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Control of a nonlinear and linearized model of self-balancing electric motorcycle

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Abstract. Self-Balancing Electric Motorcycle (SBEM) is a dynamic and nonlinear electromechanical system. In this paper, the process of mathematical modelling and linearization of SBEM is presented. The model of the control system in Matlab environment is implemented. The control system using the PID controller is designed. The operation of particular structures of the PID controller on the simulation model is compared. Due to simulation research, the most appropriate structure and parameters of the PID controller are chosen.

Keywords: PID controller, control system, mathematical modelling, self-balancing electric motorcycle

1 Introduction

Electric vehicles use electric power to work. These vehicles are usually powered by electric motors which can drive each wheel separately or the whole axle. Electric vehicles such as cars, trucks, trains, bikes and bicycles are mainly used to transport people and to travel. An electric vehicle has its own power source like a battery to provide electric power. It can be recharge using solar energy or a charging station. It can also have systems that recover energy from braking to recharge the battery. The advantages of electric vehicles compared to combustion vehicles are: quiet and clean work, environmentally friendly, relative long drive distances, cheaper usage and they can be used indoors.

Self-Balancing Electric Motorcycle (SBEM) can be used to transport people in the desired direction. The advantage of SBEM is that it is possible to stand in a vertical position without using an additional kickstand.

Control and modelling of SBEM can be an interesting subject of research. It has a nonlinear and complex mathematical model shown in [1]. The position stabilization system can be realized by means of various algorithms. The PI controller is applied in [2]. Control algorithm using LQR is introduced in [3]. Self-balancing similar vehicles are common projects. The motorcycle using a flywheel to stabilize its position is shown in [4] and an autonomous bicycle is designed in [5].

The mathematical model is an abstract, simplified, mathematical construct related to a part of reality and created for a particular purpose [6]. The model allows for a better understanding of the process and predicts certain actions under certain conditions and defined input signals. It also can be used to define dangerous states of the process and critical values of signals.

The main purpose of modelling SBEM is to better understand its principle of operation. The mathematical model is used to implement a control system which uses a PID controller in Matlab environment [7]. The final structure of the PID controller and values of its parameters are indicated during the simulation test.

The paper is organized as follows. Section 2 presents the process of mathematical modelling and linearization of SBEM. In this section, the simulation results of a nonlinear and linearized model of SBEM are presented. Section 3 presents the structure of the control system and modelling of the DC motor. In this section, two methods of tuning the PID controller are described. In Section 4 the simulation tests and results analysis are conducted. Concluding remarks are listed in the last section.

2 Modelling of SBEM

2.1 Operating principle

The SBEM principle of operation is derived from the inverted reaction wheel pendulum (Fig. 1). The main element that stabilizes the structure vertically is the reaction wheel, driven by the DC (Direct Current) motor. It uses the change of its angular momentum to bring the structure to a vertical position.

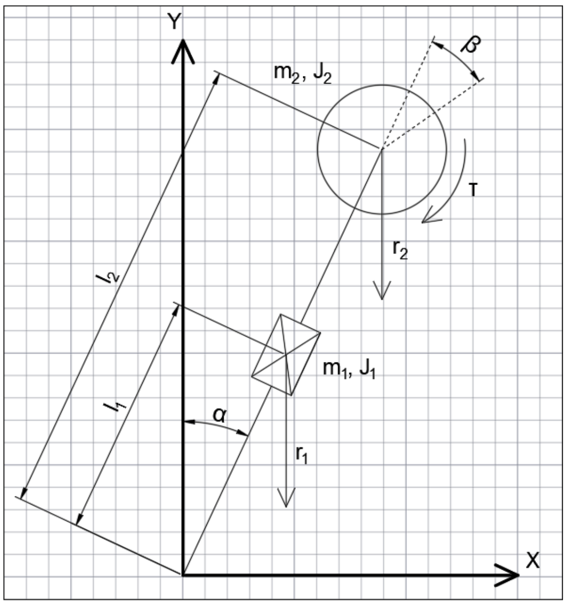

Fig. 1. Inverted reaction wheel pendulum

where m_1 – mass of the construction without mass of the reaction wheel, m_2 – mass of the reaction wheel, l_1 – distance of the centre of the mass m_1 from the coordinate system origin, l_2 – distance of the centre of the mass m_2 from the coordinate system origin, J_1 – moment of inertia of the mass m_1, J_2 – moment of inertia of the mass m_2, r_1 – length of the gravity force arm acting on the mass m_1, r_2 – length of the gravity force arm acting on the mass m_2, α – angle of the pendulum, β – angle of the reaction wheel, τ – torque applied on reaction wheel by DC motor.

