other than C_{10} are negligibly small. However, the total Sobol index of κ and α_f is higher due to mentioned interaction with C_{10}. The uncertainty of k_1 and k_2 has a negligible effect on the variance of investigated QoI.

Table 2. First S_i (on the table diagonal) and second order Sobol' indices S_{ij}

	C_{10}	k_1	k_2	κ	α_f
C_{10}	0.7607				
k_1	0.0038	0.0016			
k_2	0.0148	0.0002	0.0039		
κ	0.0695	0.0008	0.0013	0.0320	
α_f	0.0553	0.0006	0.0006	0.0097	0.0309

Table 3. Total Sobol' indices S_i^{Tot}

i	C_{10}	k_1	k_2	κ	α_f
S_i^{Tot}	0.9171	0.0107	0.0258	0.1249	0.1078

The dominant influence of C_{10} can also be noticed on scatter plots (Fig. 4) showing the QoI value versus each variable. The scatter plots were drawn on the exact model values, not with PC meta-model, in order to evaluate the sensitivity analysis outcomes. The visual evaluation of this graph indicates that sensitivity of the output to C_{10} is higher when C_{10} is high. It can also be interpreted as that the sensitivity to κ is slightly higher, when κ is closer to 0, so the fibers are close to perfect alignment. The sensitivity to α_f is slightly higher when the mean direction of fibers is in transverse direction of abdominal wall. The transverse direction of abdominal wall is known to be stiffer [7].

Figure 5 shows displacement of abdominal wall for two extreme values of C_{10} (limits of uniform distribution) with other the same parameters.

4   Conclusions

Uncertainties have been propagated in the abdominal wall and their importance has been assessed by Sobol' indices. The uncertainty of C_{10} - parameter of the
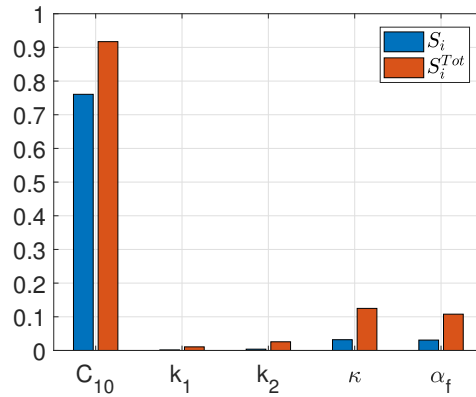
Fig. 3. Sobol indices

isotropic part corresponding to groundmatrix has the highest dominant contribution to the variance of the output. Other variables, including one related to fibers, are much less important. This outcome can be interesting in the context of some reported postulates that despite the anisotropy of the single components of abdominal wall, the entire abdominal wall can be treated as isotropic [8]. Next most important variables are the structure ones: κ and α_f due to their interaction with C_{10} (higher order sensitivity indices). Such interaction may be considered a bit surprising and needs further research. k_1 and k_2 have negligible effect in this case. These two parameters have unclear physical interpretation [14] and low importance of their uncertainty would be beneficial in further parameter identification problems. However, it should be mentioned that the limits of their input distribution, in contrast to C_{10}, was not supported by any experimental data.

