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On the problem of optimised allocation of water quality sensors and actuators in DWDS

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***Abstract*—The problems of water quality sensors and actuators placement in drinking water distribution systems (DWDSs) are addressed as separate, primarily. However, against the background of control systems theory, the nature of DWDSs dynamics indicates that these both problems are interdependent and impact the design of related water quality monitoring and control structures and algorithms. The research work presented in this paper is to investigate the state-of-the-art in this field and discuss the problems of water quality sensors and actuators placement within DWDS and to highlight the potential benefits of considering the joint task of their allocation.**

I. Introduction

One of the most important factors defining modern society is its dependence on complex, interdependent infrastructure systems, which due to their importance and role are often called critical infrastructure systems (CISs) [1]. Drinking water distribution systems (DWDSs) are one example of such systems, which aim at delivering to the consumers the desired amount of clean and safe water (satisfying the quality requirements) [2]. Also, they comprise a very specific type of networked systems, i.e., a reactive carrier–load networked system where one can distinguish a medium carrying a load of a certain quality [3]. They interact but the relationship is only one way, from the carrier (hydraulic flow) to the load (water quality) [4]. The quality of the transported load can be altered by physical or chemical mechanisms resulting from occurring processes. These processes result from DWDS operation and in the majority of cases are guided by non-linear dynamics [3]. Due to the vast impact of DWDSs on health or even life of the population served, adequate monitoring and control actions, including security, are needed to assure continuous and reliable operation in a wide range of operational states and conditions (i.e., including normal, disturbed and emergency) [5]. Achieving this goal is a demanding task, hence, adequate on-line control and monitoring systems are needed. However, to deploy effective structures and algorithms able to serve the aforementioned purposes, proper allocation of water quality (understood as a disinfectant, free chlorine, concentration) sensors and actuators, namely monitoring stations (points) and (disinfectant) booster stations, respectively, needs to be performed. Finding a satisfactory solution to this task(s), usually addressed dis-jointly, involves the selection of suitable algorithms and numerical methods.

Addressing the water monitoring system, its goal is to provide information on the state of the DWDS. Typically, such a system resembles a cascading structure comprising the water quantity (hydraulics) and quality monitoring systems, respectively. It is obvious that these systems require on-line measurements. Unfortunately, placing the water quantity and quality hard sensors at all proper elements of DWDS is not possible, e.g., due to access limitations for their installation. Henceforth, the hard measurement sensor information needs to be supplied with estimates by so-called soft-sensors. This concept for the water quantity monitoring system has been discussed, e.g., in [6]. In turn, the goal of the quality monitoring system is typically set up to provide the information on the (free) chlorine concentration throughout the DWDS. This information is typically based on the chlorine soft-sensors as well as on-line measurements acquired at the DWDS nodes, namely tanks and pipe junctions equipped with sensors of chlorine concentration [7]. According to the subject of this paper, the problem of water quality hard sensors placement has been investigated. In the literature, this issue has been presented for years, however, no universal method of determining their location has been derived [8]. Proposed allocation algorithms are typically formulated using (multi-) objective optimisation framework. For example, a mixed-integer linear programming based sensor allocation scheme targeting contamination detection has been proposed in [9], for which purpose five distinct objectives have been formulated. Another approach that is based on maximising a water demand coverage at the DWDS can be found in [10], [11]. A comprehensive survey on the topic of sensor allocation can be found in [12]. In previous authors works, an optimising approaches to the chlorine concentration sensors placement at the DWDS have been presented [12]–[14]. In general, the devised approaches achieve the desired trade-off between the sensors and their maintenance costs, under several water demand scenarios composed of the water demand patterns and the chlorine estimation performance. In particular, a single- and bi-objective optimised sensor allocation problems have been given in [13], whereas a multi-objective allocation problem has been discussed in [14]. In turn, the detailed research on modelling, estimation with the interval observer and the optimised robust placement of the hard sensors of the chlorine concentration can be found in [7], [12].

In turn, works both on DWDS structure optimisation and control optimisation in the context of quantity or quality, e.g., chlorine injection scheduling can be found in [15], [16] and [17]–[20], respectively. As it has been aforementioned, the design of water quality control system is closely linked to the allocation of booster stations. In previous authors works, an integrated approach to booster stations allocation and open-loop as well as closed-loop control system design has been addressed in [21]–[24]. In particular, a multi-objective approach with induced Pareto order in the solution space has been presented in [21]–[23]. In contrast, a joint closed-loop model-based predictive control (MPC) algorithm design and booster stations allocation problem has been introduced in [24].

It should be added that the above-mentioned methodologies are based on models of processes occurring within DWDS which include: hydraulic transport (quantity) [2] and water quality affected by the biochemical processes occurring within the transported medium. In the case of the later, several model fractions can be distinguished in the literature. These can be grouped into three categories, namely: empirically derived black-box models with parameters laboriously fit to measurement data [25]; dynamic models of first [26] or second-order [27]; non-linear dynamic models [25], [28]–[33]. First two groups of models are typically used to model the disinfectant decay. The third group extends the disinfectant decay by considering factors such as: biological regrowth [30], [31], corrosion [32] and disinfection by-products (DBPs) formation [25], [33].

