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INVERSE ANALYSIS AS A KEY ELEMENT OF SAFETY ASSESSMENT UNDER THE SNOW LOAD FOR THE LARGE SUSPENSION ROOFS STRUCTURE

K. ZOLTOWSKI[1], M. DRAWC[2]

The paper presents a concept and realization of monitoring system for the Silesian Stadium in Chorzow. The idea of the system lies in fusion of structure monitoring with a calibrated numerical FEM model [1]. The inverse problem is solved. On the base of measured selected displacements, the numerical FEM model of the structure combined with iterative method, develops the current snow load distribution. Knowing the load, we can calculate the forces and stresses in each element of the structure and thanks to this we can determine the safety thresholds and asses the owner. Test results and conclusions are presented.

Key words: Structural health monitoring, inverse problem, snow load, safety of the structure, FEM model

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1. INTRODUCTION

The Silesian Stadium in Chorzow (60 thousand spectators) was finally equipped by one of the largest roofs in Europe (surface ~43 thousand m2). The suspension prestressed cable construction is very light and susceptible [2]. Snow is the major load and its impact on safety of the structure is dominating. The Silesian province governor, having in mind a tragic disaster on January 2006 when the structure of exhibition hall failed under snow load killing 65 people [3], ordered to prevent the roof structure from unexpectable snow load.

Removing the snow from the roof is a standard winter maintenance action. However, because of sensitive cladding of the roof, each snow clearing must be done by hand. So, it is dangerous for workers and, additionally, an incompetent snow removal can cause damage to roof surface. In that case 20 thousand m2 should be cleaned. Therefore, a snow load and snow distribution on the roof is a key information to the owner in context of the safety and maintenance. A well-designed monitoring of roof under snow load can notify us of potential hazards and protect against unnecessary roof snow removal. In practice, the real value and the distribution of the snow load on such a vast area is impossible to predict due to the many random parameters, such as wind direction and severity, temperature, and general weather. As we cannot predict the snow load (value and distribution) we cannot indicate the place on the construction which will be meaningful to measure the state of construction and assess the safety. Additionally, structure is heavy nonlinear. The proposed monitoring system solves this problem. The invers procedure is developed. On the base of measured displacements, a snow load and snow distribution is identified.

The invers technique is popular in SHM. Representative examples are published in [4],[5],[6],[7],[8]. They are generally focused to identification of potential failure in structure. Only in [7] and [8] an inverse analysis is used for identification of load. However, the identification of snow load on the big, heavy geometric nonlinear arena roof using invers method seems to be not practiced.

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2. SILESIAN STADIUM IN CHORZOW

2.1. GENERAL DESCRIPTION

The Silesian Stadium structure is based on the idea of a spoke wheel [2]. The main parts of the structure are two compression rings, one tension ring, and forty radial cable girders, spread out between them (Fig. 1). Cable girders consist of a tensioning cable (lower cable), carrying cable (upper cable), and hangers which are connecting them (Fig. 2).

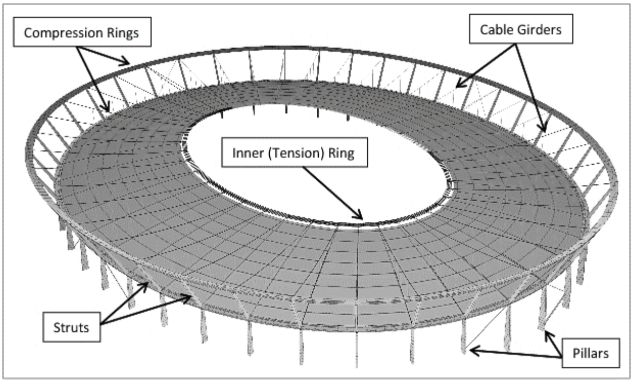

Fig. 1. Structural visualization of the roof over Silesian Stadium in Chorzow

The compression rings and their connection struts are made of steel box sections, while the inner (tension) ring is composed of eight circumferential cables connected by clamps in the axis of each girder. The lower compression ring is based on forty reinforced concrete pillars. These connections have free radial sliding, in order to minimize the influence of temperature. Roof cladding is made of transparent polycarbonate panels which are laid on the secondary steel grid suspended to cable girders on the level of the lower cables.

