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Hybrid Method Analysis of Unshielded Guiding Structures

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***Abstract*—A combination of mode matching, finite element methods and generalized impedance matrix is presented in a context of propagation problems for open guiding structures. The computational domain is divided into two regions: the first one is a circular cylinder containing whole guiding structure and the second one surrounds this artificial cylinder. The impedance matrix is calculated with the use of finite element method in the first region and fields outside are expressed by analytical functions. As a last step propagation coefficients are obtained with the use of global roots and poles finding algorithm. The results for simple dielectric ridge waveguides are presented and compared with alternative solutions.**

***Index Terms*—finite element method, open waveguides, generalized impedance matrix, mode matching method**

I. Introduction

Analysis of propagation problems is a relevant topic in microwaves and optical design process. The complexity of the issue increases if the guiding structure is open and leaky or complex modes are an object of interest. There are many methods of analyzing such unshielded guides, depending on their geometrical complexity. For very simple structures such as guides with circular or elliptical cross sections fields outside the rod can be described by Hankel or Mathieu functions. In these particular cases the problem can be solved by utilizing the mode matching (MM) technique [1]–[5]. The main advantage of this method is its high efficiency (low computer resources requirements). However, it is limited to very few cases of simple geometries. Guides with an arbitrary but convex cross section can be analyzed with the use of field matching method [6], [7]. The efficiency and accuracy of this technique is lower than in the previous method, however, the spectrum of different shapes is much wider. More universal techniques are integral equation methods [8]–[11] which can be utilized to analyze rods with arbitrary cross sections. Their efficiency strongly depends on a choice of proper current bases. Furthermore, the usage of Green's functions can be problematic due to singularity points in computational domain. For arbitrary cross sections discrete methods can also be applied. Although methods such as finite element method (FEM) or finite difference (FD) method are predisposed to solve these kinds of problems, modeling of infinite space can be very problematic and affect accuracy. There are many different ways of "proper" domain truncation. The most famous ones are transparent boundary conditions (TBC) [12], [13] and a perfectly matched layer (PML) [13], [14]. However, both have their disadvantages. First one gives much less accurate results than the other, however, the PML requires an additional domain extension. Moreover, utilization of the PML may result in appearance of artificial modes (Berenger's modes [15]), which can be difficult to identify. To decrease numerical costs and increase accuracy hybrid techniques can be utilized [16]. They combine advantages of discrete techniques such as their versatility and advantages of analytical methods such as high accuracy and short computation time. Recently, a method that merges FEM, impedance matrix and MM method was proposed in a context of scattering problems [17].

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In this article similar hybrid technique (a combination of impedance matrix, FEM and MM method) is developed to analyze propagation problems. The computational domain is divided into two regions: the first one is a circular cylinder containing the whole guiding structure and the second one surrounds this artificial cylinder. In the inner region the FEM is utilized and generalized impedance matrix is obtained on the surface of the cylinder. The MM is applied in the outer region of the domain. Eventually, global roots and poles finding algorithm is utilized to calculate complex propagation coefficients. The approach has been verified with alternative methods and has been deemed compatible.

II. Formulation Of The Problem

The considered structure is a dielectric guide with an arbitrary cross section. Let us assume that the object is homogeneous in one direction so it can be analyzed as a 2.5D problem (Fig. 1). In this approach an artificial surface is added, which separates two regions. In the first region the FEM is utilized to calculate the impedance matrix on the artificial surface and in the next step it is combined with external field expressed by a series of Hankel functions. Then,

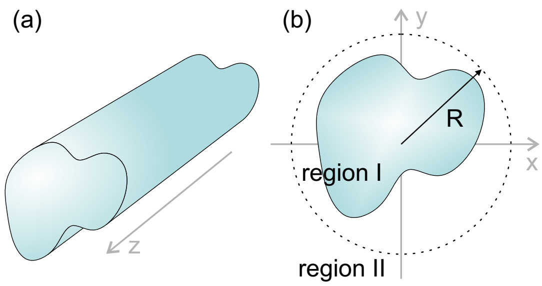

Fig. 1. A dielectric waveguide (a) the geometry (b) cross section.

a homogeneous system of equations is created and propagation coefficients can be obtained from its nontrivial solution.

A. Definition of impedance matrix

The aforementioned impedance matrix is defined on the boundary of region I and describes a relation between tangential components of electric and magnetic fields. Individual components of fields are defined as follows

$$\vec{E}_z(R,\varphi,z) = \sum_{m=-m}^{M} V_{zm}\vec{e}_{zm}(\varphi,z), \tag{1}$$

$$\vec{E}_\varphi(R,\varphi,z) = \sum_{m=-m}^{M} V_{\varphi m}\vec{e}_{\varphi m}(\varphi,z), \tag{2}$$

$$\vec{H}_\varphi(R,\varphi,z) = \sum_{m=-m}^{M} I_{\varphi m}\vec{h}_{\varphi m}(\varphi,z), \tag{3}$$

$$\vec{H}_z(R,\varphi,z) = \sum_{m=-m}^{M} I_{zm}\vec{h}_{zm}(\varphi,z). \tag{4}$$