Instead, in this paper, a joint approach to the problem of allocation of chlorine sensors and booster stations at a DWDS is presented. Clearly, an integrated optimisation task to the water quality sensors and actuators placement has been formulated. Hence, the main contribution of this paper is that it aggregates the state-of-the-art solutions to the monitoring and MPC systems design for DWDSs and indicates potential new research areas in the field supported by initial observations made.

II. DWDS DESCRIPTION

In general, a DWDS is a collection of links and nodes. Typically, links in a DWDS description are to represent physical infrastructure elements such as: pipes ($\omega_{\mathrm{P}} \in \Omega_{\mathrm{P}}$), valves ($\omega_{\mathrm{V}} \in \Omega_{\mathrm{V}}$) and pumps or pump stations ($\omega_{\mathrm{M}} \in \Omega_{\mathrm{M}}$). In turn, nodes comprise a collection of: physical piping connections – joints ($\omega_{\mathrm{J}} \in \Omega_{\mathrm{J}}$), mixing the inflows and redirecting medium into outflows including the user demand; tanks ($\omega_{\mathrm{T}} \in \Omega_{\mathrm{T}}$); and reservoirs ($\omega_{\mathrm{R}} \in \Omega_{\mathrm{R}}$) that represent, e.g., water treatment works or very large, in terms of capacity, medium storage. It should be noticed that the provided description of a DWDS allows one to interpret this plant in terms of a directed graph, which immediately suggests a possible set of methods and tools applicable to the problem. However, it should be added that conceptually, the problem is 'multi-layered' in nature as the graph describing the hydraulic behaviour has a time-dependent structure as the flows may vary not only in terms of their magnitude but the direction as well. Also, the hydraulic flow interacts with the quality process in the bio-chemical flow reactors represented by the links and flow-through type reactors represented by nodes. Following this description, the following characterisation justifies the underlying complexity of monitoring and control design tasks.

Considering that the hydraulic system description is well established in the literature [2] to improve the legibility of this work, it is worth adding a compact description of the underlying quality processes. To that goal, a typical approach is to assume that the transport mechanisms are considered only in the axial direction of the pipe and that mixing in the tanks and at the nodes of the DWDS is considered to be ideal. Thereafter, the phenomena involved in the transportation of medium in pipes are guided by the advective-reactive scheme given, for each $\omega_{\mathrm{P}} \in \Omega_{\mathrm{P}}$, by [28]:

$$\partial_t C_{i_{\mathrm{P}}}^{i_{\mathrm{sc}}} = \begin{cases} -v_{i_{\mathrm{P}}} \partial_{z_{i_{\mathrm{P}}}} C_{i_{\mathrm{P}}}^{i_{\mathrm{sc}}} + \Xi_{\mathrm{P}}^{i_{\mathrm{sc}}} & \text{, in bulk flow} \\ \Xi_{\mathrm{P}}^{i_{\mathrm{sc}}} & \text{, at pipe wall} \end{cases}, \quad (1)$$

where: $\partial_{(\cdot)}$ denotes a partial derivative with respect to $(\cdot)$; $v_{i_{\mathrm{P}}} \stackrel{\text{def}}{=} v_{i_{\mathrm{P}}}(t)$ is the medium linear velocity; $z_{i_{\mathrm{P}}}$ denotes the distance from the beginning of the i_{P}th pipe and $z_{i_{\mathrm{P}}} \in [0; z_{i_{\mathrm{P}}}^{max}]$ where $z_{i_{\mathrm{P}}}^{max}$ is the i_{P}th pipe length; $C_{i_{\mathrm{P}}}^{i_{\mathrm{sc}}} \stackrel{\text{def}}{=} C_{i_{\mathrm{P}}}^{i_{\mathrm{sc}}}(t, z_{i_{\mathrm{P}}}, v_{i_{\mathrm{P}}})$ stands for the i_{sc}th (chemical, biological or bio-chemical) species concentration in the i_{P}th pipe; $\Xi_{\mathrm{P}}^{i_{\mathrm{sc}}} \stackrel{\text{def}}{=} \Xi_{\mathrm{P}}^{i_{\mathrm{sc}}}(\mathbf{C}_{i_{\mathrm{P}}}, v_{i_{\mathrm{P}}}, s_{i_{\mathrm{P}}})$ represents the quality processes dependent on all species concentration and $s_{i_{\mathrm{P}}}$ is the contact surface of the pipe wall; t is time instant. It should be noted, that the interaction between the bulk and wall species is integrated into the $\Xi_{\mathrm{P}}^{i_{sc}}$ [28].