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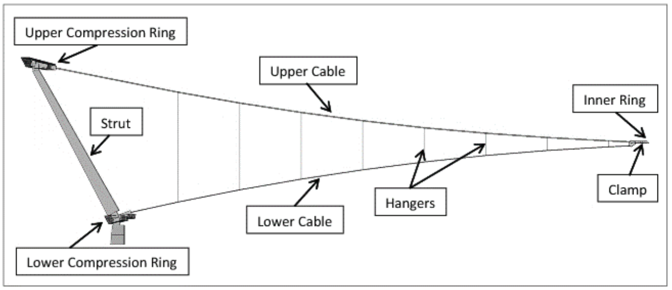

Fig.2. Elements of the cable girder.

2.2. CONCEPT OF THE MONITORING SYSTEM

We can measure deformations and inner forces in every element of the roof system. But the reasonability of such solution is doubtful. The presented idea of the snow load monitoring is based on relation between the finite number of sensors and the FEM model of the structure [1]. A special procedure of the iteration process approaches a definite snow load and snow distribution which determine in the FEM model, values equal to the measured by sensors on structure (with given accuracy). The complete system consists of 4 main parts (see Fig. 3).

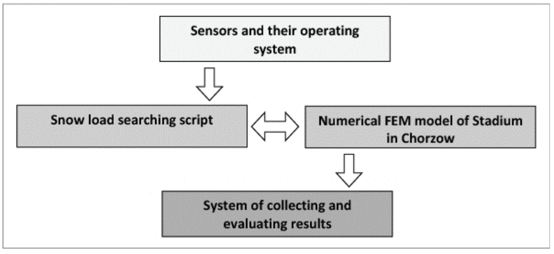

Fig. 3. Four main parts of the monitoring system.

The roof structure of the stadium in Chorzow, due to its light weight and a large surface area, is highly susceptible (up to 1.2 m vertical displacement under the snow load). In that case, a little variation in snow distribution can bring up significant differences in the displacements of the roof surface. After many tests on FEM models, it was found that the shape of surface displacements under snow load resembles this load (intensity and distribution). By tracking displacements, we can recognize the roof snow load. Displacement measurement sensors and temperature sensors transmit data to the Snow Load Identification Processor (SLSS). SLSS assumes approximate load and measured temperature distribution and calculates displacements and compares them with roof displacements actual at

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selected points. SLSS is a simple iterative algorithm that constantly approximates snow loads in the FEM model until the displacements obtained from the model coincide with those of the real structure (Fig. 5). Snow load generated by the script is divided into areas (Fig. 4). Each area has a designated value of the uniform load. Areas with no load are possible, as well the number of areas is equal to the number of sensors. Each sensor is assigned to one surface. The accuracy depends directly on the number of sensors. After several tests a minimum number of sensors was developed (Fig. 4). SLSS was prepared in the FEM SOFiSTiK, environment using its powerful text-editor and script language (CADINP). The script takes advantage of CADINP capabilities such as local/global variables, loops, logic queries and, most importantly, direct access to the SOFiSTiK database. The database is a place where SOFiSTiK stores all the results of calculations and the model information, such as materials and geometry.

