Equilibrium. Quarterly Journal of Economics and Economic Policy
Volume 15 Issue 3 September 2020
p-ISSN 1689-765X, e-ISSN 2353-3293
www.economic-policy.pl

ORIGINAL ARTICLE

Citation: Kot, S. M. (2020). Estimating the parameter of inequality aversion on the basis of a parametric distribution of incomes. *Equilibrium. Quarterly Journal of Economics and Economic Policy, 15*(3), 391–417. doi: 10.24136/eq.2020.018

Contact: skot@zie.pg.gda.pl; Gdansk University of Technology, Department of Economic Sciences, ul. Narutowicza 11/12, 80-233 Gdansk, Poland

Received: 02.03.2020; Revised: 04.08.2020; Accepted: 25.08.2020; Published online: 25.09.2020

Stanislaw Maciej Kot
Gdansk University of Technology, Poland
orcid.org/0000-0002-5875-6498

Estimating the parameter of inequality aversion on the basis of a parametric distribution of incomes

JEL Classification: *C18; D31; D63; I31*

Keywords: *income inequality; inequality aversion; estimation; income distribution*

Abstract

Research background: In applied welfare economics, the constant relative inequality aversion function is routinely used as the model of a social decisionmaker's or a society's preferences over income distributions. This function is entirely determined by the parameter, ε, of *inequality aversion*. However, there is no authoritative answer to the question of what the range of ε an analyst should select for empirical work.
Purpose of the article: The aim of this paper is elaborating the method of deriving ε from a parametric distribution of disposable incomes.
Methods: We assume that households' disposable incomes obey the generalised beta distribution of the second kind GB2($\underline{a,b,p,q}$). We have proved that, under this assumption, the social welfare function exists if and only if ε belongs to $(0,ap+1)$ interval. The midpoint ε_{mid} of this interval specifies the inequality aversion of the median social-decisionmaker.
Findings & Value added: The maximum likelihood estimator of ε_{mid} has been developed. Inequality aversion for Poland 1998–2015 has been estimated. If inequality is calculated on the basis of disposable incomes, the standard inequality–development relationship might be complemented by inequality aversion. The "augmented" inequality–development relationship reveals new phenomena; for instance, the stage of economic development might matter when assessing the impact of inequality aversion on income inequality.

Introduction

In this paper, we propose a new method of estimating the parameter, ε, of *the constant relative inequality aversion function* (CRIA) (Atkinson, 1970). In applied welfare economics, CRIA is the routinely-used mathematical tool of encompassing societal preferences over income distributions. The expected value of CRIA, i.e. *the social welfare function* (SWF), is the basic maximand of social policy. The parameter ε measures *inequality aversion,* i.e. the rate at which a society trades-off economic efficiency for income equality.[1] However, ε cannot be directly measured because it concerns *unobservable* social preferences.

In the literature, there is no consensus among economists concerning what empirical data can convincingly reflect a social attitude toward income inequality and how to elicit ε from such data. In Section 2, we present a review of some recent answers to these questions.

In this paper, we retrieve ε form the distribution of disposable income (DDI). The societal redistributive system transforms the distribution of market income (wages and capital interests) into DDI (market income minus tax, plus social transfers). Note that the c*urrent* redistributive policy has no impact on the current distribution of market income; the policy shapes only current DDI. Thus social inequality aversion manifests itself in the form of the current DDI.

To be more specific, suppose m competitive redistributive policies which guaranty the same maximum SWF, but they differ concerning the level of inequality aversion, $\varepsilon_1,\ldots,\varepsilon_m$ say. Thus the policies offer different solutions of the efficiency-equality trade-off. However, only one policy, say lth, wins such a competition, according to the legally binding rules of social choice, $l=1,\ldots,m$. One may ask the question: What would ε_l be if the current DDP was the result of the wining redistributive policy?

To answer this question, we assume that DDP obeys the generalised beta distribution of the second kind (GB2) (MacDonald, 1984). Then, SWF will be the expected value of CRIA, with respect to GB2. We prove that SWF exists if and only if ε lies in a finite interval. We propose the midpoint of this interval as the estimate of social aversion to inequality. We develop the maximum likelihood estimator of ε.

To assess the usefulness of our method to retrieve unobservable inequality aversion, we estimate the parameter ε and related normative characteristics for Poland for the years 2000–2015. We use micro-data on DDP from

[1] "How much efficiency and how much equality" is the fundamental dilemma of Economics (Okun, 1975).

the Polish Household Budget Surveys (PHBS). Then, we confront our empirical findings with relevant facts predicted by economic theories.

We organise the remainder of this paper as follows. At the beginning of Section 2, we introduce the basic welfare terms. Next, we review recent approaches to retrieving ε. In Section 3, we present the details of our method of estimating inequality aversion. Section 4 contains empirical results, namely the estimates of ε and related normative characteristics for Poland, for the years 2000-2015. In Section 5, we assess the usefulness of our method to retrieve inequality aversion. Here, we also verify some prominent economic hypotheses. Section 6 concludes.

Literature review

Welfare frameworks

Suppose that a positive valued random variable X describes income distribution.[2] The standard SWF is the mean value of personal welfare $u(x)$, where $u(x)$ is the utility of income x. When X is of the discrete type, with the probability mass function $P(X=x_i)=1/n$, $n<\infty$, SWF will have the form

$$SWF = \sum_{i=1}^{n} u(x_i)\,\frac{1}{n},\ i=1,\ldots n \tag{1}$$

(Lambert & Naughton, 2009). The authors interpret SWF (1) as "(…) a person's expected utility, measured from behind a 'veil of ignorance', which is specified in a thought experiment in such a way that the person may be identified with any one of the individuals populating the income distribution with the same probability."

When X is of the continuous type, with the density function $f(x)$ (X~$f(x)$, for short), SWF will have the form

$$SWF = \int_0^{\infty} u(x) f(x) dx \tag{2}$$

(Lambert & Naughton, 2009). Note that SWF (2) exists if and only if the integral on the right side is absolute convergent and finite, namely,

[2] Hereafter, we reserve capital letters for random variables and lower-case letters for the values of random variables.

$$\int_0^\infty |u(x)| f(x) dx < \infty \tag{3}$$

(Fisz, 1967, p. 64).

CRIA, whose single parameter ε is the object of our interest, has the form

$$u(x|\varepsilon) = \begin{cases} \frac{x^{1-\varepsilon}}{1-\varepsilon}, for\ \varepsilon \neq 1 \\ log\ x, for\ \varepsilon = 1 \end{cases} \tag{4}$$

(Atkinson, 1970), where *log x* is a natural logarithm of x. In the literature, the name 'inequality aversion' is commonly used for ε.

Geometrically, ε reflects the curvature of CRIA. When $\varepsilon<0$, $u(x|\varepsilon)$ is convex and represents an inequality-loving society. When $\varepsilon=0$, $u(x|\varepsilon)$ is linear and characterises an inequality neutral society. Such a society does not care about inequality, preferring one distribution X_1 over another X_2 if and only if under X_1 the mean income is higher than under X_2 (Lambert, 2001, p. 99). If $\varepsilon>0$, $u(x|\varepsilon)$ is strictly concave and represents an inequality-averse society.

It is worth adding that two functions $u_1(x)$ and $u_2(x)$ are *equivalent as utilities* if there exist constants α and $\beta>0$ such that $u_1(x)=\alpha+\beta u_2(x)$ for all x (Pratt, 1964). Actualy, Atkinson (1970) and other economists have used $u^*(x|\varepsilon)=\alpha+\beta u(x|\varepsilon)$, where $u(x|\varepsilon)$ has the form of (4). For $\varepsilon\neq 1$, the function $u^{**}(x|\varepsilon) = \frac{x^{1-\varepsilon}-1}{1-\varepsilon}$ guaranties convergence to logarithm case when $\varepsilon \to 1$.[3] It is easy to see that $u^{**}(x|\varepsilon)$ and $u(x|\varepsilon)$ (4) are equivalent as utilities when assuming $\alpha=-1/(1-\varepsilon)$ and $\beta=1$.