Each field can take a form of a sum of weights and following basis functions:

$$\vec{e}_{zm}(\varphi,z) = e^{-\gamma z}e^{\mathrm{j}m\varphi}\vec{i}_z, \quad \vec{e}_{\varphi m}(\varphi,z) = -e^{-\gamma z}e^{\mathrm{j}m\varphi}\vec{i}_\varphi,$$
$$\vec{h}_{zm}(\varphi,z) = e^{-\gamma z}e^{\mathrm{j}m\varphi}\vec{i}_z, \quad \vec{h}_{\varphi m}(\varphi,z) = e^{-\gamma z}e^{\mathrm{j}m\varphi}\vec{i}_\varphi.$$

A correlation between fields can be written as follows:

$$\begin{bmatrix}\mathbf{V}_z\\ \mathbf{V}_\varphi\end{bmatrix} = \mathbf{Z}\begin{bmatrix}\mathbf{I}_\varphi\\ \mathbf{I}_z\end{bmatrix} = \begin{bmatrix}\mathbf{Z}_{TM,TM} & \mathbf{Z}_{TM,TE}\\ \mathbf{Z}_{TE,TM} & \mathbf{Z}_{TE,TE}\end{bmatrix}\begin{bmatrix}\mathbf{I}_\varphi\\ \mathbf{I}_z\end{bmatrix}, \tag{5}$$

where $V_z, V_\varphi, I_z, I_\varphi$ are coefficients of electric and magnetic fields, respectively. All the coefficients are the weights of the aforementioned basis functions. The matrix defined in this way has size $2M \times 2M$, where M is a number of modes under consideration. Such approach can also be applied for anisotropic media.

B. FEM in impedance matrix calculations

The relation between electric and magnetic fields, previously named as a generalized impedance matrix, can be obtained with the use of the FEM. All the steps were described in [17]. However, this approach was applied only for scattering problems, where the propagation coefficient γ was known. In propagation issues this coefficient is a point of interest, which implies necessary modifications.

C. Utilization of MM method

Consideration of unshielded structures is associated with modeling of field radiation to infinite space. To model such outgoing waves in second region, fields can be written in following forms

$$E_z^{II}(\rho,\varphi,z) = \sum_{m=-M}^{M} b_m^E H_m^{(2)}(\kappa\rho)e^{-\gamma z}e^{\mathrm{j}m\varphi} \tag{6}$$

and

$$H_Z^{II}(\rho,\varphi,z) = \sum_{m=-M}^{M} b_m^H H_m^{(2)}(\kappa\rho)e^{-\gamma z}e^{\mathrm{j}m\varphi}, \tag{7}$$

where b_m^E, b_m^H are unknown coefficients, $H_m^{(2)}(\cdot)$ is Hankel function of the second kind and mth order, whereas parameter $\kappa^2 = \omega^2\mu_0\varepsilon_0 + \gamma^2$. To ensure the continuity of the tangential components for both inner and outer regions the relation takes the following form:

$$(\mathbf{M}_B^E - \mathbf{Z}\mathbf{M}_B^H)\mathbf{B} = \mathbf{N}(\gamma)\mathbf{B} = 0, \tag{8}$$

where the matrices are expressed by

$$\mathbf{M}_B^E = \begin{bmatrix}\mathbf{H} & 0\\ -\mathbf{H}^\beta & -\mathbf{H}^\mu\end{bmatrix}, \quad \mathbf{M}_B^H = \begin{bmatrix}\mathbf{H}^\varepsilon & \mathbf{H}^\beta\\ 0 & \mathbf{H}\end{bmatrix} \tag{9}$$

and the submatrices are defined as

$$\begin{aligned}
\mathbf{H} &= diag(H_{-M}^{(2)}(\kappa R), ..., H_M^{(2)}(\kappa R)),\\
\mathbf{H}^\beta &= diag(\frac{\mathrm{j}\gamma M}{R\kappa^2}H_{-M}^{(2)}(\kappa R), ..., \frac{-\mathrm{j}\gamma M}{R\kappa^2}H_M^{(2)}(\kappa R)),\\
\mathbf{H}^\mu &= \frac{\mathrm{j}\omega\mu_0}{\kappa}diag(H'^{(2)}_{-M}(\kappa R), ..., H'^{(2)}_M(\kappa R)),\\
\mathbf{H}^\varepsilon &= -\frac{\mathrm{j}\omega\varepsilon_0}{\kappa_1}diag(H'^{(2)}_{-M}(\kappa R), ..., H'^{(2)}_M(\kappa R))
\end{aligned}$$

and R is a radius of the artificial surface surrounding the waveguide, $\mathbf{B} = [\; b_{-M}^E, \; \ldots, \; b_M^E, \; b_{-M}^H, \; \ldots, \; b_M^H]^T$ and $\mathbf{Z}$ is an impedance matrix. Nontrivial solutions exist only if the determinant of $\mathbf{N}$ matrix vanishes for γ representing a propagation coefficient of the considered mode

$$\det(\mathbf{N}(\gamma)) = 0. \tag{10}$$

The roots of the determinant can be found with the use of global complex roots and poles finding algorithm [18].