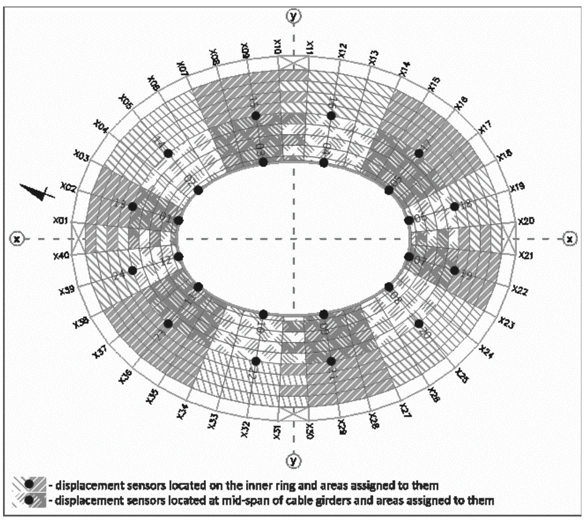

Fig. 4. Displacement sensors and the corresponding areas

In conclusion, we can say that SLSS is a steering code for the FEM computation. Each iteration step requires a nonlinear geometric procedure in FEM computation. After the iteration process the real snow load is identified and SLSS gives then required information about the calculated internal forces in structure. The final step is to collect, store and evaluate, the results in terms of safety of the structure and people. The threshold must be specified for the snow removal and emergency evacuation. The whole process is repeated over and over with the period approximately equal to time of calculation

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needed to determine the snow load and to find the current state of elements. Computation times obtained in our tests did not exceed 20 minutes (standard PC Core I7) which means that the current state of the construction we could check approximately every 20 minutes. The accuracy of the presented procedure depends of two basic elements:

- Number of sensors on the structure (must give sufficient data to SLSS).
- FEM model of the structure (must be verified in by validation process).

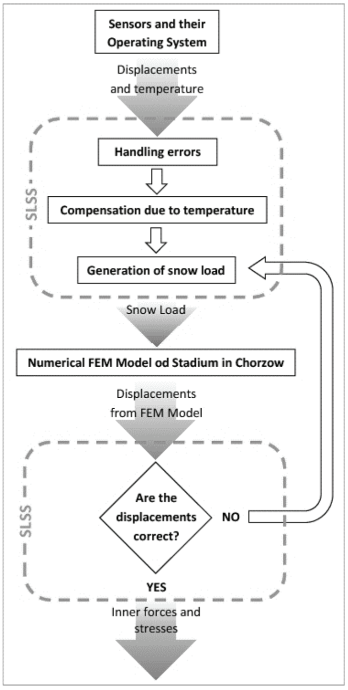

Fig. 5. Snow load search script - flowchart.

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3. VALIDATION OF FEM MODEL

3.1. METHODOLOGY

Validation of the numerical model is crucial to obtain a realistic behavior of the structure under various combinations of environmental loads, and to gain reliable values of deflections and stresses in structural elements of the roof. Validation would rely on the comparison of frequencies and mode shapes of roof cables identified in nature with the theoretical values. This would allow us for a critical evaluation of the model, verification of implementation of the construction, and adjustment of the numerical model by matching the calculated frequencies to the reality. The adjustment would be to change the value of initial prestressing forces of cables.