The stabilization mechanism is based on Newton's second law for rotational motion. According to this principle, the derivative of the angular momentum of a rigid body equals the torque acting on it. It is given by equation (1).

$$\frac{dL(t)}{dt} = J \cdot \frac{d\omega(t)}{dt} = \tau(t) \tag{1}$$

where $L(t)$, J, $\omega(t)$, $\tau(t)$ are the angular momentum, the moment of inertia, the angular velocity and the torque, respectively.

When the equilibrium of the system is disturbed, voltage is applied to the DC motor and the torque of the motor is applied to the reaction wheel causing it to accelerate. According to equation (1), the torque acting on the reaction wheel is created. The reaction wheel in turn according to Newton's third law applies the equal amount of the torque to the DC motor, but in the opposite direction. Because it is mounted to motorcycle body, the torque acts on the whole construction, bringing it back to the vertical position [1].

By controlling this reaction torque the motorcycle body can be balanced. The torque of the reaction wheel DC motor should correspond to the moment of gravitational force acting on the vehicles center of mass when deflected from equilibrium point [3].

2.2 Nonlinear model of SBEM

For the mathematical model of SBEM Euler-Lagrange equations are applied [8]. A description of the total kinetic and potential energy is required. It is given by the following equations:

$$E_{tp}(t) = g \cdot cos\alpha(t) \cdot (m_1 \cdot l_1 + m_2 \cdot l_2) \tag{2}$$

$$\begin{aligned} E_{tk}(t) = {} & \frac{1}{2} \cdot (m_1 \cdot l_1^2 + m_2 \cdot l_2^2 + J_1 + J_2) \cdot \left(\frac{d\alpha(t)}{dt}\right)^2 \\ & + J_2 \cdot \frac{d\alpha(t)}{dt} \cdot \frac{d\beta(t)}{dt} + \frac{1}{2} \cdot J_2 \cdot \left(\frac{d\beta(t)}{dt}\right)^2 \end{aligned} \tag{3}$$

where $E_{tp}(t)$, g, $E_{tk}(t)$ are the total potential energy, the gravity constant and the total kinetic energy of the system, respectively.

Euler-Lagrange equations use Lagrangian given by equation (4). It is a difference between the total kinetic energy and the total potential energy.

$$L\left[\alpha(t),\beta(t),\frac{d\alpha(t)}{dt},\frac{d\beta(t)}{dt},t\right] = E_{tk}(t) - E_{tp}(t) \tag{4}$$

where L is Lagrangian and t is time.

Euler-Lagrange equations are given by the following equation:

$$\frac{\partial L}{\partial y_i(x)} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_i(x)}\right) = \tau_i(t) \tag{5}$$

where $y_i(x)$ is the function depended on variable x and $\tau_i(t)$ is the generalized torque in the $y_i(x)$ direction.

Lagrangian is given by equation (6).

$$\begin{gathered} L = \frac{1}{2}\cdot(m_1\cdot l_1^2 + m_2\cdot l_2^2 + J_1 + J_2)\cdot\left(\frac{d\alpha(t)}{dt}\right)^2 + J_2\cdot\frac{d\alpha(t)}{dt}\cdot\frac{d\beta(t)}{dt} + \\ +\frac{1}{2}\cdot J_2\cdot\left(\frac{d\beta(t)}{dt}\right)^2 - g\cdot cos\alpha(t)\cdot(m_1\cdot l_1 + m_2\cdot l_2) \end{gathered} \tag{6}$$

Euler-Lagrange equations are given by the following equations:

$$\begin{cases} \dfrac{\partial L}{\partial \alpha(t)} - \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\alpha}(t)}\right) = 0 \\ \dfrac{\partial L}{\partial \beta(t)} - \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\beta}(t)}\right) = \tau(t) \end{cases} \tag{7}$$

where $\tau(t)$ is the torque provided by DC motor.