The tanks of the DWDS are included by considering, for each $\omega_{\mathrm{T}} \in \Omega_{\mathrm{T}}$ [28]:

$$\mathrm{d}_t \left(C_{i_{\mathrm{T}}}^{i_{\mathrm{sc}}} V_{i_{\mathrm{T}}} \right) = F_{\mathrm{in}} C_{\mathrm{in}\, i_{\mathrm{T}}}^{i_{\mathrm{sc}}} - F_{\mathrm{out}} C_{i_{\mathrm{T}}}^{i_{\mathrm{sc}}} + V_{i_{\mathrm{T}}} \Xi_{i_{\mathrm{T}}}^{i_{\mathrm{sc}}}, \quad (2)$$

where: $\mathrm{d}_{(\cdot)}$ denotes a derivative with respect to $(\cdot)$; F_{in}, F_{out} are the inflow and outflow, respectively; $C_{\mathrm{in}\, i_{\mathrm{T}}}^{i_{\mathrm{sc}}}$ is the concentration of the i_{sc}th species in the i_{T}th tank inflow; $V_{i_{\mathrm{T}}} \stackrel{\text{def}}{=} V_{i_{\mathrm{T}}}(t)$ stands for the volume of the medium contained in the i_{T}th tank; $\Xi_{i_{\mathrm{T}}}^{i_{\mathrm{sc}}} \stackrel{\text{def}}{=} \Xi_{i_{\mathrm{T}}}^{i_{\mathrm{sc}}}(\boldsymbol{C}_{i_{\mathrm{T}}})$ signify the quality processes dependent on all species concentration and it is assumed that may be different than $\Xi_{\mathrm{P}}^{i_{\mathrm{sc}}}$. Moreover, (2) allows one to formulate model for junction nodes as well.

The reaction processes for the pipes and tanks share the same structure and are described by: $\boldsymbol{\Xi}_{(\cdot)} = \boldsymbol{\Lambda}\, \boldsymbol{r}_{(\cdot)}$, where: $(\cdot) \in \{\mathrm{P}, \mathrm{T}\}$ is to linguistically indicate either a tank or pipe; $\boldsymbol{\Xi}_{(\cdot)} \in \mathbb{R}^{n_{(\cdot)}}$ and $n_{(\cdot)}$ denotes the total number of processes; $\boldsymbol{\Lambda}$ is the stoichiometric coefficient matrix; $\boldsymbol{r}_{(\cdot)}$ is the fundamental (non-linear) process rate vector and $\boldsymbol{r}_{(\cdot)} \in \mathbb{R}^{n_{\mathrm{z}}}$ where n_{z} denotes the total number of fundamental processes [28].

In consequence, the feasibility of the monitoring or control system design task is strictly determined by the interdependency of the aforementioned DWDS dynamics with the placement of the sensors or actuators. It should be noted that the dominant role is related to system dynamics which is

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characterised by spatial distribution and non-linearity. Moreover, the influence of periodically time-varying disturbances as well as the requirement for the capability to operate in multiple operating states and conditions makes the task even more challenging.

III. Resource allocation problem

This section is structured as follows. Subsection III-A presents a generic (multi-objective) formulation of resource allocation task. Consequently, in subsections III-B and III-C, the generic formulation is refined towards design of monitoring system and sensor placement, and control system and actuator placement, respectively. Finally, a joint task is formulated in subsection III-D.

A. Generic formulation

Take $\boldsymbol{x}$ to denote a vector of the decision variables, set up to represent the resources to be allocated. This vector is selected as a binary word:

$$\boldsymbol{x} \stackrel{\text{def}}{=} [x_1, x_2, \ldots, x_{n_\text{f}}]^T, \tag{3}$$

where: $n_\text{f} = \overline{\overline{\Omega}}_\text{J f}$ is the length of $\boldsymbol{x}$; $\overline{\overline{(\cdot)}}$ denotes a cardinal number of $(\cdot)$; $\Omega_\text{J f}$ signifies an ordered set of feasible admissible junction nodes for resource allocation. The $\Omega_\text{J f}$ is understood in terms of $\Omega_\text{J} = \Omega_\text{J f} \cup \Omega_\text{J uf} \wedge \Omega_\text{J f} \cap \Omega_\text{J uf} = \emptyset$ with $\Omega_\text{J uf}$ standing for the set of locations assumed unfeasible. It should be noted that in case of booster stations, the information on location, in general, is considered to be coupled with the related control input trajectories in the form of disinfectant injections patters (a one-to-one relation). Moreover, the location of the reservoirs is known *a priori*, however, the sensors or actuators location is only known *a posteriori*. The later fact is important to the methods targeting the design of booster stations location and the related injection patterns.