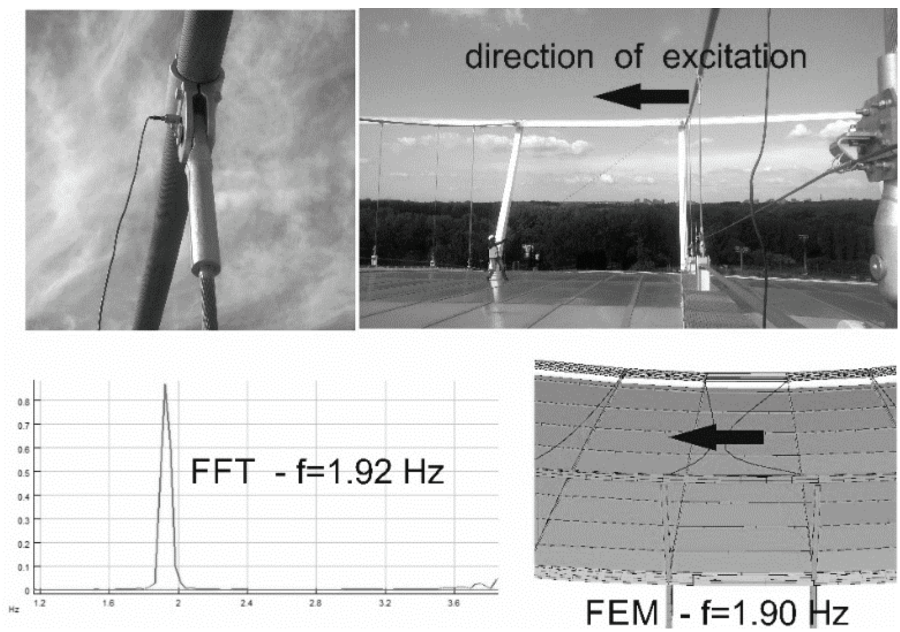

Fig. 6. Validation of FEM model. Excitation of upper cable, FFT and FEM result

The prestress of the roof structure gives 70% of cable load capability. We assume that the distribution of masses and the stiffness matrix are known and in line with the technical documentation. The main part of the validation process concerned a dynamic testing of the real structure, just after the completion of roof cladding. All upper cables and ring cables were excited, response was measured and eigen-frequencies were identified (Fig. 6).

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3.1. RESULTS

The identified forces are close to FEM model results.

In upper cables are in the range of 100% to 108% (medium 103%), and the standard deviation is 1.64%, with the respect to the theoretical (FEM model).

In the tension ring the values are within of 117% to 91% (medium 104%), and the standard deviation is 5.38% with the respect to the theoretical (FEM model).

The identification of forces in the tension ring by the vibration method is much less precise than for the upper cable - due to the short free span between clamps and because of the unknown bending stiffness of cable.

4. NUMERIC TEST OF SLSS

4.1. METHODOLOGY

Tests were performed using two identical numerical FEM models of Stadium in Chorzow. The first model acted as the actual structure named later as “real”. The second one was a part of the SLSS. The “real” snow loads were defined and analyzed. The displacements in place of the planed sensors were read and treated as measured on the “real” structure. These displacements (as “effect of monitoring”) were input to SLSS, as a data from the real structure. Snow load, the displacement of the roof and the forces in the key elements of structure obtained from the SLSS, were compared with those defined as “real”. In theory an inverse problem of big nonlinear static system (22380 unknowns) on the base data from 24 sensors was solved. The presented tests are based on the snow loads borrowed from work " Silesian Stadium in Chorzow near Katowice, Poland: Determination of snow drift / accumulation effects by means of wind tunnel study" done by the WACKER INGENIEURE WIND ENGINEERING company [2]. These are designated in the wind tunnel, with snow loads for four directions of wind $\beta = 0°$, 45°, 90°, and 270°. These wind directions correspond to the main wind directions at site (South, South-West, West and East).

4.2. RESULTS

Load designated by the monitoring is obviously very simplified (Fig. 7). Despite this, there is similarity. Percentage difference between the sum of the vertical reactions of the actual and determined snow load has not exceeded 3.5% (Fig. 8).