The aim of the study was o asses the importance of each structure and material parameter of the GOH model in application to modelling of human abdominal wall. Some limitations of the presented study should be mentioned. Firstly, due to the lack of large data sets or existing recommendations, distribution of random variables was assumed by our own judgement. Nevertheless, the obtained sensitivity analysis results indicate which data could be important for proper identification of probabilistic models. Secondly,as a simplification, the material and structure parameters are assumed to be constant in space and just one fiber family is considered. In reality, human abdominal wall is constructed from various components with various properties and fiber orientation. Rectus muscles covered by rectus sheath and linea alba in the midline and lateral part composed of three muscles each covered by aponeuroses, differs from each other in that sense. Spacial distribution of isotropic hyperlastic materials was presented in [8]. Abdominal wall could also be divided into regions with various properties, one corresponding to rectus muscles and one corresponding to oblique muscles
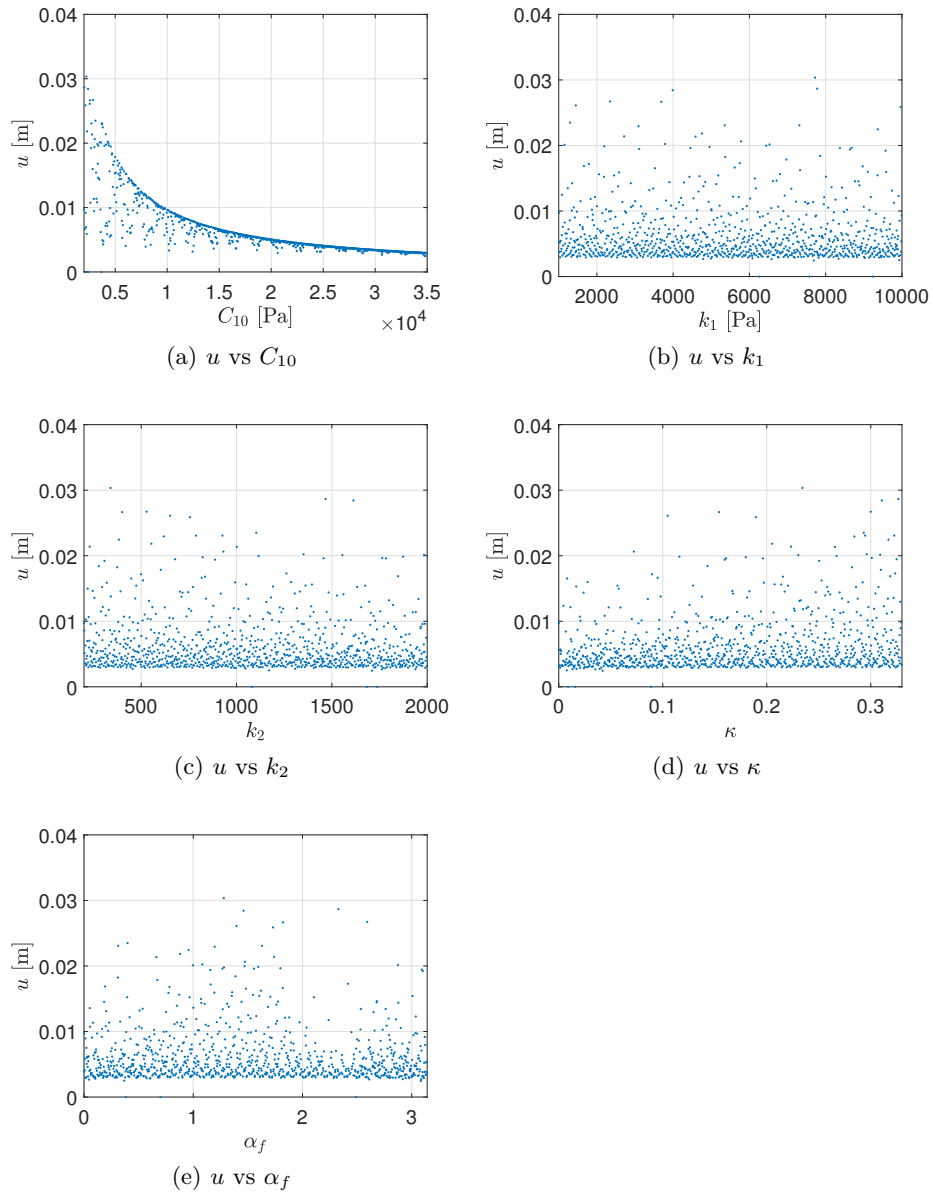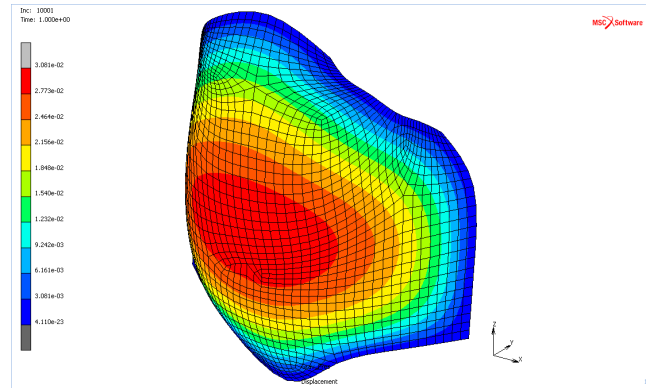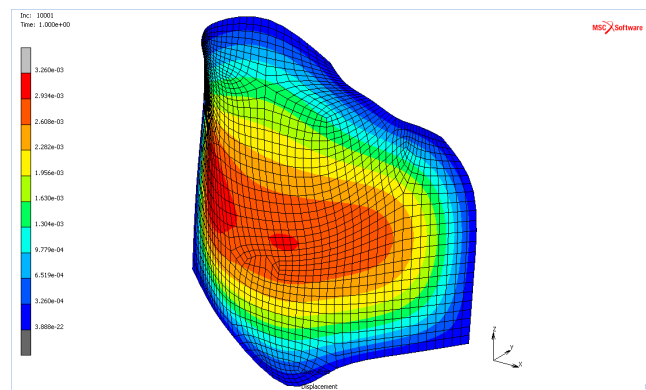
(a) u vs C_{10}