Using equations (6) and (7) the mathematical model of SBEM is derived:

$$\begin{cases} g\cdot sin\alpha(t)\cdot(m_1\cdot l_1 + m_2\cdot l_2) - (m_1\cdot l_1^2 + m_2\cdot l_2^2 + J_1 + J_2)\cdot\dfrac{d^2\alpha(t)}{d^2t} - J_2\cdot\dfrac{d^2\beta(t)}{d^2t} = 0 \\ J_2\cdot\dfrac{d^2\alpha(t)}{d^2t} + J_2\cdot\dfrac{d^2\beta(t)}{d^2t} = \tau(t) \end{cases} \tag{8}$$

The system of equations (8) can be reduced to a single equation of the following form:

$$\frac{d^2\alpha(t)}{dt^2} = -\frac{1}{m_1\cdot l_1^2 + m_2\cdot l_2^2 + J_1}\cdot\tau(t) + \frac{g\cdot(m_1\cdot l_1 + m_2\cdot l_2)}{m_1\cdot l_1^2 + m_2\cdot l_2^2 + J_1}\cdot\sin\alpha(t) \tag{9}$$

2.3 Linearization of SBEM model

The mathematical model of SBEM is nonlinear due to trigonometric function *sinα(t).* Because of that, it is difficult for a wide operating range using a PID controller. The process of linearization is needed [9].

The first step is to determine the static duty point that sets out values of system parameters in a steady-state. The static duty point is described as follows:

$$S_0 = \left(\alpha_0, \frac{d^2\alpha_0}{dt^2}, \tau_0\right) = (0,0,\tau_0) \tag{10}$$

where S_0 is the static duty point, α_0 is the angle of the pendulum in the steady-state and τ_0 is the torque applied by DC motor in the steady-state.

The value of the torque in the steady-state can be derived from equation (9) using values of parameters from equation (10):

$$f(0,0,\tau_0) = -\frac{1}{m_1 \cdot l_1^2 + m_2 \cdot l_2^2 + J_1} \cdot \tau_0 + \frac{g \cdot (m_1 \cdot l_1 + m_2 \cdot l_2)}{m_1 \cdot l_1^2 + m_2 \cdot l_2^2 + J_1} \cdot \sin 0 = 0 \tag{11}$$

where f is the function which describes the equation of static characteristic.

The value of the torque in a steady-state equals zero. The next step is to expand equation (9) into a Taylor series with the operating point and neglect higher-order terms. The linearized equation of SBEM mathematical model is given by equation (12).

$$\left(\frac{d^2\alpha(t)}{dt^2} - 0\right) = -\frac{1}{m_1 \cdot l_1^2 + m_2 \cdot l_2^2 + J_1} \cdot (\tau(t) - \tau_0) + \\ + \frac{g \cdot (m_1 \cdot l_1 + m_2 \cdot l_2)}{m_1 \cdot l_1^2 + m_2 \cdot l_2^2 + J_1} \cdot (\alpha(t) - \alpha_0) \tag{12}$$

The final step is to use increment variables to describe equation (12). It is given by equation (13).

$$\Delta \frac{d^2\alpha(t)}{dt^2} = -\frac{1}{m_1 \cdot l_1^2 + m_2 \cdot l_2^2 + J_1} \cdot \Delta\tau(t) + \frac{g \cdot (m_1 \cdot l_1 + m_2 \cdot l_2)}{m_1 \cdot l_1^2 + m_2 \cdot l_2^2 + J_1} \cdot \Delta\alpha(t) \tag{13}$$

The nonlinear (9) and linearized (13) mathematical model of SBEM was implemented in Matlab environment.

2.4 Simulation of SBEM model

The models implemented in Matlab were tested to determine the accuracy of the linearized model of SBEM. For this purpose, the unit-step responses of derived models were examined and the linearization errors were calculated. The linearization error is the difference between nonlinear and linearized model response.

All values of models parameters are: m_1 = 0.59 kg, m_2 = 0.11 kg, l_1 = 0.06 m, l_2 = 0.12 m, J_1 = 3.26e-3 kg·m^2, J_2 = 403e-6 kg·m^2, g = 9.81 m/s^2.