Thus, the feasible problem set (Ω_FPS), comprising the task constraints, is set up not only to include the information on DWDS dynamics but also in order to constrain the possible choice of $\boldsymbol{x}$ to its admissible values ($\boldsymbol{x} \in \Omega_\text{FPS}$). The set of admissible resource location ($\Omega_\text{J f}$) is, in principle, dependent on the structure of a given DWDS. Typically, the criteria used to assess the allocation scheme ($\boldsymbol{x}$) aim at optimising the economical cost factors ($\boldsymbol{J}_\text{c} \in \mathbb{R}^{n_\text{c}}$) while assuring high performance ($\boldsymbol{J}_\text{q} \in \mathbb{R}^{n_\text{q}}$) of the system under design. Often, the performance of a given monitoring or control system is increased by reducing the related estimation or control error ($\boldsymbol{J}_\text{e} \in \mathbb{R}^{n_\text{e}}$). Hence, $\boldsymbol{J}_\text{q} \propto \alpha\, \boldsymbol{J}_\text{e}^{-1}$, where α is to indicate that the relation is not necessarily direct[1].

Finally, a general statement of a multi-objective resource allocation task reads:

$$\Omega_\text{OSS} \stackrel{\text{def}}{=} \min_{\boldsymbol{x} \in \Omega_\text{FPS}} \boldsymbol{J}(\boldsymbol{x}), \tag{4}$$

where: $\boldsymbol{J} \stackrel{\text{def}}{=} \left[\boldsymbol{J}_\text{c}^T, \boldsymbol{J}_\text{e}^T\right]^T$ and $\Omega_\text{OSS} \subset \Omega_\text{FSS}$ is an optimal solution set contained within a feasible solution set ($\Omega_\text{FSS} \stackrel{\text{def}}{=} \boldsymbol{J}(\Omega_\text{FPS})$) optimised under prescribed, typically Pareto, order. The exact characterisation of the above task, including DWDS specific details provided in section II and relation to a specific resource allocation task is discussed in the following subsections.

[1]Depending on the case considered, α might represent a mapping.

B. Monitoring system and sensors placement

As it has been mentioned previously, the proposed water quality monitoring system delivers information about disinfectant (chlorine) concentration in the form of measurements and estimates throughout the whole DWDS. The source of measurements is chlorine concentration hard sensors, whereas the chlorine concentration estimates (soft-sensors) are based on chlorine concentration measurements and appropriate mathematical models. Therefore, an optimal allocation of chlorine concentration sensors (monitoring stations) should provide the desired trade-off between the whole costs of sensors and the defined accuracy of chlorine concentration estimates. Moreover, the wide range of system operation modelled, e.g., by different water demand scenarios should be taken into account. Thus, for monitoring stations placement purposes, according to (3), the following notation is introduced: $n_\text{sf} \stackrel{\text{def}}{=} n_\text{f}$ to indicate the length of decision vector allocating hard sensors to the feasible junction nodes belonging to the set of admissible junction nodes: $\Omega_\text{J sf} \stackrel{\text{def}}{=} \Omega_\text{J f}$. Moreover, taking $\mu_{(\cdot\cdot)}(\cdot)$ to indicate a membership of $(\cdot)$ in $(\cdot\cdot)$ first constraint is formulated as a set of junction nodes where the hard sensors have been allocated (Ω_s). This set is identified by selecting nodes such that $\forall i_\text{sf} \in \overline{1, n_\text{sf}} : \mu_{\Omega_\text{s}}(\omega_{\text{J sf}\, i_\text{sf}}) = x_{i_\text{sf}}, \omega_{\text{J sf}\, i_\text{sf}} \in \Omega_\text{J sf}$. Practical considerations, e.g., available investment funds or installation possibilities, give rise to a second constraint, namely the bounds on the total number of monitoring stations to be allocated ($n_\text{s}(\boldsymbol{x}) \stackrel{\text{def}}{=} \overline{\overline{\Omega}}_\text{s}$), where: $n_\text{s}(\boldsymbol{x}) \in [n_{\text{s}\min}, n_{\text{s}\max}]$.

The last of the constraints required to construct the Ω_FPS is the plant model (Section II) and the observer dynamics, where the latter, for legibility of the work, is introduced in the following lines.

In the considered case, the following objectives are defined to assess the economy of the allocation and the related performance of the monitoring system, such that $\boldsymbol{J}_\text{c} \equiv J_\text{c}$ and $\boldsymbol{J}_\text{e} = \left[J_{\text{e}\,1}, \ldots, J_{\text{e}\,n_\mathfrak{z}}\right]^T$. Thus, J_c is given by [12], [14]:

$$J_\text{c}(\boldsymbol{x}) \stackrel{\text{def}}{=} n_\text{s}(\boldsymbol{x}). \tag{5}$$

