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Experimental certification of an informationally complete quantum measurement in a device-independent protocol

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Minimal informationally complete positive operator-valued measures (MIC-POVMs) are special kinds of measurement in quantum theory in which the statistics of their d^2-outcomes are enough to reconstruct any d-dimensional quantum state. For this reason, MIC-POVMs are referred to as standard measurements for quantum information. Here, we report an experiment with entangled photon pairs that certifies, for what we believe is the first time, a MIC-POVM for qubits following a device-independent protocol (i.e., modeling the state preparation and the measurement devices as black boxes, and using only the statistics of the inputs and outputs). Our certification is achieved under the assumption of freedom of choice, no communication, and fair sampling. © 2020 Optical Society of America under the terms of the OSA Open Access Publishing Agreement

1. INTRODUCTION

A minimal informationally complete positive operator-valued measure (MIC-POVM) [1,2] is a measurement on a d-dimensional quantum system that: (i) is informationally complete (IC) (i.e., its statistics determine completely any quantum state and allow for a simple state reconstruction) and (ii) is minimal, since it has the minimum number of outcomes a measurement must have to be IC, namely, d^2 [3]. MIC-POVMs are fundamental in quantum information. For example, they are crucial for optimal quantum state tomography [4,5], quantum key distribution with an optimal trade-off between security and key rate [6], device-independent certification of optimal randomness from one bit of entanglement [7,8], and optimal entanglement detection [9]. Arguably, MIC-POVMs are the standard measurements in quantum information [10] and thus have a privileged role in information-theoretic reconstructions of quantum theory [10].

Experimentally, MIC-POVMs have been aimed at in photonic experiments of qubit [11], qutrit [12,13], and two-qubit [14] tomography; quantum key distribution [11]; and generalized measurements using quantum walks [14,15]. However, all these experiments made assumptions about the state preparation and the functioning of the measurement devices and therefore have limited applicability to cryptographic scenarios.

Here, we address the problem of experimentally certifying a MIC-POVM for qubits following a device-independent (DI) protocol [7,8,16,17]. That is, modeling the state preparation and the measurement devices as black boxes and using only the statistics of the inputs and outputs obtained within a Bell inequality experiment. The idea behind the experiment is to integrate the MIC-POVM within a Bell inequality experiment and use it to produce correlations that, according to quantum theory, are only attainable with a MIC-POVM for qubits. Our results not only allow us to certify a MIC-POVM for qubits in a DI protocol, but also constitute what we believe is the first experimental observation of "qubit correlations that can only be explained by four-outcome nonprojective measurements." [18]

2. CERTIFICATION METHODS

To certify a four-outcome MIC-POVM in a DI way, we use the bipartite Bell scenario shown in Fig. 1. There, in the middle, is a source emitting pairs of particles. One of the particles is measured by one party, Alice, and the other particle is measured by the other party, Bob. Alice chooses her measurement from a set of four measurements: three two-outcome measurements A_x, with $x \in \{1, 2, 3\}$, and one four-outcome measurement A_4. Bob chooses his measurement from a set of four two-outcome measurements B_y, with $y \in \{1, 2, 3, 4\}$. The possible outcomes of the two-outcome measurements are $+1$ and -1, and the possible outcomes of the four-outcome measurements are 1, 2, 3, and 4. The outcomes of A_x and B_y are denoted by a and b, respectively. From the experimental results, we obtain the set of conditional probabilities $\{P(a, b|x, y)\}$.

In our experiment, we are interested in the conditional probabilities appearing in the Bell inequality introduced in Ref. [7] as

$$\beta_{\mathrm{el}}^{\mathrm{m}} = \beta_{\mathrm{el}} - k \sum_{i=1}^{4} P(a = i, b = +1 | x = 4, y = i), \quad (1)$$

where β_{el} is the Bell operator of the so-called "elegant Bell inequality" [7,8,19]; namely,

$$\beta_{\mathrm{el}} = E_{11} + E_{12} - E_{13} - E_{14} + E_{21} - E_{22} + E_{23} - E_{24}$$
$$+ E_{31} - E_{32} - E_{33} + E_{34}, \quad (2)$$