The unit-step responses of the models are shown in Fig. 2.

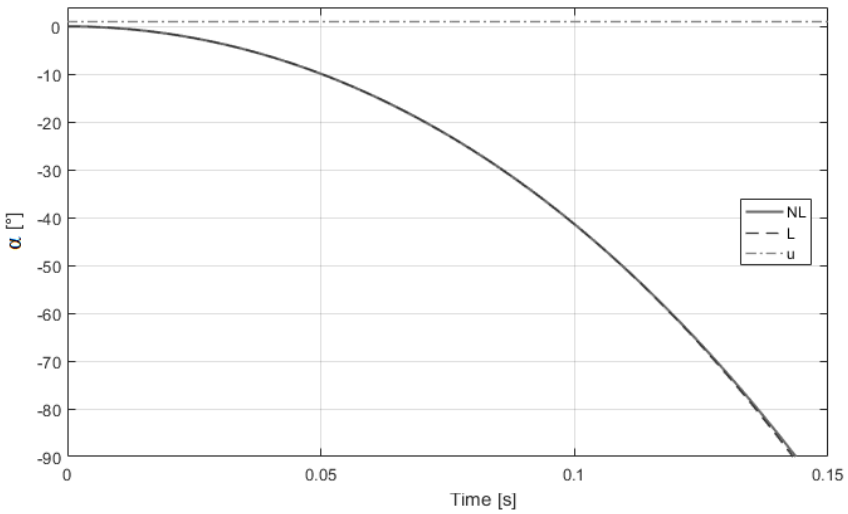

Fig. 2. Unit-step responses of nonlinear and linearized models (*NL* – nonlinear model, *L* – linearized model, *u* – unit-step input)

In Fig. 2 an angle of -90° indicates a situation in which the motorcycle is completely tilted to one side. Linearization error is illustrated in Fig. 3.

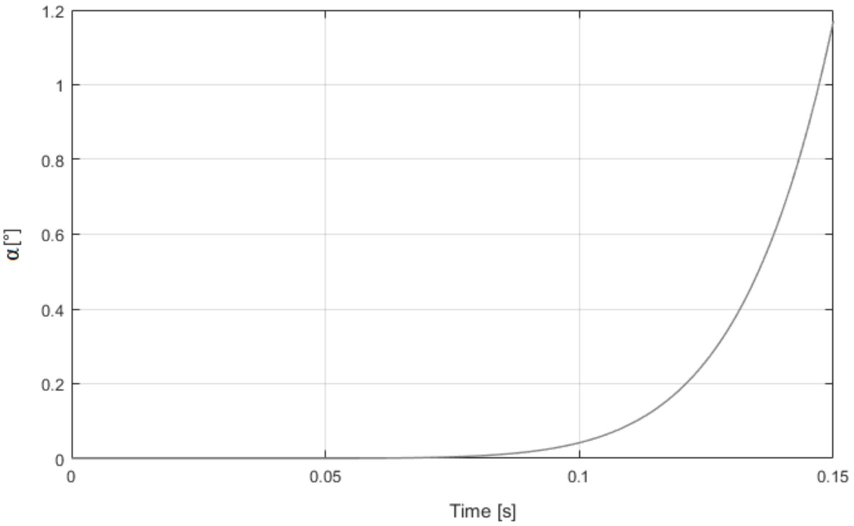

Fig. 3. Linearization error

The linearization errors are due to the fact that the linearized model is only an approximation of a nonlinear model in the specified area of a duty point. This means that it will only behave like a nonlinear model within a certain range of deviations from the duty work point. The further away from the duty point, the greater the linearization error.

3 Design of control system

3.1 Structure of the control system

The control structure of the vertical position of the SBEM is illustrated in Fig. 4.