Atkinson (1970) proposed the normative ("ethical") index of inequality $A(\varepsilon,\mu)$

$$A(\varepsilon, \mu) = 1 - \frac{\mu_\varepsilon}{\mu}, \tag{5}$$

where μ is the mean income, and μ_ε is *the equally distributed equivalent income* (EDEI). EDEI is the income that if received by all individuals, provides the same value of SWF as the current distribution (Kolm, 1969; Atkinson, 1970; Sen, 1973, p. 42). We may recognise the normative index $A(\varepsilon,\mu)$ as the socially accepted level of income inequality.

[3] I am grateful to an anonimous referee for pointing out this fact.

In general, μ_ε is the solution to the equation: $u(\mu_\varepsilon)$=SWF. For utility function (4) and SWF (1), μ_ε has the form

$$\mu_\varepsilon = \begin{cases} \left(\frac{1}{n}\sum_{i=1}^{n} {x_i}^{1-\varepsilon}\right)^{1/(1-\varepsilon)}, for\ \varepsilon \neq 1 \\ \exp\left\{\frac{1}{n}\sum_{i=1}^{n} log\, x_i\right\}, \quad for\ \varepsilon = 1 \end{cases} \tag{6}$$

For utility function (4) and SWF (2), μ_ε is equal to

$$\mu_\varepsilon = \begin{cases} \left(\int_0^\infty x^{1-\varepsilon} f(x)dx\right)^{1/(1-\varepsilon)}, for\ \varepsilon \neq 1 \\ \exp\left\{\int_0^\infty \log x\, f(x)dx\right\}, \quad for\ \varepsilon = 1 \end{cases} \tag{7}$$

provided that the integrals on the right-hand side of (7) are absolute convergent and finite.

The trade-off between equality and economic efficiency (Okun, 1975) is apparent in *the abbreviated social welfare functions* (ASWF) (see Lambert, 2001, Chapter 5, for a full presentation). The Atkinson ASWF is equal to EDEI, namely,

$$\mu_\varepsilon = \mu(1 - A(\varepsilon, \mu)) \tag{8}$$

The following ASWF is the descriptive counterpart of (8), namely

$$\widetilde{SS} = \mu(1 - G), \tag{9}$$

where G is the Gini index of income inequality. The $\widetilde{SS}$ was proposed by Sheshinski (1972) and popularised by Sen (1973). Equations (8) and (9) show that efficiency, as measured by μ, can be traded-off for equity, as measured by 1-$A(\mu,\varepsilon)$, or 1-G. The disincentive effects of redistributive taxation can be more than offset by the gains to the poor (Lambert 2001: 107). The trade-off explains why politicians do not reduce inequality to the extent higher than that observed.

Recent methods of inequality aversion estimation

In typical applications of $A(\varepsilon,\mu)$ for the comparisons of inequality in distinct income distributions, an analyst assumes a fixed value for ε and uses this value to all compared distributions. However, little theoretical or empirical ground exists to impose such an approach (Aristei & Perugini, 2016). Moreover, there is no consensus among economists regarding the range of ε an analyst should select. The literature offers various methods of establishing ε.

In experimental economics, two approaches for retrieving ε can be observed (see Clark & D'Ambrosio, 2015, for a broader presentation). In the first approach, ε is elicited from data yielded by *the leaky bucket experiment* (Okun, 1975). When a transfer of an income, e.g., \$1, is made from a person with income x_1 (a rich person) to a person with income x_2 (a poor person), a certain fraction of it, say d, is lost because of administrative costs. The basis of eliciting ε is the extent of losses, or leakages, which are accepted by participants of an experiment.

Formally, the leaky bucket experiment consists in deriving the post-transfer SWF and equating it to the pre-transfer SWF. The rate, d, of leakage that preserves the initial SWF will be equal to

$$d = 1 - \left(\frac{x_2}{x_1}\right)^{\varepsilon} \tag{10}$$

(Atkinson, 1980). Note that d depends on the ratio x_2/x_1 of incomes. The participants assess an acceptable leakage d of income for various levels of the ratio. Inequality aversion ε is the solution to Eq. (10).

The leaky bucket experiments have usually provided relatively low estimates of ε. Amiel *et al.* (1999) experimented with large groups of students from various universities and found that the median of ε was between 0.1 and 0.22. Pirttilä and Uusitalo (2007) found the median of ε below 0.5 when performing the leaky bucket experiment in a representative survey of Finnish people.

In the second approach to eliciting ε, participants of an experiment choose between distinct income distributions in hypothetical societies. In research with Swedish students, Carlsson *et al.* (2005) found the median of ε between 1 and 2. Notably, 7% of respondents reported $\varepsilon<0$. Pirttilä and Uusitalo (2007) found the median ε larger than 3.

Experimental economics has provided ambiguous estimates of ε (see Levitt & List 2007, for a broader discussion). Beckman *et al.* (2004, p. 19) noted that the apparent shortcoming of the methods is "(…) what people say in response to hypothetical questions and what they actually do when income is at stake may be quite different." Moreover, the methods in question are impractical in a retrospective analysis of inequality aversion; current economic experiments cannot provide data on revealed preferences over *the past* income distributions unless independence of time is assumed.

In the literature, ε has also been retrieved from tax policies. Richter (1983), Vitaliano (1977), and Young (1987) have estimated ε based on *the equal sacrifice model*. This model assumes that income taxes are set such that the loss in individual utility is equated across all income levels, given a plausible utility function of income. Suppose *t(x)* denotes a tax schedule that expresses the tax liability of a person with income x. The tax schedule is *an equal absolute sacrifice* for the utility function *u(x)* if and only if, for all x and some constant $c>0$, the following identity holds:

$$u(x) - u[x - t(x)] = c \tag{11}$$

For utility function (1), Cowell and Gardiner (1999) demonstrated that (11) can be expressed as

$$-\ln[1 - t'(x_i)] = \varepsilon ln \frac{x_i}{x_i - t(x_i)} \tag{12}$$

where *t'* is the first derivative of *t*. The ordinal least squares method can be applied to estimate ε, assuming a null intercept.

Stern (1977) used (12) and found ε=1.97 for the UK fiscal year 1973/74. Cowell and Gardiner (1999) presented lower estimates of ε for the UK, namely, 1.43 and 1.41 for the respective fiscal years 1998/99 and 1999/2000 when using data on personal income tax. The estimates of ε are substantially lower (1.29 and 1.28, respectively) when based on aggregated data from fiscal files and National Insurance Contributions.

Young (1990) fit model (11) to federal US income taxes in the years 1957, 1967, and 1977 and obtained values of ε equal to 1.61, 1.52, and 1.72, respectively. These estimates are much higher than those provided by the leaky bucket experiment. Young's estimates of ε for other countries are also higher than 1, for instance, 1.59 for Japan in 1987, 1.63 for West Germany in 1984, and 1.40 for Italy in 1987. Gouveia and Strauss (1994) estimated the equal sacrifice model for the United States for 1979 and 1989 and found ε values between 1.72 and 1.94. Aristei and Perugini (2016) estimated the model in question for former communist countries and found ε ranged from 0.93 to 1.68.

However, the estimation of ε based on the equal sacrifice criterion poses some problems. Young (1990) and Mitra and Ok (1996) have demonstrated that the criterion may be violated in practice. Also, the equal sacrifice model does not account for the possible reduction of income inequality by social transfer policies.

Lambert *et al.* (2003) estimate countries' inequality aversion assuming the *natural rate of subjective inequality* (the NRSI hypothesis). The authors ask the following question: What would a country-specific ε be if subjective inequality were established at a given level A_0?

Because the authors measure subjective inequality by the Atkinson index $A(\varepsilon,\mu)$ (5), ε will be the solution to the equation: $A(\varepsilon,\mu)=A_o$. For $\varepsilon\neq1$ and μ_ε (6), one can express this equation as

$$1-\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{1-\varepsilon}\right)^{1/(1-\varepsilon)}=A_o \tag{13}$$

Given A_o, and country's incomes x_i, $i=1,\ldots,n$, Lambert *et al.* (2003) solve (13), with respect to ε, numerically. The authors obtain the estimates of ε for 96 countries while assuming various levels of NRSI. The estimates varied from 0.157 (A_0=0.1) to 139.3 (A_0=0.4).