where $E_{xy} = \sum_{a,b} a\, b\, P(a, b|x, y)$. For local hidden variable theories, β_{el} is upper-bounded by 6. In contrast, in quantum theory β_{el} is upper-bounded by $4\sqrt{3} \approx 6.928$. The quantum maximum can be achieved with pairs of qubits in state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, and the following projective measurements,

$$\begin{aligned}
A_1 &= \sigma_x, & B_1 &= \tfrac{1}{\sqrt{3}}(\sigma_x - \sigma_y + \sigma_z), \\
A_2 &= \sigma_y, & B_2 &= \tfrac{1}{\sqrt{3}}(\sigma_x + \sigma_y - \sigma_z), \\
A_3 &= \sigma_z, & B_3 &= \tfrac{1}{\sqrt{3}}(-\sigma_x - \sigma_y - \sigma_z), \\
& & B_4 &= \tfrac{1}{\sqrt{3}}(-\sigma_x + \sigma_y + \sigma_z),
\end{aligned} \quad (3)$$

where σ_i are the Pauli matrices.

According to quantum theory, our target Bell operator $\beta_{\mathrm{el}}^{\mathrm{m}}$ is also upper-bounded by $4\sqrt{3}$. This quantum maximum can be attained with state $|\Phi^+\rangle$ and the measurements in Eq. (3). However, in this case, the second term in Eq. (1) is zero if and only if A_4 is a qubit symmetric MIC-POVM whose elements are

anti-aligned with Bob's measurements B_y in Eq. (3). That is, if A_4 is the four-outcome POVM that is defined by

$$A_{4,1} = \frac{1}{2} \begin{pmatrix} \alpha & -\beta(1+i) \\ \beta(-1+i) & 1-\alpha \end{pmatrix},$$

$$A_{4,2} = \frac{1}{2} \begin{pmatrix} 1-\alpha & \beta(-1+i) \\ -\beta(1+i) & \alpha \end{pmatrix},$$

$$A_{4,3} = \frac{1}{2} \begin{pmatrix} 1-\alpha & \beta(1-i) \\ \beta(1+i) & \alpha \end{pmatrix},$$

$$A_{4,4} = \frac{1}{2} \begin{pmatrix} \alpha & \beta(1+i) \\ \beta(1-i) & 1-\alpha \end{pmatrix}, \quad (4)$$

where $\alpha = \frac{3-\sqrt{3}}{6}$ and $\beta = \frac{\sqrt{3}}{6}$. In this case, the extremes of the four unit vectors associated to the elements of A_4 define a regular tetrahedron within the Bloch sphere.

Any measurement different than the one defined in Eq. (4) will produce a smaller value for $\beta_{\mathrm{el}}^{\mathrm{m}}$. Although certifying a symmetric MIC-POVM requires ideal conditions, we can use the property above to test whether a genuine, four-outcome MIC-POVM has indeed been realized in the experiment by computing the maximum of $\beta_{\mathrm{el}}^{\mathrm{m}}$ that can be obtained using three-outcome measurements. To do this, let us consider the following generalization of $\beta_{\mathrm{el}}^{\mathrm{m}}$,

$$\sum_{x=1}^{3} \sum_{y=1}^{4} \gamma_{xy} E_{xy} - k \sum_{y=1}^{4} \sum_{a=1}^{4} \sum_{b=\pm 1} \gamma_{bxy} P(a, b|4, y). \quad (5)$$

We compute the maximum value of Eq. (5) that can be obtained using three-outcome measurements; i.e., the maximum value of

$$\max_{j=1,2,3,4} \left[\sum_{x=1}^{3} \sum_{y=1}^{4} \gamma_{xy} E_{xy} - k \sum_{y=1}^{4} \sum_{a \neq j} \sum_{b=\pm 1} \gamma_{bxy} P(a, b|4, y) \right]. \quad (6)$$

Each of the maximizations within Eq. (6) are taken with a constraint that the jth outcome of Alice's A_4 measurement has probability 0. The larger the gap between the experimental value of Eq. (5) and the maximum possible value of Eq. (6), the more confident we can be that indeed a genuine four-outcome POVM has been performed.

Most crucially, as we show in Supplement 1, four irreducible outcomes in Dimension 2 imply information completeness, which is arguably the most important feature of a quantum measurement.