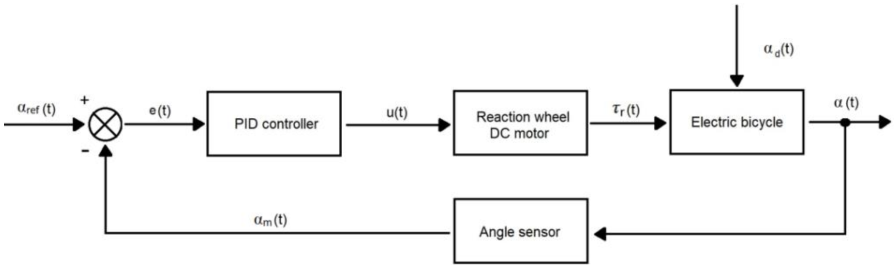

Fig. 4. SBEM control system

where $\alpha_{ref}(t)$ – reference angle, $e(t)$ – error signal, $u(t)$ – voltage signal, $\tau_r(t)$ – reaction torque, $\alpha_d(t)$ – equilibrium disturbance, $\alpha(t)$ – angle, $\alpha_m(t)$ – measured angle.

The reference value is the vertical angle of SBEM. In this case, the reference value equals 0°. The error signal is created by subtracting the reference angle and the measured angle. Next, the PID controller generates a control signal fed to the reaction wheel DC motor. When the reaction wheel starts spinning, the reaction torque is generated and the electric bicycle is balanced. The actual angle of SBEM is measured by an angle sensor. The equilibrium disturbance is an external force which causes deviation from the vertical axis of SBEM. Two types of disturbances were considered: impulse and constant. The first corresponds to the application of the force for a short time. It is an equivalent of a short push. The second type corresponds to the application of force for a long time. It can be regarded as placing the mass on one side of SBEM.

3.2 Model of reaction wheel DC motor

To design a control system in Matlab the DC motor mathematical model is needed. To obtain the mathematical model the equivalent scheme of the DC motor is used (Fig. 5).

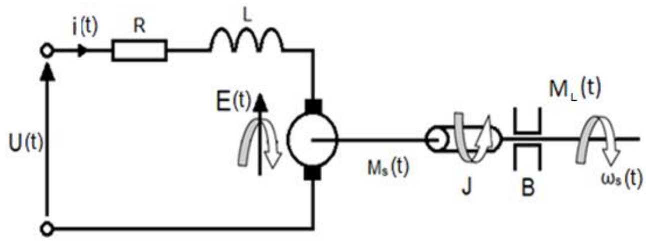

Fig. 5. DC motor equivalent scheme

where *U(t)* – rotor power supply voltage, *i(t)* – motor current, *R* – rotor winding resistance, *L* – rotor winding inductance, *E(t)* – electromotive force of induction, $M_s(t)$ – rotor torque, *J* – rotor shaft moment of inertia, *B* – viscous friction coefficient, $M_L(t)$ – load torque, $\omega_s(t)$ – angular velocity.

Mathematical model adequate to DC motor scheme (Fig. 5) is given by equations (14).

$$\begin{cases} U(t) = R \cdot i(t) + L \cdot \dfrac{di(t)}{dt} + k_e \cdot \omega_s(t) \\ k_m \cdot i(t) = J \cdot \dfrac{d\omega_s(t)}{dt} + B \cdot \omega_s(t) + M_L(t) \end{cases} \tag{14}$$

where k_e is the electromotive constant and k_m is torque constant.

To implement the model of the DC motor in Matlab the Laplace transform is applied. By using the Laplace transform in equations (14) and reorganizing the output equation, the DC motor model is obtained.

$$\Omega_s(s) = \frac{1}{J \cdot s + B} \cdot \left(\frac{U(s) - k_e \cdot \Omega_s(s)}{R + L \cdot s} \cdot k_m - M_L(s) \right) \tag{15}$$

The equation (15) was implemented in Matlab environment. All values of DC motor parameters are: R = 5.71 Ω, L = 380e-6 H, k_m = 0.80 N·m/A, k_e = 0.13 V·s/rad, J = 3.6e-6 kg·m/s^2, B = 3.69e-4 N·m·s/rad, M_L = 0.0 N·m.

3.3 Tuning of PID controller

To tune the PID controller the second Ziegler-Nichols tuning method is used. The required parameters to calculate the PID controller such as critical gain K_{cr} and period of sustained oscillation T_{cr} are indicated. Using these two parameters, the parameters of the PID controller are computed and presented in Table 1.