The main shortcoming of this method is that the estimates of ε are *conditional* on NRSI. In other words, Eq. (13) does not specify a single value of ε, but *the family* $\{\varepsilon(A)\}_{A\epsilon I}$ of ε, indexed by $A\epsilon I$, where I is the set of unknown NRSI.

Lambert *et al*. (2003) predict the following empirical consequence of NRSI: "We present evidence consistent with the existence of a natural rate of subjective inequality by verifying that countries with low (high) tolerance for inequality have low (high) inequality as measured by the Gini coefficient as well." We shall verify the NRSI hypothesis in Section 4.2.

Kot (2017) proposed the method of recovering parameter ε from the psychophysical measurement of household welfare, originated by Kot (1997). In a survey, a respondent is supposed to imagine a situation in which his/her actual household income (y) increases (decreases) by \$1, \$2, etc., until he/she would notice a *just perceptible change* in welfare. Denoting by t_l and t_u, the respective lower- and upper-income thresholds, the parameter ε of the utility function (4) is the solution to the nonlinear equations

$$\begin{cases} pt_l^{1-\varepsilon}+(1-p)t_u^{1-\varepsilon}-y^{1-\varepsilon}=0, for\ \varepsilon\neq1 \\ t_l^{p}t_u^{1-p}-y=0, for\ \varepsilon=1 \end{cases} \tag{14}$$

where $0<p<1$.

Kot (2017) developed criteria for the predetermined selection of version (14), namely, $\varepsilon<0$, $\varepsilon=0$, $0<\varepsilon<1$, $\varepsilon=1$ and $\varepsilon>1$, based on thresholds t_l, t_u, and y, for all p. Eq. (14) can be solved numerically. Assuming p=0.5[4], Kot

[4] Parameter p is necessary to obtain a unique solution of Eq. (9), for $\varepsilon\neq1$. Since $u(y)$ is somewhere between $u(t_l)$ and $u(t_u)$, p=0.5 is justified by the maximum entropy criterion.

(2017) estimated ε using archival statistical data from the survey conducted among Polish households by The Public Opinion Research Centre in October 1999. The author found that Polish households are predominantly inequality averse. Inequality aversion larger than 1 dominates other levels of ε. Only 2 per cent of households reveal inequality aversion in the interval (0,1). Notably, 7.64% of households show $\varepsilon<0$; namely, they appeared to be inequality-lovers. This figure is surprisingly close to Carlsson's et al. (2005) 7% of inequality loving respondents. The existence of non-positive inequality aversion suggests the violation of the Principle of Transfers (see also Amiel et al. 2004).

Conducting a specially designed survey is a practical shortcoming of Kot's method. Moreover, further investigations are necessary to specify the 'share' parameter *p*.

Estimating ε when disposable income obey the GB2 distribution

Let the positive valued continuous random variable *X,* with the density function *f(x),* describe DDP. Suppose *m* competitive redistributive policies which provide the same maximum SWF, but they differ concerning the level of inequality aversion $\varepsilon_1,\ldots,\varepsilon_m$. In other words, every policy offers the different solution of the efficiency-equality trade-off. Thus, there could be *m* competing DDPs, $f(x|\varepsilon_1),\ldots,f(x|\varepsilon_m)$, as the result of redistribution. Every *i*th policy promises the same social welfare SWF (2) equal to

$$SWF = \begin{cases} \frac{1}{1-\varepsilon_i}\int_0^\infty x^{1-\varepsilon_i} f(x|\varepsilon_i)dx, for\ \varepsilon_i \neq 1 \\ \int_0^\infty logx f(x|\ \varepsilon_i)dx, for\ \varepsilon_i = 1 \end{cases} \quad , i=1,\ldots,m, \quad (15)$$

under constraint (3). However, only one policy, *l*th say, $l=1,\ldots,m$, wins the competition, according to the legally binding rules of social choice. We may recognise ε_l as the social norm of redistribution. Thus the *current* DDP, the 'winner', with the density function $f(x|\varepsilon_l)$, embodies the redistributive norm ε_l. We ask the following question: What would the level of ε_l be if the current DDP was $f(x|\varepsilon_l)$?

Lerner (1944, p.9) was the first to propose the mean value solution to the problem of assigning a utility function to a person while assuming a state of ignorance.

To answer this question, we assume that DDP obeys the GB2 distribution with the density function

$$f(x) = \frac{ax^{ap+1}}{b^{ap}B(p,q)\left[1+\left(\frac{x}{b}\right)^{a}\right]^{p+q}}, x>0 \tag{16}$$

where a, *b, p, q* are positive parameters, and *B(p,q)* is Eulers' Beta function (McDonald, 1984). We also assume that the mean of *X* exists, namely, the condition $aq>1$ holds (Kleiber & Kotz, 2003, p. 188).

The GB2 distribution is now widely acknowledged to provide an excellent description of income distributions while including many other models as particular or limiting cases (Jenkins, 2007a). The GB2 with a=1 is the beta distribution of the second kind. When p=1, the GB2 takes the form of the Burr (1942) XII-type or the Singh-Maddala (1976) distribution. The GB2 with q=1 is the Burr (1942) III-type, or the Dagum (1977) distribution. When p=q =1, GB2 will become the log-logistic or the Fisk (1961) distribution. Also, the log-normal distribution (Aitchison & Brown, 1957) is a limiting case of the GB2 with a=1 and $q\to\infty$.

Proposition 1. Suppose $u(x|\varepsilon)$ is given by (4), for $\varepsilon\neq 1$, and $f(x|\varepsilon)$ has the form (16). Let the mean income in the GB2 exist. Then, SWF (15) exists if and only if $\varepsilon \in (0,ap+1)$.

Proof: For proof, it is sufficient to demonstrate that inequality (3) holds. Integral (3) can be expressed as

$$\int_0^\infty \left|\frac{x^{1-\varepsilon}}{1-\varepsilon}\right| \frac{ax^{ap+1}}{b^{ap}B(p,q)\left[1+\left(\frac{x}{b}\right)^{a}\right]^{p+q}}dx = \frac{1}{|1-\varepsilon|}\int_0^\infty x^{1-\varepsilon}\frac{ax^{ap+1}}{b^{ap}B(p,q)\left[1+\left(\frac{x}{b}\right)^{a}\right]^{p+q}}dx.$$

The integral on the right side specifies the partial/negative moment $E_f[X^{1-\varepsilon}]$ of order 1-ε of the GB2 distribution. Kleiber (1997) showed that the moment exists if and only if $\varepsilon \in (max\{0, 1\text{-}aq\}, ap+1)$. As $1\text{-}aq<0$, $max\{0, 1\text{-}aq\}=0$. Then, we obtain $\varepsilon \in (0,ap+1)$. QED.

Proposition 1 states that a social decisionmaker would have inequality aversion within the interval (0,ap+1) if and only if he/she performed a conclusive appraisal of social welfare, namely, if and only if he/she operated with a finite SWF. Thus the proposition excludes unrealistic policies, which would promise infinite social welfare.

When a decisionmaker acts 'behind a veil of ignorance', ε will have the uniform distribution within the interval $(0,ap+1)$.[5] Aristei and Perugini (2016) argue that the ε value revealed by redistributive policies should correspond to the preferences of the voter in the *median position* of the inequality aversion ladder. The median position corresponds to the midpoint of the uniform distribution of inequality aversion within $(0,ap+1)$, i.e.

$$\varepsilon_{mid} = \frac{1}{2}(ap + 1) \tag{17}$$

We propose ε_{mid} (17) as the estimate of socially tolerable inequality aversion.[6]

We can get the midpoint estimate of inequality aversion also for the particular cases of the GB2 distribution. For the Dagum distribution, the midpoint formula (17) is valid. For the Singh-Maddala distribution and the Fisk distribution, we get $\varepsilon_{mid} = \frac{1}{2}(a + 1)$. For the beta distribution of the second kind, we get $\varepsilon_{mid} = \frac{1}{2}(p + 1)$.