D. Global complex roots and poles finding algorithm

The algorithm [18] is flexible and can be applied even for functions with branch cuts and singularities. Moreover, it does not require any preliminary knowledge about a root or a pole. It consists of two stages: preliminary estimation and self-adaptive mesh refinement. In the first step the whole domain is covered by a triangular mesh and function is discretized in the vertices of the triangles. In this step each edge is analyzed and has proper quadrant difference assigned; candidates are also evaluated and verified. The last step of the algorithm is mesh refinement, which prevents an improper convergence.

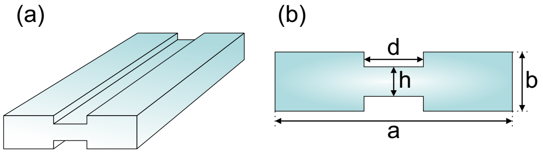

Fig. 2. A dielectric H-structure (a) the geometry (b) cross section.

The algorithm was recently improved [19] and can reduce computational time. Both previous and improved algorithms are easy to parallelize.

III. Numerical Results

In order to verify the validity of the presented approach a few numerical examples are demonstrated. Firstly, a rectangular waveguide with dimensions $a = 6$ cm, $b = h = 1.5$ cm and relative permittivity $\varepsilon_r = 4$ is analyzed at frequency 6 GHz. The convergence of two modes (leaky and guided) is tested and the results are presented in Table I.

TABLE I
Convergence of the method for the structure from Fig. 2 for $h = b$

M	leaky mode	guided mode
5	0.13751 + 0.77479i	1.27030i
7	0.14122 + 0.77562i	1.27069i
9	0.14258 + 0.77598i	1.27073i
11	0.14303 + 0.77603i	1.27072i
13	0.14312 + 0.77603i	1.27072i
15	0.14310 + 0.77604i	1.27070i

According to the convergence table sufficient accuracy is obtained for $M = 11$ and in further simulations this number of modes is set. Four different modes found with the use of current algorithm are confirmed with use of field matching method [6] and two of them are compared to results obtained with commercial software. All the results are collected in Table II.

TABLE II
Normalized coefficients of the investigated waves

	FEM+MM	FM	HFSS
mode A (guided)	1.270i	1.264i	1.281i
mode B (guided)	1.428i	1.432i	1.434i
mode C (leaky)	0.143 + 0.776i	0.146 + 0.761i	
mode D (leaky)	0.197 + 1.062i	0.193 + 1.053i	

As a second example a structure presented in Fig. 2 is investigated. The results are obtained for a guide with dimensions $a = 6$ cm and $b = 1.5$ cm as in a previous example and $d = 1.5$ cm with different length h. The relative permittivity is $\varepsilon_r = 4$ and all the simulations are performed at frequency 6 GHz. The attenuation and phase coefficients are shown in Fig. 4 and Fig. 3, respectively. It can be observed that the propagation coefficients change as a function of length h. The biggest impact on the propagation coefficients can be seen for leaky modes, so they can be considered as the most sensitive.

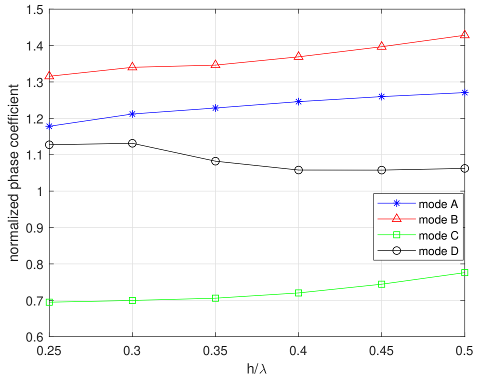

Fig. 3. Normalized phase coefficients of the modes presented in Table II.

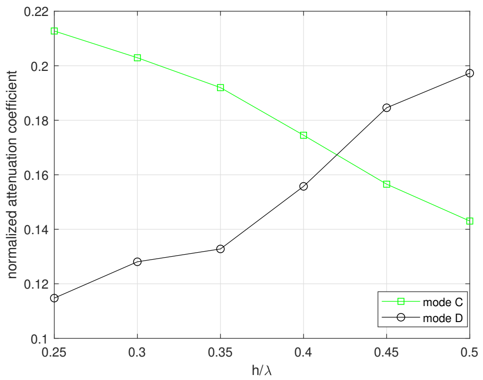

Fig. 4. Normalized attenuation coefficients of the modes presented in Table II.

IV. Conclusion

The combination of the generalized impedance matrix, FEM, MM technique and global roots/poles finding algorithm has been applied to find propagation coefficients for open guiding structures of arbitrary cross sections. The discrete domain is limited only to close vicinity of the waveguide and there is no need for any absorbing conditions, which is the main advantage of this method. Furthermore, this method is free from problems of integral equation methods, i.e. it does not require neither current basis functions nor an integration of Green's functions. Obtained results are compatible with those obtained with field matching technique.

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