Table 1. Parameters for PID controller

Controller structure	K_p	T_i	T_d
P	7.0	-	-
PI	6.3	0.63	-
PID	8.4	0.38	0.09

where K_p, T_i, T_d are proportional gain, integral time and derivative time, respectively.

The second method of tuning the PID controller is based on simulation tests. To determine controller parameters, the following quality control indicators were considered:

- permissible control error < 1% of disturbance amplitude,
- settling time t_s below < 1s – it is the time required by the response to reach and steady below $|\varepsilon| = 1°$,
- permissible overshoot < 10% – calculated as a ratio of the absolute value of second peak A_2 to the absolute value of first peak A_1 in percentages:

$$OS_{\%} = \left|\frac{A_2}{A_1}\right| \cdot 100\% \tag{16}$$

All parameters needed to calculate quality control indicators are shown in Fig. 6.

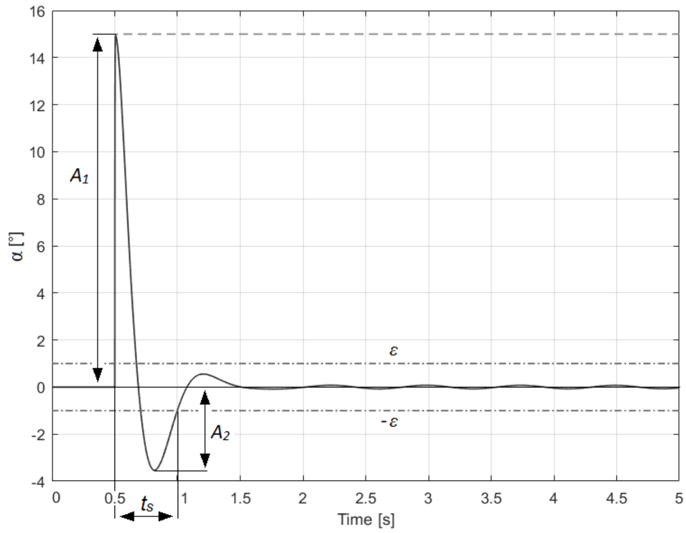

Fig. 6. Parameters of step response needed to calculate quality control indicators

On the basis of the simulation tests, the parameters of the PID controller that meet the mentioned requirements were determined. These values are shown in Table 2.

Table 2. Parameters for PID controller

Controller structure	K_p	T_i	T_d
P	31.8	-	-
PI	32.2	0.04	-
PID	32.0	0.06	0.1

4 Simulation tests and results analysis

In this section, simulation tests were carried out. By analyzing simulation results the most appropriate structure of the PID controller was chosen. The PID structure was determined on the basis of tests.

Simulation tests of the control system of the nonlinear and linearized model of SBEM using two sets of PID controller parameters were conducted. Two types of disturbances: step and impulse were examined. The amplitude of 15° was taken as the maximum deviation. The control results of the second Ziegler-Nichols tuning method are shown in Fig. 7.