Finally, while Bell scenarios can generally be useful for measurement certification, it is important to point out that the Bell inequality in the protocol above is tailored to the specific measurement targeted in our certification. To use the same procedure for an arbitrary measurement, a different Bell inequality would usually be required. In fact, finding the optimal Bell inequality for certifying in a device-independent protocol a given generalized measurement is, in general, a difficult problem.

3. EXPERIMENT

A. Experimental Setup

A type-I spontaneous parametric down-conversion source is used to generate entangled photon pairs in state $|\Phi^+\rangle =$
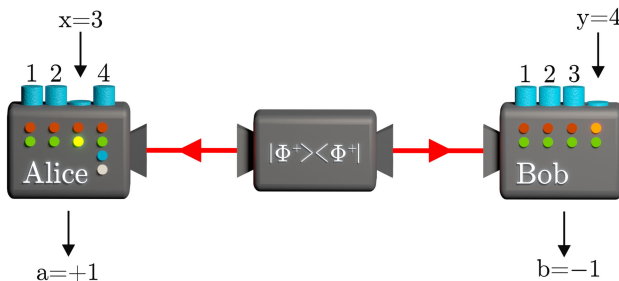

Fig. 1. Scenario considered in our experiment consists of two parties, Alice and Bob, who perform local measurements on maximally entangled pairs of qubits. See further details in the text.

$1/\sqrt{2}(|HH\rangle + |VV\rangle)$, where H and V denote horizontal and vertical polarization, respectively. Pairs of entangled photons at 780 nm are produced in two orthogonally oriented 2 mm thick beta barium borate (BBO) crystals, pumped with a femto-second laser at 390 nm. As shown in Fig. 2, these photons go through 1 nm spectral bandpass filters (SF), and are then coupled into single-mode fibers (SMF) to have a perfect spatial mode overlap between the two polarizations. These SMFs then bring the photons to Alice's and Bob's measurement stations. Whenever projective measurements are performed on both sides (i.e., whenever $x \in \{1, 2, 3\}$ and $y \in \{1, 2, 3, 4\}$), the two measurement

theory. To obtain an upper bound on its value, we use the semidefinite programming method of Navascués, Pironio, and Acín (NPA) [20] implemented in the Python package Ncpol2spda [21].

To identify the values of γ_{xy} and γ_{bxy} that provide the largest gap between the experimental value of Eq. (5) and the maximum possible value of Eq. (6), we performed a series of numerical optimizations using the Nelder–Mead method [22], with the target function defined as the difference between the experimental value of Eq. (5) and a bound of Eq. (6), with variable coefficients γ_{xy} and γ_{bxy} and fixed values of k. As a result, we obtained the following Bell operator,

$$
\begin{aligned}
\beta_{\text{IC}} = {} & 0.9541 E_{11} + 0.9917 E_{12} - 0.9767 E_{13} - 1.0064 E_{14} \\
& + 0.9514 E_{21} - 0.9921 E_{22} + 0.8211 E_{23} - 1.0237 E_{24} \\
& + 1.0641 E_{31} - 1.0044 E_{32} - 1.0579 E_{33} + 1.1563 E_{34} \\
& - 3[1.2068 P(1, 1|4, 1) - 0.0374 P(1, 2|4, 1) - 0.0034 P(2, 1|4, 1) + 0.0140 P(2, 2|4, 1) \\
& + 0.0006 P(3, 1|4, 1) + 0.0268 P(3, 2|4, 1) - 0.0163 P(4, 1|4, 1) - 0.0155 P(4, 2|4, 1) \\
& - 0.0033 P(1, 1|4, 2) + 0.0184 P(1, 2|4, 2) + 1.1156 P(2, 1|4, 2) - 0.0046 P(2, 2|4, 2) \\
& - 0.0125 P(3, 1|4, 2) + 0.0401 P(3, 2|4, 2) - 0.0175 P(4, 1|4, 2) - 0.0240 P(4, 2|4, 2) \\
& - 0.0108 P(1, 1|4, 3) + 0.0153 P(1, 2|4, 3) - 0.1195 P(2, 1|4, 3) + 0.1752 P(2, 2|4, 3) \\
& + 0.6201 P(3, 1|4, 3) + 0.0149 P(3, 2|4, 3) - 0.0399 P(4, 1|4, 3) + 0.0527 P(4, 2|4, 3) \\
& + 0.0058 P(1, 1|4, 4) - 0.0149 P(1, 2|4, 4) + 0.0025 P(2, 1|4, 4) + 0.0205 P(2, 2|4, 4) \\
& + 0.0150 P(3, 1|4, 4) + 0.0212 P(3, 2|4, 4) + 0.9565 P(4, 1|4, 4) - 0.0023 P(4, 2|4, 4)].
\end{aligned}
\tag{7}
$$