In turn, the objectives $\boldsymbol{J}_\text{e}$, $\forall i_\mathfrak{z} \in \overline{1, n_\mathfrak{z}}$ reads [12], [14]:

$$\begin{aligned} J_{\text{e}\,i_\mathfrak{z}} \stackrel{\text{def}}{=} & \sum_{m_\text{j}=1}^{\overline{\overline{\Omega}}_\text{MJ}} \sum_{k=1}^{K} \left[C^{+}_{m_\text{j}, i_\mathfrak{z}}(k)(\boldsymbol{x}) - C^{-}_{m_\text{j}, i_\mathfrak{z}}(k)(\boldsymbol{x})\right] \\ & + \sum_{i_\text{T}=1}^{\overline{\overline{\Omega}}_\text{T}} \sum_{k=1}^{K} \left[C^{+}_{i_\text{T}, i_\mathfrak{z}}(k)(\boldsymbol{x}) - C^{-}_{i_\text{T}, i_\mathfrak{z}}(k)(\boldsymbol{x})\right], \end{aligned} \tag{6}$$

subject to: $C^{\pm}_{m_\text{j}, i_\mathfrak{z}}(k)(\boldsymbol{x}), C^{\pm}_{i_\text{T}, i_\mathfrak{z}}(k)(\boldsymbol{x}) = S^{\pm}(\boldsymbol{x})$, where: the mark $\pm$ is to distinguish between the upper and lower bounds, the former is given by taking variables and operators with upper parts of the mark while the latter by taking their lower

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parts; $i_{\mathfrak{z}}$ relates to estimation accuracy for an individual water demand scenario where $\Omega_{\mathfrak{z}}$ signifies a set of all considered water demand scenarios and $n_{\mathfrak{z}} = \overline{\overline{\Omega}}_{\mathfrak{z}}$; Ω_{MJ} is a set of monitored junction nodes where there are not hard sensors; $C^{\pm}_{m_{\mathrm{j}},i_{\mathfrak{z}}}(k)(\boldsymbol{x}) \stackrel{\mathrm{def}}{=} C^{\pm,i_{\mathrm{sc}}}_{m_{\mathrm{j}},i_{\mathfrak{z}}}(k)(\boldsymbol{x})$, $C^{\pm}_{i_{\mathrm{T}},i_{\mathfrak{z}}}(k)(\boldsymbol{x}) \stackrel{\mathrm{def}}{=} C^{\pm,i_{\mathrm{sc}}}_{i_{\mathrm{T}},i_{\mathfrak{z}}}(k)(\boldsymbol{x})$ denote envelopes bounding the unknown chlorine concentration (i_{sc}th specie concentration) at the m_{j}th junction node and i_{T}th tank at the discrete time instant k for $i_{\mathfrak{z}}$th water demand scenario, respectively; $k = 1, 2, ..., K$ is a discrete time instant imposed by the quality sampling interval (T_{QP}) to produce the estimates at these time instances and $K = \frac{T}{T_{\mathrm{QP}}}$ where T signifies a considered time horizon; $S^{\pm}(\boldsymbol{x})$ represents the interval observer.

Hence, the accuracy of estimation is defined by the width of envelopes bounding the unknown chlorine concentrations. Therefore, the tighter the bounding intervals are the more accurate the estimates are. The upper and lower estimates are provided by the devised interval observer. This observer uses a linear water quality model as well as direct and indirect state measurements. Taking into account the considerations presented in section II it is possible to derive the following model of chlorine concentration dynamics throughout the entire DWDS [7]:

$$\mathrm{d}_t\, \boldsymbol{x}_{\mathrm{s}}(t) = \boldsymbol{A}(t)\boldsymbol{x}_{\mathrm{s}}(t) + \boldsymbol{b}(t), \tag{7}$$

where: $\boldsymbol{x}_{\mathrm{s}}(t) \in \mathbb{R}^n$ denotes the quality state variables vector which represents chlorine concentrations at all pipe segment ends and in tanks and n is the number of quality state variables at the DWDS; $\boldsymbol{A}(t) \in \mathbb{R}^{n\times n}$ signifies the time-varying state matrix with elements containing the hydraulic quantities, lengths of pipe segments and reaction rate coefficients; $\boldsymbol{b}(t) \in \mathbb{R}^n$ stands for the time-varying vector of inputs with elements containing the hydraulic quantities, lengths of pipes segments, chlorine concentrations in reservoirs and injection of chlorine at the booster stations. The detailed derivation of the model (7) can be found in [7].

In turn, the direct state measurements ($\boldsymbol{x}_{\mathrm{s},2}(t)$) are information from chlorine concentration hard sensors located in junction nodes supplied by only one pipe. Whereas, if the sensor is placed at the junction node with several connected pipes the measurement is called indirect or pseudo-measurement ($\widetilde{\boldsymbol{x}}_{\mathrm{s},2}(t)$). For more details, see [7]. Hence, the vector of measurements yields: $\overline{\boldsymbol{x}}_{\mathrm{s},2}(t) = [\boldsymbol{x}_{\mathrm{s},2}(t) \quad \widetilde{\boldsymbol{x}}_{\mathrm{s},2}(t)]^{\mathrm{T}}$.