We can derive the maximum likelihood (ML) estimator of ε_{mid} using the ML estimators of the parameters of the GB2 distribution (16).

Proposition 2. Let the random variables *A* and *P* be the ML estimators of the parameters *a* and *p* of the GB2 distribution (16). Let cov_{ap} be the covariance between *A* and *P*. Then, the ML estimator $\hat{\varepsilon} = \frac{AP+1}{2}$ of ε_{mid} (17) will have the mean equal to

$$E[\hat{\varepsilon}] = \frac{1}{2}(ap + cov_{ap} + 1) \tag{18}$$

and the standard deviation equal to

$$D[\hat{\epsilon}] = \frac{1}{2}\left\{a\sigma_p^2 + p^2\sigma_a^2 + 2ap \cdot cov_{ap} + [cov_{ap}]^2\right\}^{1/2}, \tag{19}$$

where σ_a^2 and σ_p^2 are the variances of *A* and *P*, respectively.

[5] Note that such a decisionmaker is in the state of maximum entropy. The uniform distribution on the interval $[a,b]$ is the maximum entropy distribution among all continuous distributions which are supported in the interval $[a, b]$ (Cover & Thomas, 1991, p. 269).

[6] Kot (2012, p. 81) derived formula (17) from the mathematical conditions of the existence of EDEI (7) in the GB2 distribution.

Proof. The ML estimators A and P have the asymptotic normal distribution, namely, $A \sim N(a,\sigma_a)$ and $P \sim N(p,\sigma_p)$, respectively. Ware and Lad (2003) developed the moment-generating function of the product $Z=X_1 \cdot X_2$ of two correlated and normally distributed random variables, i.e. $X_1 \sim \mathrm{N}(\mu_1,\sigma_1)$ and $X_2 \sim \mathrm{N}(\mu_2,\sigma_2)$. The authors obtained $E[Z]=\mu_1\mu_2+\rho\sigma_1\sigma_2$ and $D^2[Z]=\mu_1^2\sigma_2^2+\mu_2^2\sigma_1^2+2\rho\mu_1\mu_2\sigma_1\sigma_2+\rho^2\sigma_1^2\sigma_2^2$, where ρ is the coefficient of the correlation between X_1 and X_2. As $cov_{ap}=\rho\sigma_a\sigma_p$, we get (18) and (19). QED.

The distribution of the product $A \cdot P$ is crucial for making statistical inference concerning ε. Aroian *et al.* (1978) demonstrated that if either μ_1/σ_1 or μ_2/σ_2 or both approach infinity then $Z=X_1X_2$ will be asymptotically normal.[7] This observation justifies Proposition 3:

Proposition 3. If either a/σ_a or p/σ_p, or both, tend to infinity, $\hat{\varepsilon}$ will have the asymptotic normal distribution with the mean (18) and the standard deviation (19).

Proposition 3 enables obtaining the asymptotic confidence interval for ε.

For the sake of convenience, we shall refer to the proposed method of estimating inequality aversion as *the parametric method* (PM).

Empirical results for Poland for 2000–2015

Estimates of GB2 distribution

We estimate the parameters of the GB2 distribution using statistical microdata data from the PHBS 2000–2015. The household monthly disposable incomes, in constant 2010 prices, are adjusted by household sizes, which provides incomes per capita. We omit null and negative incomes. We use household sizes as weights.

We estimate the parameters of the GB2 distribution by the ML method by using our programme written in Fortran 99 because the gb2fit Stata module (Jenkins, 2007b) failed to converge for some years. We calculate the matrix of variances–covariances using Brazauskas' (2002) exact formula for the Fisher information matrix. The results are presented in Table 1.

Assessing goodness of fit of the GB2 distribution poses a severe problem. We apply the Pearson chi-squared test using 20 equiprobable cells. However, Chernoff and Lehmann (1954) demonstrated that the test is not χ^2

[7] Recently, Cui *et al.* (2016) obtained the exact distribution of $Z=X_1 \cdot X_2$ where the generalised Bessel function of the second kind is involved.

and depends on the true values of the parameters when applying the ML method for ungrouped (raw) data. D'Agostino and Stephens (1986, p. 68) noticed: "All that can be said in general is that the correct critical points fall between those of $\chi^2(k\text{-}h\text{-}1)$ and those of $\chi^2(k\text{-}1)$.", where k is the number of cells and h is the number of estimated parameters.

In our case, the critical values of the test are $\chi^2(20\text{-}1)=30.144$ and $\chi^2(20\text{-}4\text{-}1)=24.966$, for the 5% significance level. Thus the chi-squared test prescribes rejecting the GB2 distribution as the theoretical model of Polish DDP. This result is typical in applications involving large sample sizes (McDonald, 1984).[8] According to our knowledge, other tests of goodness of fit, for example, the tests based on empirical distribution functions, have not been established for composite hypotheses concerning the GB2 distribution.

We can check the validity of the GB2 parameter estimates indirectly by comparing some empirical characteristics of DDP with their GB2 predictions. Table 2 shows the results of the comparison of the mean, the Gini index and $\widetilde{SS}$, i.e. the Sheshinski-Sen ASWF (9).

Examining Table 2 shows that GB2 distribution predicts the selected characteristics of DDP quite accurately. Regression functions, presented in Table 3, confirm this qualification.

Examining Table 3 shows that the GB2 distribution predicts the empirical characteristics of DDP almost perfectly because the values of the adjusted R^2 are very close to one. It is worth adding that the time series of the characteristics do not exhibit linear trends. Thus we may neglect the effect of a 'hidden third variable' ('Year') on R^2.

The estimates of inequality aversion and related normative characteristics

Having the estimates of GB2 parameters presented in Table 1, we calculate the mid-point estimates $\hat{\varepsilon}$ of inequality aversion (18), and its standard errors $D[\hat{\varepsilon}]$ (19). As the ratios $\hat{\varepsilon}/D[\hat{\varepsilon}]$ are very large; we can calculate the bounds of 95% confidence intervals, according to Proposition 3. Table 4 and Fig. 1 show the results of the calculations.

Examining Table 4 and Fig. 1 shows two remarkable features of inequality aversion. First, the estimates of ε for Poland are greater than zero. Thus Polish society was inequality averse in the years 2000–2015. Moreover, all estimates of ε are greater than one. Thus our method of retrieving ε provides higher levels of inequality aversion than those offered by the leaky

[8] Bandourian *et al.* (2003) fitted the GB2 distribution to income data of 23 countries and 82 country-year cases. The chi-squared test rejected this distribution in all but five cases.

bucket experiments discussed in Section 2. Second, inequality aversion varies over time. According to our model of the competitive redistributive policies, every year, a society may promote a distinct policy, according to current challenges of an economic and social environment.

Table 5 presents some normative characteristics of DDP.

Besides the Atkinson ASWF, μ_ε, and the Atkinson inequality index, $A(\varepsilon,\mu)$, Table 5 shows two additional characteristics, namely the absolute and relative benchmark incomes x^* and z^*. Hoffman (2001) observed that a small increase in low incomes decreases inequality, whereas a small increase in high incomes enhances inequality. Therefore, there must exist a specific income level, x^*, which separates these effects. The author referred to x^* as *the relative poverty line* or *the dividing line* between the rich and the poor.

Lambert and Lanza (2006) proved the existence of x^* for a general class of inequality indices. The authors call x^* *the absolute benchmark level of income*. The *relative benchmark level of income* equals to $z^*=x^*/\mu$. When one measures inequality by the Atkinson index (5), the benchmark income x^* has the following form

$$x^* = \begin{cases} \mu(1-A(\mu,\varepsilon))^{(\varepsilon-1)/\varepsilon} = \mu\left(\frac{\mu_\varepsilon}{\mu}\right)^{(\varepsilon-1)/\varepsilon}, for\ \varepsilon \neq 1 \\ \mu, for\ \varepsilon = 1 \end{cases} \tag{20}$$

Therefore, the relative benchmark, z^*, has the form

$$z^* = \begin{cases} (1-A(\mu,\varepsilon))^{(\varepsilon-1)/\varepsilon} = \left(\frac{\mu_\varepsilon}{\mu}\right)^{(\varepsilon-1)/\varepsilon}, for\ \varepsilon \neq 1 \\ 1, for\ \varepsilon = 1 \end{cases} \tag{21}$$

(Lambert & Lanza, 2006).