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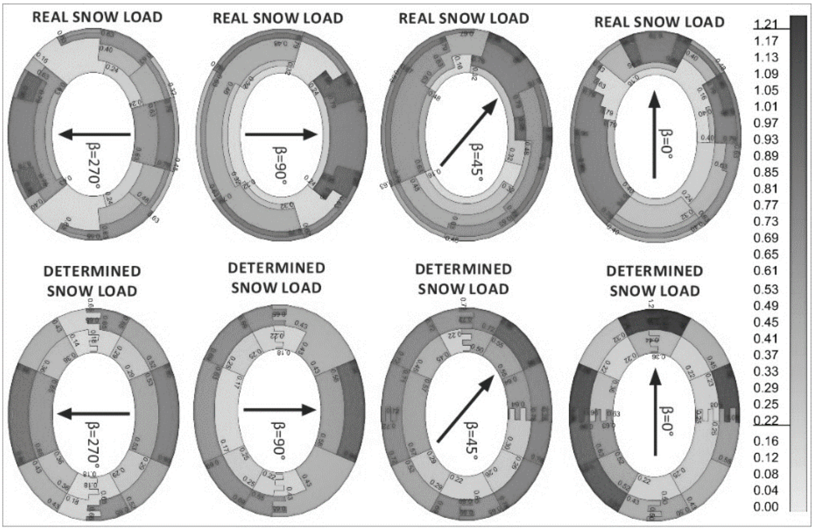

Fig. 7. Comparison of real and determined snow load distribution.

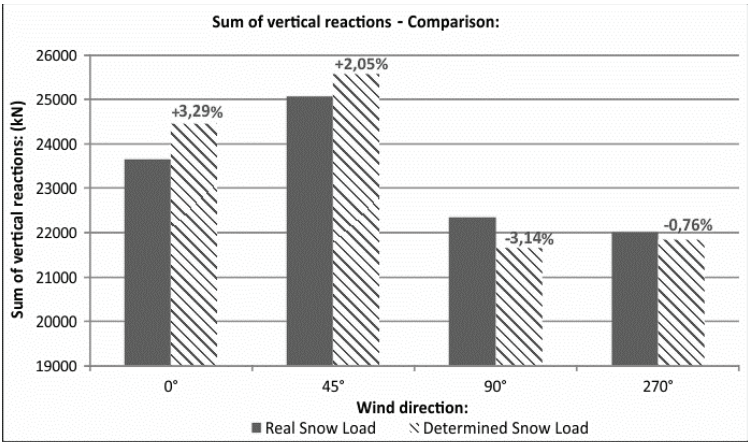

Fig.8. Sum of vertical reactions - comparison.

When it comes to displacements, SLSS shows very good accuracy at the points where the displacements are large. However, in the case of very small deflections (less than 10mm), the differences can be big.

- Maximum error expressed in millimeters did not exceed 35 mm, minimum -38 mm,

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- Average error is 4 mm.
- Standard deviation is 15 mm.

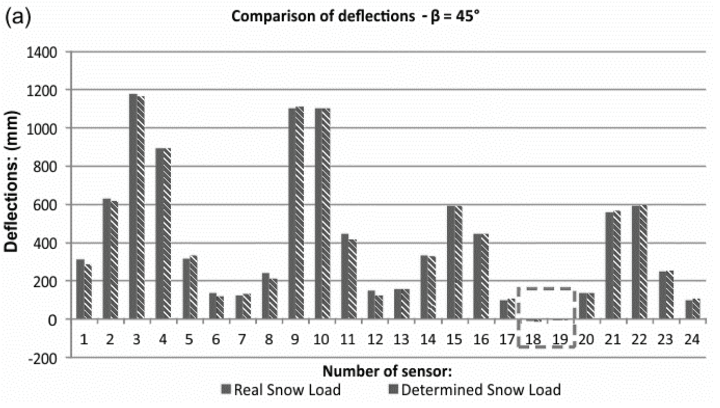

Fig. 9. Comparison of deflections β = 45° [mm]