Thus, the devised interval observer $S^{\pm}(\boldsymbol{x})$ reads [7]:

$$\begin{cases} \mathrm{d}_t\boldsymbol{w}^{\pm}(t) = \boldsymbol{A}^{\pm}_{11}(t)\boldsymbol{w}^{\pm}(t) + \boldsymbol{N}_1\boldsymbol{A}^{\pm}_{12}(t)\overline{\boldsymbol{x}}^{\pm}_{\mathrm{s},2}(t) + \boldsymbol{M}\boldsymbol{v}^{\pm}(t) \\ \boldsymbol{w}^{\pm}(0) = \boldsymbol{N}\boldsymbol{x}^{\pm}_{\mathrm{s}}(0) \\ \hat{\boldsymbol{x}}^{\pm}_{\mathrm{s},1}(t) = \boldsymbol{N}_1^{-1}\boldsymbol{w}^{\pm}(t) \end{cases}, \tag{8}$$

where: $\boldsymbol{x}_{\mathrm{s}}(t) = [\boldsymbol{x}_{\mathrm{s},1}(t) \quad \boldsymbol{x}_{\mathrm{s},2}(t)]^{\mathrm{T}} \in \mathbb{R}^n$ and $\boldsymbol{x}_{\mathrm{s},1}(t) \in \mathbb{R}^s$, $\boldsymbol{x}_{\mathrm{s},2}(t) \in \mathbb{R}^m$ are the vectors of unmeasured and measured state variables, respectively and $s = n - m$; $\hat{\boldsymbol{x}}^{\pm}_{\mathrm{s},1}(t)$ denote the upper and lower bounds on the estimated state variables; $\boldsymbol{w}(t)$ is the auxiliary variable, defined as: $\boldsymbol{w}(t) = \boldsymbol{N}\boldsymbol{x}_{\mathrm{s}}(t)$; $\boldsymbol{N} = [\boldsymbol{N}_1 \quad \boldsymbol{0}]$; $\boldsymbol{N}_1 = \eta\boldsymbol{I} \in \mathbb{R}^{s\times s}$ denotes the invertible matrix proportional to the identity matrix and η is the real, arbitrary, positive and constant parameter; $\boldsymbol{M} = [\boldsymbol{N}_1 \quad \boldsymbol{0} \quad \boldsymbol{0}]$; $\boldsymbol{v}^{\pm}(t) = \left[\boldsymbol{b}^{\pm}_1(t) \quad \frac{1}{2}\boldsymbol{B}_1 \quad \pm\frac{1}{2}\boldsymbol{B}_2\right]^{\mathrm{T}}$; $\boldsymbol{B}_1 = \boldsymbol{b}^{+}_2(t) + \boldsymbol{b}^{-}_2(t)$; $\boldsymbol{B}_2 = \boldsymbol{b}^{+}_2(t) - \boldsymbol{b}^{-}_2(t)$; $\boldsymbol{A}_{11}(t) = \{a_{\alpha,\beta}\}$, $\boldsymbol{A}_{12}(t) = \{a_{\alpha,\gamma}\}$, $\boldsymbol{A}_{21}(t) = \{a_{\gamma,\beta}\}$, $\boldsymbol{A}_{22}(t) = \{a_{\gamma,\gamma}\}$, $\forall\alpha,\beta \in \overline{1,s}$, $\forall\gamma \in \overline{s+1,m}$ are suitable parts of the matrix $\boldsymbol{A}(t)$ structured by the measurement state variables: $\boldsymbol{A}(t) = \begin{array}{c} \scriptstyle est. \\ \scriptstyle meas. \end{array} \begin{bmatrix} \boldsymbol{A}_{11}(t) \in \mathbb{R}^{s\times s} & \boldsymbol{A}_{12}(t) \in \mathbb{R}^{s\times m} \\ \boldsymbol{A}_{21}(t) \in \mathbb{R}^{m\times s} & \boldsymbol{A}_{22}(t) \in \mathbb{R}^{m\times m} \end{bmatrix}_{n\times n}$; $\boldsymbol{b}_1(t) = \{b_\alpha\}$, $\boldsymbol{b}_2(t) = \{b_\gamma\}$ denote suitable parts of the vector $\boldsymbol{b}(t)$ structured by the measurement state variables: $\boldsymbol{b}(t) = \begin{array}{c} \scriptstyle est. \\ \scriptstyle meas. \end{array} \begin{bmatrix} \boldsymbol{b}_1(t) \in \mathbb{R}^s \\ \boldsymbol{b}_2(t) \in \mathbb{R}^m \end{bmatrix}_n$; $\overline{\boldsymbol{x}}^{\pm}_{\mathrm{s},2}(t)$ denote the upper and the lower bounds of the direct and indirect measured state variables given by: $\boldsymbol{x}^{-}_{\mathrm{s},2}(t) \le \boldsymbol{x}_{\mathrm{s},2}(t) \le \boldsymbol{x}^{+}_{\mathrm{s},2}(t)$ and $\widetilde{\boldsymbol{x}}^{-}_{\mathrm{s},2}(t) \le \widetilde{\boldsymbol{x}}_{\mathrm{s},2}(t) \le \widetilde{\boldsymbol{x}}^{+}_{\mathrm{s},2}(t)$; $\le$ is understood element-wise.