It is worth adding that x^* does not seem to be a right candidate for a poverty line as Hoffman's (2001) term 'the relative poverty line' suggests. Kot (2009) argues that a poverty line, z, should satisfy the inequality $z \leq$ EDEI. A policy operating with $z >$ EDEI would promise eradication of inequality on the price of overall poverty. It is easy to see that $x^* >$EDEI$=\mu_\varepsilon$.

The appraisal of the parametric method PM

For assessing the usefulness of PM to retrieve ε, we estimate regression functions which link selected normative variables, based on ε, with corresponding descriptive counterparts. We fit the regression functions to the time series of the variables in question. As the time series do not exhibit linear trends, we may ignore a possible impact of the time variable (*Year*) on R^2. Table 6 presents the estimates of the parameters of regression functions.

Model 1 in Table 6 shows the relationship between the normative Atkinson ASWF, μ_ε, (8) and the descriptive ASWF, $\widetilde{SS}$, (9). R^2 close to one means that μ_ε predicts $\widetilde{SS}$ almost perfectly. In economic terms, the ranking of income distributions, according to μ_ε, is the same as the ranking according to $\widetilde{SS}$. If the rankings differed remarkably, our method of estimating ε would be questionable.

We can also appraise the usefulness of PM basing upon a particular consequence of Lambert and Lanza's (2006) theory of the benchmark incomes. Lambert and Lanza's (2006) *Theorem* 3 specifies a general relationship between x^* and inequality aversion. We reformulate this theorem in terms of the Atkinson index (5).

Theorem 3. Let $A(\mu_1,\varepsilon_1)$ and $A(\mu_2,\varepsilon_2)$ be the Atkinson inequality indices (5), where $\varepsilon_1>\varepsilon_2$. Then, for all unequal income distributions X_1 and X_2, $x_1^* < x_2^*$. However, a general conclusion that x^* is a declining function of inequality aversion seems to be unambiguous only for $\varepsilon>1$. To show this, we differentiate the logarithm of (20) with respect to ε, namely

$$\frac{\partial \log x^*}{\partial \varepsilon} = \frac{1}{\mu_\varepsilon}\frac{\varepsilon-1}{\varepsilon}\frac{\partial \mu_\varepsilon}{\partial \varepsilon} + \frac{1}{\varepsilon^2} log\frac{\mu_\varepsilon}{\mu} \qquad (22)$$

Note that the sign of $\frac{\partial \log x^*}{\partial \varepsilon}$ is the same as the sign of $\frac{\partial x^*}{\partial \varepsilon}$ because x^* is positive. For $\varepsilon>1$, the sign of $\frac{\partial \log x^*}{\partial \varepsilon}$ is negative since $\frac{\partial \mu_\varepsilon}{\partial \varepsilon}<0$ (Lambert, 2001, p. 99) and $\mu_\varepsilon/\mu<1$. In this case, x^* is a declining function of ε. For $\varepsilon=1$, $\frac{\partial \log x^*}{\partial \varepsilon} = 0$. However, for $0<\varepsilon<1$, the sign of $\frac{\partial \log x^*}{\partial \varepsilon}$ may be either negative or positive. Eq. (22) also holds for the relative benchmark z^*.

As all our estimates of ε are greater than 1, x^* and z^* ought to be a declining function of ε. Models (2) and (3) in Table 6 demonstrate that our findings are consistent with this theoretical consequence. One can also see that the model (3) for z^* fits the data better than the model (2) for x^*.

We can also use our estimates of inequality aversion for the verification of some prominent economic hypotheses. Frisch (1959) hypothesised higher inequality aversion in poorer countries. Contrary to Frisch, Atkinson (1970) hypothesised higher inequality aversion in rich countries. Suppose that GDP per capita measures countries' economic development. Thus, Frish's hypothesis states that ε is a *declining* function of GDP per capita, whereas Atkinson's hypothesis states that ε is an *increasing* function of GDP per capita. Examining model 4 in table 6 shows that Frisch's hypothesis is true in the case of Poland for the years 2000–2015.

A relationship between the Gini index G and the Atkinson index $A(\mu,\varepsilon)$ (5) is crucial for Lambert's *et al.* (2003) NRSI hypothesis presented in Section 2.2. We can verify this hypothesis empirically. Model 5 in Table 6 shows that there is a statistically significant positive linear relationship between G and $A(\mu,\varepsilon)$.[9]

Lambert et al. (2003) also hypothesised that the Gini index is a declining function of ε. Hereafter, we shall refer to this hypothesis as LMS, according to authors' names, to wit Lambert Millimet and Slottje. However, LMS has a competitor in the form of the well-known inequality–development relationship (IDR). IDR was originally proposed by Kuznets (1955), who presented the famous *inverted-U hypothesis*: during the development, inequality first increases and then declines.

Many theoretical studies have supported Kuznets's hypothesis (e.g. Robinson, 1976; Galor & Tsiddon, 1996; Aghion & Bolton, 1997; Dahan & Tsiddon, 1998). However, the empirical support of this hypothesis is sometimes ambiguous (see Tuominen, 2015, for a broad review).

Unfortunately, the standard IDR cannot be applied directly for testing LMS because Kuznets and his followers have applied inequality in the distribution of market income, thus ruling out all redistributive issues. As we argued in Section 1, a society's attitude towards inequality shapes DDI, not the distribution of market income.

We can verify LMS using the Gini index for DDP and complementing the standard IDR by ε. In this manner, we obtain *the augmented inequality–development relationship* (AIDR). More specifically, AIDR links the Gini index with GDP per capita and with inequality aversion ε.

We shall analyse AIDR non-parametrically using graphical visualisations. The impact of a single dimension on inequality can be determined when respecting the ceteris paribus rule. Thus, for a given degree of inequality aversion, we shall obtain the standard IDR curve. For a given level

[9] See Harvey (2003) and Sarabia and Azpitarte (2012) for some empirical findings supporting NRSI hypothesis.

of economic development, we shall get G as a function of ε. We shall use this function for testing LMS.

Figure 2 displays AIDR in the three-dimensional space, whereas Fig. 3 displays the contours of AIDR. We fit the surface of the Gini index by splines in order to avoid troubles with a parametric specification of AIDR. We use the graphic module of Statistica, 3.3, TIBICO Software Inc.

Figure 3 shows that inequality seems to be a declining function of inequality aversion for GDP/capita above 1300 [PLN], *ceteris paribus*. For lower levels of GDP, inequality seems to trace out the U-shaped curve along with increasing inequality aversion, ceteris paribus. Thus the LMS hypothesis appears to be true only for a high stage of economic development. We also observe in Figs. 2 and 3 that inequality traces out the classical inverted U-shaped curve along with the development, ceteris paribus.

Conclusions

In this paper, we propose a parametric method, PM, of estimating a society's inequality aversion, ε, assuming that disposable income distribution, DDP, obeys the GB2($x;a,b,p,q$) distribution. We argue that DDP embodies societal aversion to inequality. We prove that the social welfare function, SWF, takes on a finite value if and only if ε lies in the interval $(0,ap+1)$. The values of ε outside this interval would characterise unrealistic policies offering infinite social welfare. We propose the midpoint, ε_{mid}, of this interval as the estimate of societal aversion to inequality. We develop the maximum likelihood estimator of ε_{mid}, which enables calculating the standard errors and the confidence intervals of inequality aversion.