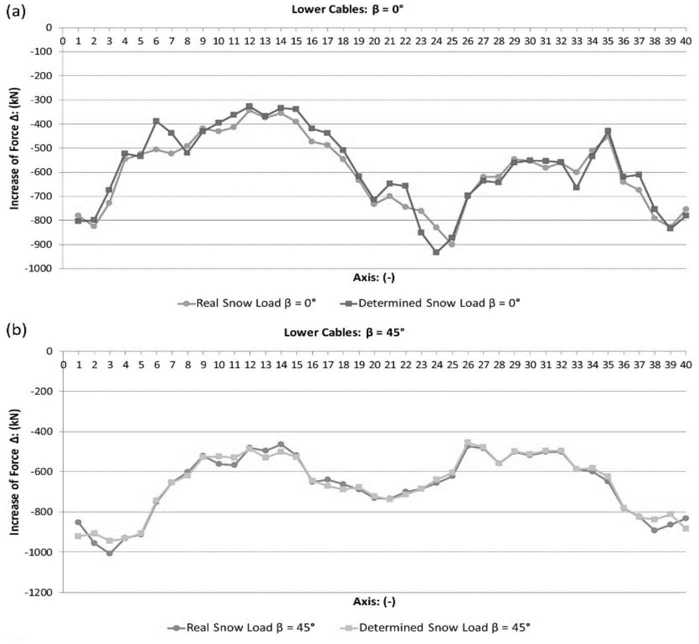

Figure 10. Increase of force in lower cables: (a) β = 0°, (b) β = 45°

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From technical point of view this is a good result because the maximum deflection of the characteristic snow load (variant full snow) specified by standards is about 1300 mm (Fig.9). The charts show the increments of "real" and predicted forces in upper cables caused by the analyzed snow loads (Fig. 10).

4.2. DISCUSSION

The extreme obtained force increments under the snow load was 68% to 129% of a real one for a single lower cable and 75% to 113.7 % of a real one for an upper cable (see Table 1). These significant inaccuracies are related to only a few cables of a set of 40. They result from simplified snow load adopted by the monitoring and do not apply to every snow load. Some loads are imitated better than others. Most of the cable forces has a good compatibility with reality. Max. sd (standard deviation) reaches value 12.6% but the global SD calculated from sd components presented in Tab. 1 reaches value 3.55%.

Table 1. Relative difference between force increments obtained from the real and the determined snow load.

		Cables:			Outer Rings:	
		Upper Cables:	Lower Cables:	Inner Ring Cables:	Upper Ring:	Lower Ring:
β = 0°	Max:	113.7%	113.0%	106.0%	102.0%	101.0%
	Min:	86.0%	77.0%	102.0%	98.0%	89.0%
	sd	**6.0%**	**7.4%**	**1.7%**	**1.5%**	**4.5%**
β = 45°	Max:	106.0%	108.0%	101.0%	0.9%	102.0%
	Min:	93.0%	93.0%	99.0%	0.1%	98.0%
	sd	**2.9%**	**3.7%**	**0.2%**	**0.6%**	**1.6%**
β = 90°	Max:	105.0%	111.0%	95.0%	96.0%	98.0%
	Min:	75.0%	70.0%	93.0%	94.0%	94.0%
	sd	**7.3%**	**9.0%**	**0.5%**	**0.7%**	**2.0%**
β = 270°	Max:	113.0%	129.0%	97.0%	98.0%	100.0%
	Min:	76.0%	68.0%	96.0%	97.0%	97.0%
	sd	**8.9%**	**12.6%**	**0.5%**	**0.4%**	**1.1%**

The differences in the obtained values were subtle. In addition to the presented tests many others have been carried out during the test phase of the system, mainly on the quasi-random loads. Obtained accuracy did not differ much from those described in the article. Referring to the safety of the structure [10], it should be noticed that the maximum increase in forces in upper cables under the code snow load is reaching 21% of the total force in this cable. In ring cables this ratio is 8.8%. The rest of the force comes mainly from the prestressing of the structure.

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Described idea was implemented to monitoring system of the roof [14]. Displacement monitoring was carried out using two Leica Nova TM50 instruments with technical measurement accuracy of ~2 mm operating in automatic mode. In addition, 20 temperature and strain sensors were placed on the cable structure. The effectiveness of the implemented solutions will be assessed after a trial period of system operation. Unfortunately, in recent years there was practically no snow in Silesia and there was no chance to check effectiveness of SLSS.