The interval observer (8) generates stable and robust upper $\hat{\boldsymbol{x}}^{+}_{\mathrm{s},1}(t)$ and lower $\hat{\boldsymbol{x}}^{-}_{\mathrm{s},1}(t)$ envelopes, bounding the unmeasured state variables $\boldsymbol{x}_{\mathrm{s},1}(t)$ despite the uncertainty in: $\boldsymbol{A}(t)$, $\boldsymbol{b}(t)$, $\overline{\boldsymbol{x}}_{\mathrm{s},2}(t)$ and $\boldsymbol{x}_{\mathrm{s}}(0)$. The proof can be found in [7].

It should be added that according to definition $\boldsymbol{x}_{\mathrm{s}}(t)$ the interval observer $S^{\pm}(\boldsymbol{x})$ returns the unknown chlorine concentration estimates only in tanks ($C^{\pm}_{i_{\mathrm{T}},i_{\mathfrak{z}}}$) and at the DWDS junction nodes ($C^{\pm}_{m_{\mathrm{j}},i_{\mathfrak{z}}}$) that are supplied by one pipe. Whereas, for the junction nodes with several connected pipes the chlorine concentrations are linear combinations of the proper state variables. These concentrations can be calculated based on the estimates either by solving suitable optimisation problems or using interval analysis [34].

A comprehensive description of solving the problem of monitoring stations placement at the DWDS has been presented in [12].

C. Control system and actuators placement

Similarly, as the monitoring system design is dependent on the sensor placement, the control system design is strongly related to booster stations allocation. Moreover, not all of the nodes are feasible to be considered for booster station location. Furthermore, only a certain combination of booster stations allow generating admissible controls under prescribed *a priori* control algorithm. Finally, it should also be noticed that the change in the operational states and conditions of a DWDS as well as its structural change, e.g., due to city expansion, may lead to partial controllability or even its total loss. To that goal, a closed-loop multi-objective placement of actuators has been introduced in [24]. The closed-loop term is to indicate that the allocation procedure was invoked together with control design for which purpose an MPC approach was utilised. The proposed approach is such that it achieves trade-off between the capital cost due to the cost of a booster station and operational cost of the controller. Technical details specifying the task under the general resource allocation scheme (4) has been presented in the following lines.

Analogically to section III-B, the following notation is introduced: $n_{\mathrm{bf}} \stackrel{\mathrm{def}}{=} n_{\mathrm{f}}$ and $\Omega_{\mathrm{J\,bf}} \stackrel{\mathrm{def}}{=} \Omega_{\mathrm{J\,f}}$. Moreover, a set of junction nodes where the booster stations have been

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allocated ($\Omega_{\rm b}$) is identified by selecting nodes such that $\forall i_{\rm bf} \in \overline{1, n_{\rm bf}}: \mu_{\Omega_{\rm b}}(\omega_{\rm J\,bf\,\mathit{i}_{bf}}) = x_{i_{\rm bf}}, \omega_{\rm J\,bf\,\mathit{i}_{bf}} \in \Omega_{\rm J\,bf}$. Thus, the bounds on the total number of booster stations to be allocated ($n_{\rm b}(\boldsymbol{x}) \overset{\rm def}{=} \overline{\overline{\Omega}}_{\rm b}$) yields: $n_{\rm b}(\boldsymbol{x}) \in [n_{\rm b\,min}, n_{\rm b\,max}]$.

The dynamics of the closed-loop system ($\Sigma_{\rm cl}$), is considered a final constraint for the actuator allocation task. Since DWDS is to operate in multiple operating states and conditions to enable numerical treatment of the problem these are to be represented by a finite number of scenarios contained in $\Omega_{\mathfrak{z}}$ [24]. Hence, $\forall i_{\mathfrak{z}} \in \overline{1, n_{\mathfrak{z}}}: \Sigma_{{\rm cl}\, i_{\mathfrak{z}}}(\boldsymbol{x})$ is considered such that:

$$\begin{cases} \mathcal{F}\left(\boldsymbol{C_0}, \boldsymbol{C}(k+1), \boldsymbol{C}(k), \boldsymbol{q}(k), \boldsymbol{u}(k), \mathfrak{z}_{i_{\mathfrak{z}}}(k)\right) = 0 \\ \boldsymbol{u}(k)(\boldsymbol{x}) = \mathfrak{R}\left(\hat{\mathcal{F}}, \boldsymbol{C}(k), \boldsymbol{x}\right) \\ (k, i, \mathfrak{z}_{i_{\mathfrak{z}}}(k)) \in ([0, \ldots, H_{\rm E}], [1, \ldots, H_{\rm p}-1], \Omega_{\mathfrak{z}}) \end{cases}, \quad (9)$$