stations are identically composed by a lambda half-wave plate (HWP), a lambda quarter-wave plate (QWP), and a polarization beam splitter (PBS). Multimode fibers (MMF) finally collect the photons at the four outcomes and bring them to the single-photon avalanche photodiodes (APDs). In addition, Bob's station includes a phase plate (PP).

To implement the four-outcome POVM, Alice's measurement station couples the two-dimensional polarization space with a counter-propagating two-path Sagnac interferometer, which makes transformations in an effectively four-dimensional space possible using two HWPs. At the two outputs of the interferometer, PPs, HWPs and QWPs are used in combination with PBSs to perform the POVM (see Supplement 1). MMFs connected to APDs again gather photons at the four outcomes. Detection counting is performed with a coincidence unit (CU) using 1.6 ns coincidence windows.

A two-photon rate of about 150 coincidences per second was maintained throughout the experiment. Each measurement setting lasted 30 s, and all measurements were repeated a total of 23 times. This was done to decrease the repeatability error of the motorized wave plate mounts.

B. Results

The maximization of Eq. (6) for fixed coefficients γ_{xy} and γ_{bxy} should be made over the set of probabilities allowed by quantum

The upper bounds on the maximum possible value of β_{IC} in Eq. (7), obtained using the third level of the NPA method, in the case of three-outcome measurements and in the case of any quantum measurement are

$$
\beta_{\text{IC}}^{\,3-\text{outcome}} \leq 6.8782 \overset{\text{Quantum}}{\leq} 6.9883,
\tag{8}
$$

whereas our experimental result is

$$
\beta_{\text{IC}}^{\text{exp}} = 6.960 \pm 0.007
\tag{9}
$$

(more detailed results are provided in Tables 1 and 2). Therefore, the experimental value violates the three-outcome bound by more than 11 standard deviations, certifying that Alice's A_4 measurement was indeed an irreducible four-outcome measurement, under the assumption that the system at Alice's laboratory is a qubit. To remove this assumption, we used the SWAP method [23] to calculate the two-qubit state fidelity with the maximally entangled Bell state $|\Phi^+\rangle$ certified by the experimental data contained in Table 1 for β_{el} in Eq. (2). To this end we employed the level $3 + AABB$ of the NPA hierarchy [20]. The resulting fidelity was 0.947.

This means that a qubit measurement occurs at least 94.7% of the time. The only alternative to a qubit MIC-POVM is that in 0.947 of the cases, a three-outcome measurement on a qubit was used; in the remaining 0.053 of the cases, a four-outcome measurement on a higher-dimensional system was used. However, in such a case, the maximal possible value to be observed is not greater than
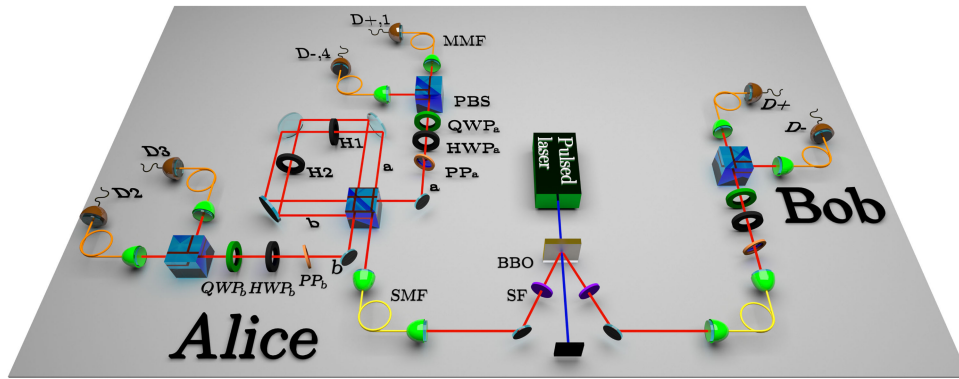
Fig. 2. Experimental setup. The following components are used: a beta-barium borate nonlinear crystal (BBO), 3 nm narrow spectral filters (SF), single-mode optical fibers (SMF), phase plates (PP), lambda half-wave plates (HWP), lambda quarter-wave plates (QWP), polarizing beam splitters (PBS), multi-mode optical fibers (MMF), and single-photon detectors (DET).