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Tab. 1. Relative difference between force increments obtained from the real and the determined snow load.

Tab. 1. Różnice w przyroście siły w cięgnach od rzeczywistych i zidentyfikowanych obciążeń śniegiem.

ROZWIĄZANIE PROBLEMU ODWROTNEGO JAKO KLUCZOWY ELEMENT OCENY BEZPIECZEŃSTWA POD OBCIĄŻENIEM ŚNIEGIEM DLA DUŻEGO ZADASZENIA WISZĄCEGO

Słowa kluczowe: Monitoring konstrukcji, problem odwrotny, obciążenie śniegiem, bezpieczeństwo, model MES

STRESZCZENIE

Stadion Śląski w Chorzowie (60 tys. widzów) został ostatecznie przykryty jednym z największych dachów w Europie (powierzchnia ~ 43 tys. m^2). Wisząca konstrukcja sprężona jest bardzo lekka i podatna (rys. 1). Śnieg jest głównym obciążeniem, a jego wpływ na bezpieczeństwo konstrukcji jest dominujący. Wojewoda Śląski, mając na uwadze tragiczną katastrofę w styczniu 2006 r., kiedy konstrukcja hali wystawowejj zawiodła pod obciążeniem śniegowym, zabijając 65 osób, nakazał zapobiegać nieoczekiwanemu obciążeniu konstrukcji dachu.

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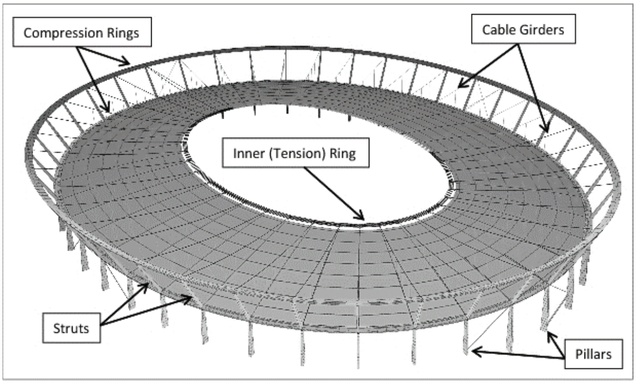

Rys. 1. Wizualizacja konstrukcji zadaszenia Stadionu Śląskiego w Chorzowie

Usuwanie śniegu z dachu jest standardowym działaniem zimowym. Jednak ze względu na delikatne poszycie dachu każde odśnieżanie należy wykonywać ręcznie. Jest to niebezpieczne dla pracowników, a ponadto niekompetentne odśnieżanie może spowodować uszkodzenie powierzchni dachu poliwęglanowego. Wg instrukcji należy wyczyścić 20 tys. m^2. Dlatego obciążenie śniegiem i rozkład śniegu na dachu to kluczowe informacje dla właściciela w kontekście bezpieczeństwa i działań utrzymaniowych. Odpowiednio zaprojektowany monitoring dachu może powiadomić o potencjalnych zagrożeniach i zabezpieczyć przed niepotrzebnym odśnieżaniem. W praktyce nie można przewidzieć rzeczywistej wartości i rozkładu obciążenia śniegiem na tak rozległym obszarze ze względu na wiele losowych parametrów, takich jak kierunek i natężenie wiatru, temperatura i ogólna pogoda. Ponieważ nie jesteśmy w stanie przewidzieć obciążenia śniegiem (wartości i rozkładu), nie możemy wskazać miejsca na konstrukcji, które będzie reprezentatywne dla oceny bezpieczeństwa. Dodatkowo konstrukcja cechuje się silną nieliniowością geometryczną. Proponowany system monitorowania rozwiązuje ten problem. Opracowano procedurę odwrotna. Na podstawie zmierzonych przemieszczeń identyfikowane jest obciążenie śniegiem i rozkład śniegu. Technika odwrotna jest popularna w systemach SHM. Metoda ta zasadniczo koncentruje się na identyfikacji potencjalnych uszkodzeń konstrukcji. Znane są wprawdzie zastosowania analizy odwrotnej do identyfikacji obciążenia. Niemniej jednak zdaniem autorów przedstawiona metoda identyfikacji obciążenia śniegiem na dużej, silnie geometrycznie nieliniowej konstrukcji dachu wiszącego nie była dotychczas stosowana.