(b) u vs k_1

(c) u vs k_2

(d) u vs κ

(e) u vs α_f

Fig. 4. Scatter plots showing sensitivity of QoI to each variable

(a) C_{10}=2kPa, maximum displacement 3.081 cm

(b) C_{10}=35 kPa, maximum displacement 0.326 cm

Fig. 5. Displacement [m] of abdominal wall in case of extreme values of C_{10}

as it was done in [10]. In the further research random fields could be applied to include the spatial variability.

Acknowledgments

References

1. Tomaszewska, A.: Mechanical behaviour of knit synthetic mesh used in hernia surgery. Acta of bioengineering and biomechanics 18(1) (2016)
2. Lubowiecka, I.: Mathematical modelling of implant in an operated hernia for estimation of the repair persistence. Computer methods in biomechanics and biomedical engineering 18(4), 438–445 (2015)
3. Junge, K., Klinge, U., Prescher, A., Giboni, P., Niewiera, M., Schumpelick, V.: Elasticity of the anterior abdominal wall and impact for reparation of incisional hernias using mesh implants. Hernia 5(3), 113–118 (2001)
4. Deeken, C.R., Lake, S.P.: Mechanical properties of the abdominal wall and biomaterials utilized for hernia repair. Journal of the mechanical behavior of biomedical materials 74, 411–427 (2017)
5. Hernández-Gascón, B., Mena, A., Pena, E., Pascual, G., Bellón, J., Calvo, B.: Understanding the passive mechanical behavior of the human abdominal wall. Annals of biomedical engineering 41(2), 433–444 (2013)
6. Pachera, P., Pavan, P., Todros, S., Cavinato, C., Fontanella, C., Natali, A.: A numerical investigation of the healthy abdominal wall structures. Journal of biomechanics 49(9), 1818–1823 (2016)
7. Song, C., Alijani, A., Frank, T., Hanna, G., Cuschieri, A.: Mechanical properties of the human abdominal wall measured in vivo during insufflation for laparoscopic surgery. Surgical Endoscopy And Other Interventional Techniques 20(6), 987–990 (2006)
8. Simón-Allué, R., Calvo, B., Oberai, A., Barbone, P.: Towards the mechanical characterization of abdominal wall by inverse analysis. Journal of the mechanical behavior of biomedical materials 66, 127–137 (2017)
9. Lubowiecka, I., Tomaszewska, A., Szepietowska, K., Szymczak, C., Lochodziejewska-Niemierko, M., Chmielewski, M.: Membrane model of human abdominal wall. simulations vs. in vivo measurements. In: Shell Structures. Theory and Applications. vol. 4, pp. 503–506 (2018)
10. Szepietowska, K., Lubowiecka, I., Magnain, B., Florentin, E.: Global sensitivity analysis of membrane model of abdominal wall with surgical mesh. In: Shell Structures. Theory and Applications. vol. 4, pp. 515–518 (2018)
11. Borzeszkowski, B., Duong, T.X., Sauer, R.A., Lubowiecka, I.: Isogeometric Shell FE Analysis of the Human Abdominal Wall. In: Innovations in Biomedical Engineering. Springer, accepted (2019)