PM has some advantages over the methods developed until now. PM provides *objective estimates* of ε, in contrast to *subjective estimates* of ε offered by the leaky bucket experiments, or by Kot's (2017) method. PM also has an advantage over the methods based on the equal sacrifice model. The methods elicit ε from tax data which are scarce and imperfect, whereas PM requires data on DDP which are available for many countries and years. One can also calculate ε_{mid} using the parameters of the GB2 distribution, already estimated in many empirical studies. For the review of such studies, see Kleiber and Kotz, (2003, pp. 195–196, 209–2010, 221–222), among many others. Obviously, the GB2 distribution, or its particular cases, should be fitted to disposable income data.[10]

[10] This requirements excludes, e.g. Bandourian's *et al.* (2003) estimates of the parameters of the GB2 distribution and its particular cases fitted to market income data.

As inequality aversion is bounded from the above, passing with ε to infinity seems to be debatable. Some authors claim that CRIA could reflect the Rawlsian maximin when $\varepsilon \rightarrow \infty$ (see, e.g. Atkinson, 1970; Lambert, 2001, pp. 99–101). From Proposition 1 it follows that such a claim is unrealistic for the major theoretical models of income distributions since it assumes implicitly infinite social welfare.

The statistical analysis of inequality aversion for Poland provides empirical results which are consistent with some theoretical predictions. Such consistency confirms the usefulness of PM to retrieve a society's aversion to inequality. The augmenter inequality-development relationship shows that the stage of economic development might matter when assessing the impact of inequality aversion on income inequality. However, further empirical studies are necessary for confirming this supposition.

References

Aghion, P., & Bolton, P. (1997). A theory of trickle-down growth and development. *Review of Economic Studies, 64*(2). doi: 10.2307/2971707.

Aitchison, J., & Brown, J. A. C. (1956). *The lognormal distribution.* Cambridge: Cambridge University Press.

Amiel, Y., Cowell, F., & Slottje, D. (2004). Why do people violate the transfer principle? evidence from educational sample surveys. *Research on Economic Inequality*, *11*.

Amiel, Y., Creedy, J., & Hurn, S. (1999). Measuring attitudes towards inequality. *Scandinavian Journal of Economics, 101*(1). doi: 10.1111/1467-9442.00142.

Aristei, D., & Perugini, C. (2016). Inequality aversion in post-communist countries in the years of the crisis. *Post-Communist Economies*, *28*(4). doi: 10.1080/14631377.2016.1224053.

Aroian, L. A., Taneja, V. S., & Cornwell, L. W. (1978). Mathematical forms of the distribution of the product of two normal variables. *Communications in Statistics - Theory and Methodology, A72*.

Atkinson, A. (1970). On the measurement of economic inequality. *Journal of Economic Theory*, *2*(3). doi: 10.1016/0022-0531(70)90039-6.

Atkinson, A. B. (1980). *Wealth, income and inequality*. Oxford: Oxford University Press.

Bandourian, R., McDonald, J. B., & Turley, R. S. (2003). A comparison of parametric models of income distribution across countries and years. *Estadistica 55*.

Beckman, S. R., Formbyand, J. P., Smith, W. J. (2004). Efficiency, equity and democracy: experimental evidence on Okun's leaky bucket. In F. Cowell (Ed.). *Inequality, welfare and income distribution: experimental approaches.* Amsterdam: Emerald Group Publishing Limited.

Brazauskas, V. (2002). Fisher information matrix for the Feller–Pareto distribution. *Statistics & Probability Letters, 59*(2). doi: 10.1016/S0167-7152(02)00143-8.

Burr, I. W. (1942). Cumulative frequency functions. *Annals of Mathematical Statistics*, 13.

Carlsson, F., Daruvala, D., & Johansson-Stenman, O. (2005). Are people inequality-averse or just risk-averse? *Economica, 72*. doi: 10.1111/j.0013-0427.2005.00421.x.

Chernoff, H., & Lehmann, E. L. (1954). The use of maximum-likelihood estimates in χ^2 test for goodness of fit. *Annals of Mathematical Statistics*, *25*(3). doi: 10.1214/aoms/1177728726.

Clark, A. E., & D'Ambrosio, C. (2015). Attitudes to income inequality: experimental and survey evidence. In A. B. Atkinson & F. Bourguignon (Eds.). *Handbook of income distribution*. Amsterdam: Elsevier.

Cover, T. M., & Thomas, J. A. (1991). *Elements of information theory*. New York: Wiley.

Cowell, F., & Gardiner, K. (1999). *Welfare weights. STICERD.* London School of Economics.

Cui, G., Yu, X., Iommelli, S., & Kong, L. (2016). Exact distribution for the product of two correlated Gaussian random variables. *IEEE Signal Processing Letters*, *23*. doi: 10.1109/LSP.2016.2614539.

D'Agostino, R .D., & Stephens, M.A. (1986). *Goodness-of-fit techniques*. New York and Basel: Marcel Dekker Inc.

Dagum, C. (1977). A new model of personal income distribution: Specification and estimation. *Economie Appliquée, 30.*

Dahan, M., Tsiddon, D. (1998). Demographic transition, income distribution, and economic growth. *Journal of Economic Growth*, *3*(1). doi: 10.1023/A:1009769930916.

Fisk, P. R. (1961). The graduation of income distribution. *Econometrica*, *29*.

Fisz, M. (1967). *Probability theory and mathematical statistics*. New York: Wiley,

Frisch, R. (1959). A complete system for computing all direct and cross-demand elasticities in a model with many sectors. *Econometrica*, *27*.

Galor, O., & Tsiddon, D. (1996). Income distribution and growth: the Kuznets hypothesis revisited. *Economica, 3*.

Gouveia, M., & Strauss, R. P. (1994). Effective federal individual income tax functions: an exploratory empirical analysis. *National Tax Journal, 47*.

Harvey, J. (2003). A note on the `natural rate of subjective inequality' hypothesis and the approximate relationship between the Gini coefficient and the Atkinson index. *Journal of Public Economics*, *89*. doi: 10.1016/j.jpubeco.2004.05.002.

Hoffmann, R. (2001). Effect of the rise of a person's income on inequality. *Brazilian Review of Econometrics*, *21*. doi: 10.12660/bre.v21n22001.2751.

Jenkins, S. P. (2007). gb2fit: Stata module to fit Generalized Beta of the Second Kind distribution by maximum likelihood. *Statistical Software Components Archive,* S456823.

Jenkins, S. P. (2007). Inequality and the GB2 income distribution. *ENCINEQ working paper*, *73*.

Kleiber, C. (1997). The existence of population inequality measures. *Economics Letters*, *57*. doi: 10.1016/S0165-1765(97)81877-0 .

Kleiber, C., & Kotz, S. (2003). *Statistical size distributions in economics and actuarial sciences*. Hoboken, NJ.: Wiley,

Kolm, S. Ch. (1969). The optimal production of social justice. In J. Margolis & H. Guitton, H. (Eds.). *Public economics*. London and New York: Macmillan.

Kot, S. M.(2009). The boundaries for inequality aversion and certain measures of income inequality. *Prace i Materiały Wydziału Zarządzania Uniwersytetu Gdańskiego, 4/2*.

Kot, S. M. (2012). *Towards the stochastic paradigm of welfare economics*. Cracow: Impuls.

Kot, S. M. (2017). Estimating inequality aversion from subjective assessments of the just noticeable differences in welfare. *Equilibrium. Quarterly Journal of Economics and Economic Policy, 12*(1). doi: 10.24136/eq.v12i1.7.

Kuznets, S. (1955). Economic growth and income inequality. *American Economic Review*, *45*.

Lambert, P. J. (2001). *The distribution and redistribution of income: a mathematical analysis.* Manchester: Manchester University Press.

Lambert, P. J., & Lanza, G. (2006). *The effect on inequality of changing one or two incomes*. *Journal of Economic Inequality, 4*(3). doi: 10.1007/s10888-006-9020-1.

Lambert, P. J., Millimet, D. L., & Slottje, D. (2003). Inequality aversion and the natural rate of subjective inequality. *Journal of Public Economics*, *87*. doi: 10.1016/S0047-2727(00)00171-7.