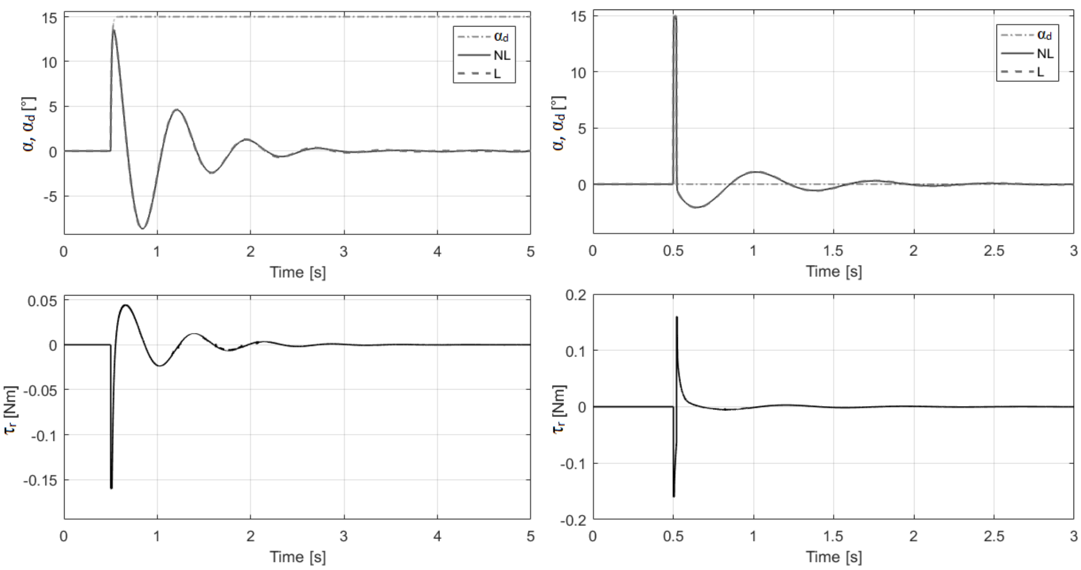

Fig. 7. Control results of the second Ziegler-Nichols tuning method

The system was found to be stable on the basis of the research conducted. However, the required control quality indicators are not met. All values of the mentioned quality indicators are shown in Table 3.

The results of experimental PID controller tuning method are shown in Fig. 8.

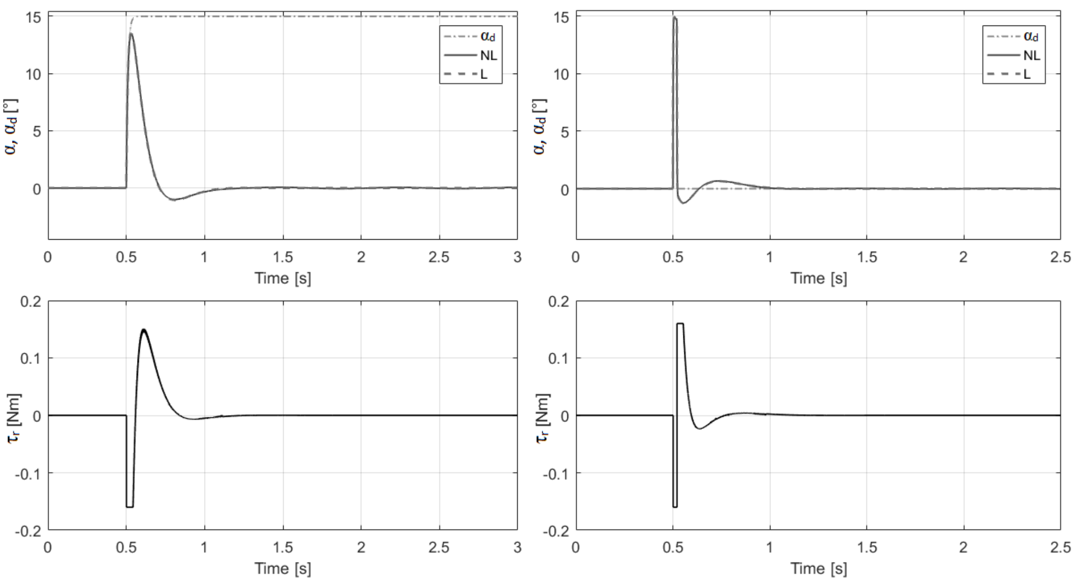

Fig. 8. Control results of the experimental tuning method

The system was found to be stable on the basis of the research conducted. The required control quality indicators are achieved. The shape of the reaction torque τ_r waveforms is caused by upper and lower limits of its value. These are the maximum torque values provided by the reaction wheel DC motor. All values of the mentioned quality indicators are shown in Table 3.

Table 3. Values of control quality indicators

Tuning method of PID	II Ziegler-Nichols		Experimental	
Input signal	Step	Impulse	Step	Impulse
Permissible control error [°]	0.08	0.01	0.05	0.02
Settling time [s]	1.53	0.23	0.31	0.27
Permissible overshoot [%]	64.23	13.78	7.46	8.33

5 Conclusions

In this paper, the processes of mathematical modelling and linearization of SBEM were presented. The implementation of the control system and control results analysis were done. Two methods of tuning the PID controller were verified. The operation of particular structures of the PID controller were compared. The most appropriate structure of the PID controller and values of its parameters were obtained. All quality control indicators were achieved using an experimental tuning method of the PID controller.

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