12. Gasser, T.C., Ogden, R.W., Holzapfel, G.A.: Hyperelastic modelling of arterial layers with distributed collagen fibre orientations. Journal of the Royal Society Interface 3(6), 15–35 (2006)
13. Sudret, B.: Global sensitivity analysis using polynomial chaos expansions. Reliability Engineering & System Safety 93(7), 964–979 (2008), http://linkinghub.elsevier.com/retrieve/pii/S0951832007001329
14. Akintunde, A.R., Miller, K.S., Schiavazzi, D.E.: Bayesian inference of constitutive model parameters from uncertain uniaxial experiments on murine tendons. Journal of the mechanical behavior of biomedical materials 96, 285–300 (2019)
15. Szymczak, C., Lubowiecka, I., Tomaszewska, A., Śmietański, M.: Investigation of abdomen surface deformation due to life excitation: implications for implant selection and orientation in laparoscopic ventral hernia repair. Clinical Biomechanics 27(2), 105–110 (2012)
16. Simón-Allué, R.: Towards the in vivo mechanical characterization of abdominal wall in animal model. Application to hernia repair. Ph.D. thesis, Universidad de Zaragoza (2016)
17. Bajuri, M., Isaksson, H., Eliasson, P., Thompson, M.S.: A hyperelastic fibre-reinforced continuum model of healing tendons with distributed collagen fibre orientations. Biomechanics and modeling in mechanobiology 15(6), 1457–1466 (2016)
18. Le Maître, O.P., Reagan, M.T., Najm, H.N., Ghanem, R.G., Knio, O.M.: A stochastic projection method for fluid flow: II. random process. Journal of Computational Physics 181(1), 9–44 (2002), http://www.sciencedirect.com/science/article/pii/S0021999102971044
19. Berveiller, M., Sudret, B., Lemaire, M.: Stochastic finite element: a non intrusive approach by regression. Revue européenne de mécanique numérique 15(1-2-3), 81–92 (2006), http://remn.revuesonline.com/article.jsp?articleId=7876
20. Szepietowska, K., Magnain, B., Lubowiecka, I., Florentin, E.: Sensitivity analysis based on non-intrusive regression-based polynomial chaos expansion for surgical mesh modelling. Structural and Multidisciplinary Optimization 57(3), 1391–1409 (2018), https://doi.org/10.1007/s00158-017-1799-9
21. Chamoin, L., Florentin, E., Pavot, S., Visseq, V.: Robust goal-oriented error estimation based on the constitutive relation error for stochastic problems. Computers and Structures 106-107, 189–195 (Sep 2012), https://hal.archives-ouvertes.fr/hal-00776138
22. Sobol, I.M.: Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. Mathematics and Computers in Simulation 55(1-3), 271–280 (2001)
23. Crestaux, T., Le Maître, O., Martinez, J.M.: Polynomial chaos expansion for sensitivity analysis. Reliability Engineering & System Safety 94(7), 1161 – 1172 (2009), special Issue on Sensitivity Analysis
24. Tran, D., Podwojewski, F., Beillas, P., Ottenio, M., Voirin, D., Turquier, F., Mitton, D.: Abdominal wall muscle elasticity and abdomen local stiffness on healthy volunteers during various physiological activities. Journal of the mechanical behavior of biomedical materials 60, 451–459 (2016)