Table 1. Experimental Values for the Combinations of Settings Needed to Test the Elegant Bell Inequality

Setting	Theory	Experiment
E_{11}	$1/\sqrt{3} \approx 0.577$	0.553 ± 0.002
E_{12}	0.577	0.573 ± 0.002
E_{13}	-0.577	-0.581 ± 0.002
E_{14}	-0.577	-0.543 ± 0.002
E_{21}	0.577	0.589 ± 0.002
E_{22}	-0.577	-0.599 ± 0.002
E_{23}	0.577	0.529 ± 0.002
E_{24}	-0.577	-0.579 ± 0.002
E_{31}	0.577	0.584 ± 0.002
E_{32}	-0.577	-0.557 ± 0.002
E_{33}	-0.577	-0.621 ± 0.002
E_{34}	0.577	0.601 ± 0.002
β_{el}	$4\sqrt{3} \approx 6.928$	6.909 ± 0.007

Table 2. Experimental Values for the Probabilities of the Outcomes of the MIC-POVM that are Most Relevant to the DI Certification Protocol [See Eq. (1)]

$P(a = i, b = +1 \mid x = 4, y = i)$	Theory	Experiment
$P(1, +1 \mid 4, 1)$	0	0.0021 ± 0.0001
$P(2, +1 \mid 4, 2)$	0	0.0020 ± 0.0001
$P(3, +1 \mid 4, 3)$	0	0.0025 ± 0.0001
$P(4, +1 \mid 4, 4)$	0	0.0025 ± 0.0001
Sum	0	0.0091 ± 0.0002

$0.947 \times 6.8782 + 0.053 \times 6.9883 \approx 6.8840$, which is smaller than the experimental value; namely, 6.960. Similarly, one can calculate that the critical fidelity η_{crit} to the two-dimensional state for MIC-POVMs is $(6.9883 - \beta_{\mathrm{exp}})/(6.9883 - 6.8782) \approx 0.257$.

Even though our DI protocol relies on this method, we can provide additional, non-DI arguments, which suggest that the actual state fidelity was considerably higher. First, we tested the quality of the polarization entanglement by performing a complete nine-measurement state tomography of the Alice–Bob system. The tomography of the joint state is shown in Fig. 3. The experimental fidelity with state $|\Phi^+\rangle$ was near optimal at $(99.6 \pm 0.1)\%$. Second, our measured value of the Bell operator of the elegant Bell Inequality was $\beta_{\mathrm{el}} = 6.909 \pm 0.007$, which corresponds to 99.7% of the quantum bound and is less than three standard deviations

away from it. As mentioned above, a result equal to the quantum bound would imply that the joint state is a maximally entangled qubit–qubit state [7,8].

As a final remark, although our protocol is DI, we have assumed freedom of choice, fair sampling, and no communication in our experiment. As we show in Supplement 1, closing the detection loophole would require overall system efficiencies above 94%, which is outside the reach of state-of-the-art photonics experiments.