Konstrukcja dachu stadionu w Chorzowie ze względu na niewielki ciężar i dużą powierzchnię jest bardzo podatna (ugięcie pod normowym śniegiem może osiągnąć wartość ~1.2 m). W takim przypadku niewielka zmienność rozmieszczenia śniegu może spowodować znaczne różnice w przemieszczeniach powierzchni dachu. Po wielu testach na modelach MES stwierdzono, że kształt deformacji powierzchni pod obciążeniem śniegiem przypomina to obciążenie (intensywność i rozkład). Dzięki śledzeniu tych przemieszczeń możemy rozpoznać obciążenie śniegiem dachu. Czujniki pomiaru przemieszczeń i temperatury przekazują dane do procesora identyfikującego obciążenia śniegiem (SLSS). SLSS przyjmuje przybliżony rozkład obciążenia oraz pomierzoną temperaturę, wylicza przemieszczenia i porównuje je z rzeczywistymi przemieszczeniami konstrukcji dachu w wybranych punktach. SLSS jest prostym algorytmem iteracyjnym, który nieustannie dokładniej przybliża obciążenia śniegiem w modelu MES, dopóki przemieszczenia otrzymane z modelu nie zbiegną się z przemieszczeniami rzeczywistej struktury. Obciążenie śniegiem generowane przez

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skrypt jest podzielone na obszary. Każdy obszar ma wyznaczoną wartość równomiernego obciążenia. Możliwe są obszary bez obciążenia, a liczba obszarów jest równa liczbie czujników. Każdy czujnik jest przypisany do jednej powierzchni. Dokładność zależy bezpośrednio od liczby czujników. Po kilku testach opracowano minimalną liczbę czujników. SLSS został przygotowany w środowisku FEM SOFiSTiK, z wykorzystaniem edytora tekstu i języka skryptowego (CADINP). Skrypt wykorzystuje możliwości CADINP, takie jak zmienne lokalne / globalne, pętle, działania logiczne i, co najważniejsze, bezpośredni dostęp do bazy danych SOFiSTiK.
Podsumowanie wyników przedstawiono w Tab. 1. (**sd** – odchylenie standardowe)

Tabela 1. Różnice w przyroście siły w cięgnach od rzeczywistych i zidentyfikowanych obciążeń śniegiem.

		Liny			Pierścienie	zewnętrzne:
		Liny nośne:	Liny napinające:	Liny pierścienia:	Górny pierścień:	Dolny pierścień:
β = 0°	Max:	113.7%	113.0%	106.0%	102.0%	101.0%
	Min:	86.0%	77.0%	102.0%	98.0%	89.0%
	sd	**6.0%**	**7.4%**	**1.7%**	**1.5%**	**4.5%**
β = 45°	Max:	106.0%	108.0%	101.0%	0.9%	102.0%
	Min:	93.0%	93.0%	99.0%	0.1%	98.0%
	sd	**2.9%**	**3.7%**	**0.2%**	**0.6%**	**1.6%**
β = 90°	Max:	105.0%	111.0%	95.0%	96.0%	98.0%
	Min:	75.0%	70.0%	93.0%	94.0%	94.0%
	sd	**7.3%**	**9.0%**	**0.5%**	**0.7%**	**2.0%**
β = 270°	Max:	113.0%	129.0%	97.0%	98.0%	100.0%
	Min:	76.0%	68.0%	96.0%	97.0%	97.0%
	sd	**8.9%**	**12.6%**	**0.5%**	**0.4%**	**1.1%**

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