Lambert, P. J., & Naughton, H. T. (2009). The equal absolute sacrifice principle revisited. *Journal of Economic Surveys, 23*. doi: 10.1111/j.1467-6419.2008.00564.x.

Lerner, A. P. (1944). *The economics of control.* London: Macmillan.

Levitt, S. D., & List, A. J. (2007). What do laboratory experiments measuring social preferences reveal about the real world? *Journal of Economic Perspectives, 21*. doi: 10.1257/jep.21.2.153.

McDonald, J. B. (1984). Some generalized functions for the size distribution of income. *Econometrica*, *52*. doi: 10.2307/1913469.

Mitra, T., & Ok, E. A. (1996). Personal income taxation and the principle of equal sacrifice revisited. *International Economic Review, 37*. doi:10.2307/2527317.

Okun, A. M. (1975). *Equality and efficiency.* Washington, DC: Brookings Institution.

Pirttilä, J., & Uusitalo, R. (2007). Leaky bucket in the real world: estimating inequality aversion using survey data. *CESifo working paper, 2026*.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica 32*.

Richter, W. F. (1983). From ability to pay to concept of equal sacrifice. *Journal of Public Economics, 20*.

Robinson, S. (1976). A note on the u hypothesis relating income inequality and economic development. *American Economic Review*, *66*.

Sarabia, J. M., & Azpitarte, F. (2012). On the relationship between objective and subjective inequality indices and the natural rate of subjective inequality. *ECINEQ working papers, 248*

Sen, A. (1973). *On economic inequality.* Oxford: Clarendon Press.

Sen, A. (1978). Ethical measurement of inequality: some difficulties. In W. Krelle & A/F. Shorrocks (Eds.). *Personal income distribution.* Amsterdam: North-Holland.

Sheshinski, E.(1972). Relation between a social welfare function and the Gini index of income inequality. *Journal of Economic Theory, 4*. doi: 10.1016/0022-0531(72)90167-6.

Singh S. K., & Maddala, G. S. (1976). A function of size distribution of income. *Econometrica*, *44*.

Stern, N. (1977). The marginal valuation of income. In M. J. Artis & A. R. Nobay (Eds.). *Essays in economic analysis*. Cambridge: Cambridge University Press.

Tuominen, E. (2015). Reversal of the Kuznets curve. Study on the inequality–development relation using top income shares data. *WIDER Working Paper 2015/036*.

Vitaliano, D. F. (1977). The tax sacrifice rules under alternative definitions of progressivity. *Public Finance Quarterly, 5*.

Ware, R., & Lad, F. (2003). Approximating the distribution for sums of products of normal variables. *Technical Report. The University of Queensland.*

World Bank (2017). *World Development Indicators 2017*. Washington, DC: World Bank.

Young, H. P. (1987). Progressive taxation and the equal sacrifice principle. *Journal of Public Economics*, *32*. doi: 10.1016/0047-2727(87)90012-0.

Young, H. P. (1990). Progressive taxation and equal sacrifice. *American Economic Review*, *80*.

Annex

Table 1. Estimates of the parameters of the GB2($x;a,b,p,q$) distribution for Poland for 2000–2015

Year	*a*	*b*	*p*	*q*	*Log L*	$\chi^2(15)$	*N*	*Persons*
2000	3.23114	741.8896	0.78741	0.97484	-842207	273.51	35952	113540
	0.01513	4.02785	0.00669	0.00931				
2001	2.64035	771.8192	1.01912	1.29302	-734423	96.31	31679	98635
	0.01143	5.51242	0.00917	0.01352				
2002	2.45454	761.8882	1.13284	1.36811	-741490	143.69	32190	99248
	0.01061	5.84388	0.01054	0.01437				
2003	2.35846	768.5532	1.18184	1.40408	-742326	162.27	32292	98923
	0.01061	5.84388	0.01054	0.01437				
2004	2.34228	756.7169	1.17746	1.37537	-739312	123.79	32054	98394
	0.01028	6.10623	0.01123	0.01450				
2005	2.70710	721.5539	0.98995	1.13633	-793294	309.08	34521	106298
	0.01277	4.79202	0.00917	0.01141				
2006	2.82992	797.1685	0.93913	1.10340	-854048	348.05	37227	113381
	0.01298	4.90792	0.00830	0.01071				
2007	3.21610	843.4617	0.82254	0.92003	-844655	449.96	37063	111016
	0.01691	4.62588	0.00766	0.00908				
2008	3.08641	964.9728	0.85551	1.01923	-837060	251.60	37042	108795
	0.01492	5.57214	0.00765	0.01005				
2009	2.98372	1007.4950	0.89214	1.06957	-826961	287.96	36966	106878
	0.01414	6.06847	0.00801	0.01065				
2010	2.98330	995.4054	0.93271	1.02267	-832110	298.13	37127	107079
	0.01513	5.96170	0.00888	0.01024				
2011	3.37254	1037.4900	0.74691	0.89654	-826206	222.84	37058	106216
	0.01748	5.59464	0.00675	0.00891				

Table 1. Continued

Year	*a*	*b*	*p*	*q*	*Log L*	*χ^2(15)*	*N*	*Persons*
2012	3.33643	1044.9590	0.76675	0.91115	-811088	391.38	37077	104195
	0.01748	5.73711	0.00705	0.00917				
2013	3.58008	1085.0750	0.67217	0.83990	-794514	392.81	36834	101741
	0.01871	5.69849	0.00599	0.00838				
2014	3.39446	1160.3810	0.74226	0.95868	-787501	354.32	36874	100607
	0.01628	6.40784	0.00644	0.00958				
2015	3.52944	1228.0340	0.71243	0.94161	-785951	354.03	36800	99997
	0.01664	6.56996	0.00607	0.00935				

Note: Standard errors below estimates. $\chi^2(15)$ – Pearson χ^2 statistics with 15 degrees of freedom based on 20 equiprobable cells.*N*-the number of households

Source: own calculations using data from PBHS, constant prices (2010=100).

Table 2. Selected empirical statistics and corresponding GB2 estimates for Poland for 2000–2015

Year	$Mean_{emp}$	$Mean_{gb2}$	$Gini_{emp}$	$Gini_{gb2}$	$\widetilde{SS}_{emp}$	$\widetilde{SS}_{gb2}$
2000	800	801	0.33318	0.33187	533	535
2001	815	815	0.33368	0.33385	543	543
2002	829	830	0.34288	0.34072	545	547
2003	847	849	0.34634	0.34669	554	555
2004	849	850	0.35169	0.35235	550	551
2005	818	820	0.34601	0.34718	535	535
2006	884	889	0.34072	0.34130	583	586
2007	972	969	0.34060	0.33897	641	640
2008	1063	1063	0.33480	0.33288	707	709
2009	1104	1104	0.33392	0.33300	735	737
2010	1150	1150	0.33799	0.33640	761	763
2011	1146	1145	0.33566	0.33555	761	761
2012	1158	1158	0.33720	0.33425	768	771
2013	1182	1176	0.33839	0.33587	782	781
2014	1219	1219	0.32516	0.32386	823	824
2015	1273	1271	0.31989	0.31798	866	867

Note: $\widetilde{SS}$ -the Sheshinski-Sen abbreviated welfare function (9).

Source: own calculations using data from PHBS, constant prices (2010=100).