C. State Tomography with the MIC-POVM

To test the tomographic capabilities of our certified MIC-POVM against the standard tomographic methods based on projective measurements, we reconstructed eight different Alice's local qubit states (those naturally occurring in our Bell scenario when we condition Alice's state to Bob's measurements and results) using both methods: first, a standard tomographic analysis from the experimental statistics of three projective measurements (in our case, A_1, A_2, and A_3; that is, σ_x, σ_y, and σ_z), and second, using only the experimental statistics of our single four-outcome measurement. The resulting reconstructed local states should be identical for both methods and, ideally, must point to the corners of a regular tetrahedron in the Bloch sphere. In the case of the MIC-POVM, a simple formula connects the four experimental frequencies produced by the single measurement with the tomographic reconstruction, so

$$\vec{s} = 3 \sum_{j=1}^{4} f_j \vec{A}_j, \tag{10}$$

where $\vec{s}$ is the unknown Bloch vector, each $\vec{A}_j$ is one of the four elements of the symmetric MIC-POVM set [see Eq. (4)], and f_j is its corresponding measured frequency [5]. On the other hand, the six experimental frequencies provided by the three projective measurements were used, through linear inversion [24], to reconstruct the same states. The results of both methods are presented in Fig. 4.

The fidelity between corresponding vectors was, in all eight cases, equal to or greater than 99.5%, indicating that the two tomographic methods yielded near optimally overlapping results, and the four-outcome POVM is informationally complete and effectively symmetric. More detailed results are provided in Supplement 1.
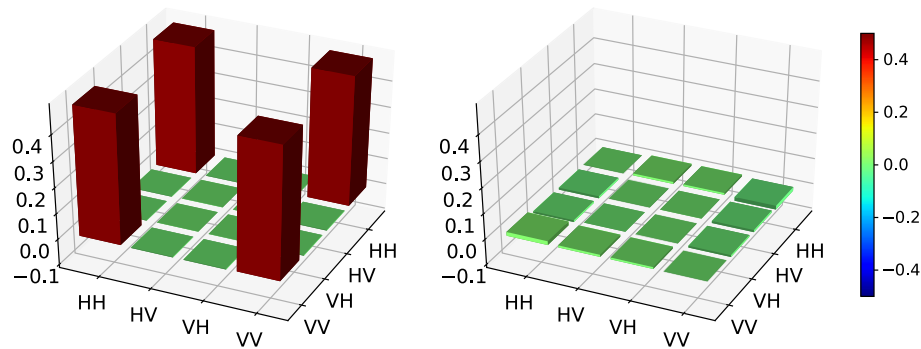
Fig. 3. Tomography of the prepared maximally entangled state. Real (left) and imaginary (right) parts.
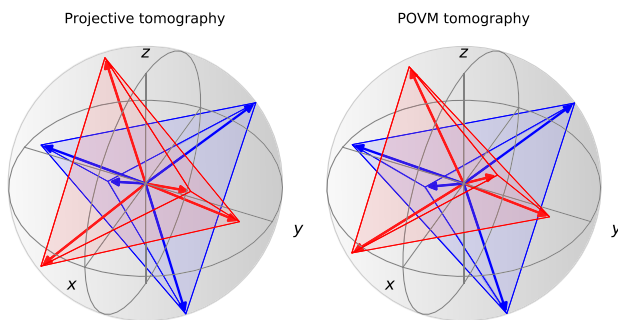

Fig. 4. Reconstruction of eight of Alice's local qubit states, conditioned on Bob's setting and outcome, as obtained from standard projective tomography (left) and MIC-POVM tomography (right).

All the experimental uncertainties reported were calculated with a priori evaluation of known sources of error and subsequent propagation in the results. The sources of error included in our analysis were photon counting statistics, the precision of wave plate rotation, detector dark counts, and higher order down-conversion events (see Supplement 1).

4. CONCLUSIONS

Quantum information identifies MIC-POVMs as the standard quantum measurements for information processing tasks because they are informationally complete and optimal for tomographic and cryptographic purposes. On the other hand, the device-independent paradigm provides the arguably optimal way to test quantum devices since it reduces the assumptions to the minimum. Device-independent tests are especially important within cryptographic scenarios and constitute, in a sense, the highest level of certification attainable with quantum theory. Here, we have reported the results of an experiment certifying for the first time a MIC-POVM for qubits following a device-independent protocol. For that, we have produced correlations between separated photons that, as we have proven, are only attributable to an informationally complete four-outcome measurement on qubits. Our results pave the way toward realistic applications that require device-independent certification of MIC-POVMs, and show how very refined concepts of quantum information are now experimentally attainable and can transform communication and information processing technologies.

Disclosures. The authors declare no conflicts of interest.

See Supplement 1 for supporting content.

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