Table 3. The regressions of **s**elected empirical statistics against GB2 estimates

	(1) $Mean_{emp}$	(2) $Gini_{emp}$	(3) $\widetilde{SS}_{emp}$
$Mean_{gb2}$	1.008^{***}		
	(0.00314)		
$Gini_{gb2}$		0.916^{***}	
		(0.0329)	
$\widetilde{SS}_{gb2}$			0.912^{***}
			(0.00974)
_cons	-7.954^{*}	0.0293^{*}	31.60^{***}
	(3.205)	(0.0111)	(6.904)
N	16	16	16
adjusted R^2	1.000	0.981	0.998

Note: Dependent variables in columns, independent variables in raws; $\widetilde{SS}$ -the Sheshinski-Sen abbreviated welfare function (9). Standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Source: own calculations using data from Table 2**.**

Table 4. Estimates of inequality aversion for Poland for 2000–2015

Year	$\hat{\epsilon}$	*D[$\hat{\epsilon}$]*	*LB.*	*UB*
2000	1.77208	.00763	1.75713	1.78703
2001	1.84538	.00927	1.82720	1.86356
2002	1.89026	.01002	1.87063	1.90989
2003	1.89362	.01024	1.87356	1.91369
2004	1.87893	.01020	1.85895	1.89892
2005	1.83991	.00909	1.82210	1.85771
2006	1.82879	.00853	1.81208	1.84551
2007	1.82264	.00838	1.80621	1.83906
2008	1.82019	.00838	1.80376	1.83661
2009	1.83091	.00861	1.81403	1.84778
2010	1.89123	.00936	1.87288	1.90958
2011	1.75945	.00776	1.74423	1.77467
2012	1.77906	.00805	1.76329	1.79483
2013	1.70317	.00727	1.68892	1.71742
2014	1.75975	.00779	1.74448	1.77501
2015	1.75720	.00766	1.74220	1.77221

Note: D[$\hat{\epsilon}$] – standard error of $\hat{\epsilon}$; *LB, UB*-the lower and upper bounds of 95% confidence interval.

Source: own calculations using data from Table 1.

Table 5. Normative characteristics for Poland for 2000–2015

Year	μ_ε	*A(ε,μ)*	*x**	*z**
2000	562	0.29819	687	0.85703
2001	561	0.31080	687	0.84323
2002	561	0.32449	690	0.83130
2003	566	0.33333	701	0.82585
2004	562	0.33926	700	0.82378
2005	553	0.32519	685	0.83564
2006	608	0.31620	749	0.84177
2007	669	0.30979	820	0.84591
2008	740	0.30350	903	0.84961
2009	767	0.30523	936	0.84766
2010	790	0.31323	963	0.83772
2011	801	0.30070	982	0.85694
2012	810	0.30069	990	0.85503
2013	828	0.29616	1017	0.86502
2014	869	0.28728	1053	0.86397
2015	915	0.27976	1103	0.86813

Note: μ_ε – EDEI (8); *A(ε,μ)* – the Atkinson index of inequality (5). *x**– the absolute benchmark (pivotal) income, *z**– the relative benchmark income.

Source: own calculations using data from Table 4 and PHBS.

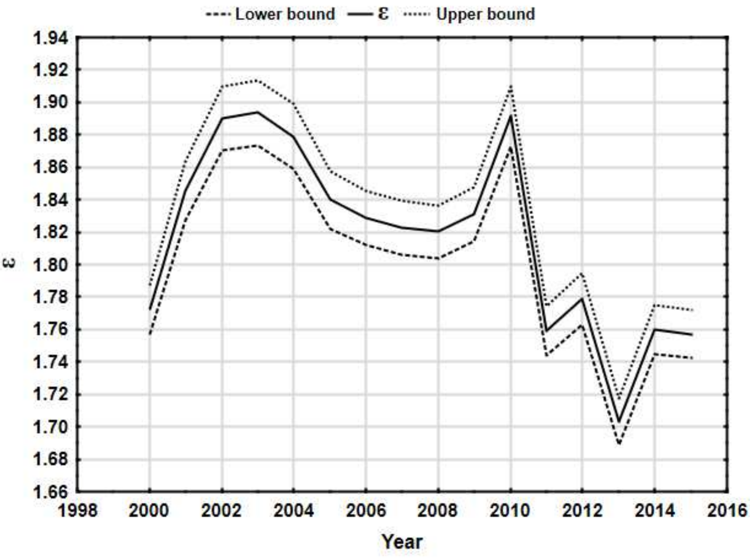
Table 6. The estimates of regression functions concerning verified hypotheses

	(1) $\widetilde{SS}$emp	(2) x^*	(3) z^*	(4) ε	(5) $Gini_{emp}$
μ_ε	0.912***				
	(0.00974)				
ε		-1714.5**	-0.224***		
		(555.8)	(0.0235)		
GDP/capita				-0.0000174**	
				(0.00000558)	
$A(\varepsilon,\mu)$					0.452***
					(0.0489)
_cons	31.60***	3969.4**	1.253***	2.015***	0.198***
	(6.904)	(1010.3)	(0.0426)	(0.0645)	(0.0151)
N	16	16	16	16	16
adjusted R^2	0.998	0.362	0.857	0.368	0.849

Note: Dependent variables in columns, independent variables in raws; $\widetilde{SS}_{\text{emp}}$ – the Sheshinsky-Sen abbreviated welfare function (9); μ_ε – the Atkinson abbreviated welfare function (8)); x^* – absolute benchmark income, z^* – relative benchmark income; Standard errors in parentheses; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

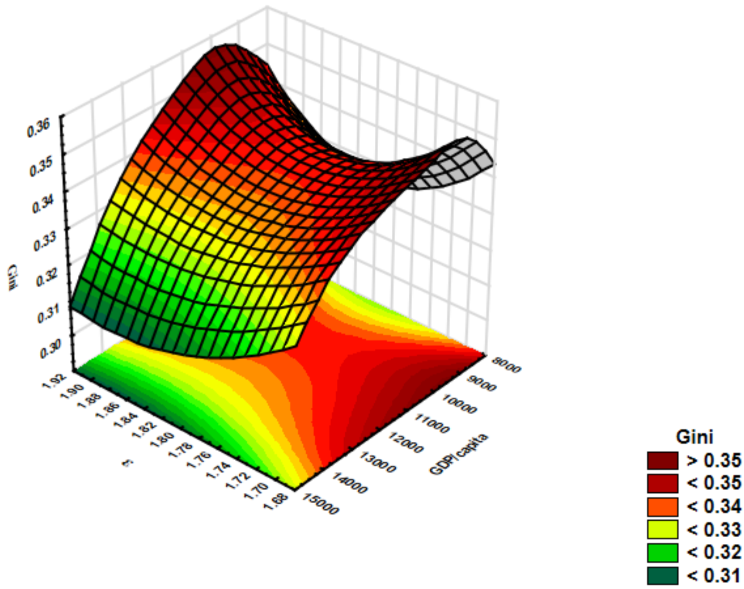
Source: own calculations using data from Table 1 and 2.

Figure 1. The estimates of inequality aversion ε and bounds of 95% confidence interval for Poland 2000–2015



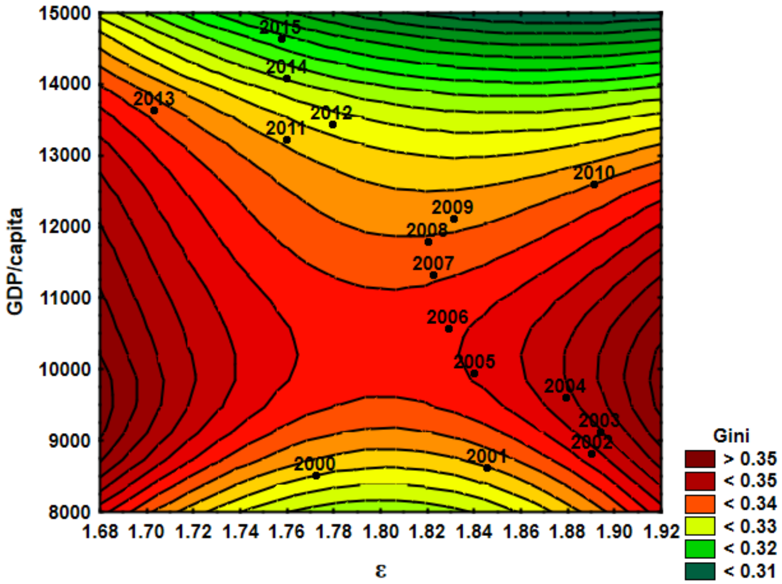
Source: own elaborating using data from Table 4.

Figure 2. The surface of the augmented inequality-development relationship



Source: own elaborating using data from Tables 2 and 4.

Figure 3. The contours of the augmented inequality-development relationship



Source: own elaborating using data from Tables 2 and 4.