where: $\forall i, k$: $\boldsymbol{C}_0 \overset{\rm def}{=} \boldsymbol{C}(0), \boldsymbol{C}(k), \boldsymbol{C}(k+1) \in \mathbb{R}^n$ denote the disinfectant concentrations; $\boldsymbol{q}(k) \in \mathbb{R}^q$ stands for water flow through DWDS; $\boldsymbol{u}(k)(\boldsymbol{x}) \in \mathbb{R}^m$ represents control signals; $\mathfrak{z}_{i_{\mathfrak{z}}}(k) \overset{\rm def}{=} (\boldsymbol{z}(k), \boldsymbol{p})_{i_{\mathfrak{z}}}$ is the $i_{\mathfrak{z}}$th disturbance scenario over the considered experiment horizon $H_{\rm E}$ paired with plant parameters. Moreover, $\mathcal{F}$ and $\mathfrak{R}$ are non-linear maps describing system dynamics and closed-loop control algorithm (predictive operator), respectively. It should be noticed that since $\mathfrak{R}$ is obtained using MPC approach $u(k)(\boldsymbol{x}) \overset{\rm def}{=} u(k|k)(\hat{C}(k), z(k+i|k), \boldsymbol{x})$. The first step control action $u(k|k)$ applied to the plant is obtained from control signal trajectory $u(k+i|k)(\hat{C}(k), z(k+i|k), \boldsymbol{x})$ generated at each k over the prediction horizon $i \in [1, \ldots, H_p-1]$. The generated control trajectory is the result of model-based optimisation task for which purpose the DWDS plant model $\hat{\mathcal{F}}$ (set up to approximate $\mathcal{F}$ that represents the DWDS dynamics as described in section II) is handled using (state) feedback ($\boldsymbol{C}(k)$) and disturbance predictions such that at each k the prediction is being updated over the $H_{\rm p}$ and $\forall i: \boldsymbol{z}(k+i|k) \in \mathbb{R}^z$. The MPC algorithm performance objectives are in line with the allocation tasks performance criteria and are defined in the following lines. Furthermore, as a consequence the provided constraints definitions altogether yield $\Omega_{\rm FPS}$.

In the considered case three objectives are defined such that $\boldsymbol{J}_{\rm c} \equiv J_{\rm c}$ and $\boldsymbol{J}_{\rm e} = [J_{\rm e\,1},\, J_{\rm e\,2}]^T$. First objective ($J_{\rm c}$) is to assess the the total number of allocated booster stations:

$$J_{\rm c}(\boldsymbol{x}) \overset{\rm def}{=} n_{\rm b}(\boldsymbol{x}). \quad (10)$$

Second objective ($J_{\rm e\,1}$) is defined as a number of consecutive disinfectant concentration limit violations:

$$J_{\rm e\,1}(\boldsymbol{x}) \overset{\rm def}{=} n_{\mathfrak{z}}^{-1} \sum_{i_{\mathfrak{z}}=1}^{n_{\mathfrak{z}}} \sum_{t=0}^{H_{\rm E}} \sum_{i_{\rm N}=1}^{\overline{\overline{\Omega}}_{\rm N}} w\left(i, t, \mathfrak{z}_{i_{\mathfrak{z}}}, \boldsymbol{x}\right), \quad (11)$$

where $\Omega_{\rm N}$ is the set of all DWDS nodes and:

$$w\left(i_{\rm N}, t, \mathfrak{z}_{i_{\mathfrak{z}}}, \boldsymbol{x}\right) \overset{\rm def}{=} \begin{cases} 1 & \text{if } c\left(i_{\rm N}, t, \mathfrak{z}_{i_{\mathfrak{z}}}, \boldsymbol{x}\right) \notin \left[\underline{c}^*_{i_{\rm N}}, \overline{c}^*_{i_{\rm N}}\right] \\ 0 & \text{otherwise} \end{cases}, \quad (12)$$

where $\underline{c}^*_{i_{\rm N}}, \overline{c}^*_{i_{\rm N}}$ indicate the acceptable range of the disinfectant concentration $c(i_{\rm N}, t, \mathfrak{z}, \boldsymbol{x})$ at the $i_{\rm N}$th DWDS node. This is an alternative approach to section III-B, where the scenarios span new objective subspace considered in sensor allocation task. Third objective ($J_{\rm e\,2}$), given by a barrier type function, is to drive the disinfectant concentration to the preferred range in the each of the network nodes (e.g. [22], [23]):

$$J_{\rm e\,2}(\boldsymbol{x}) \overset{\rm def}{=} n_{\mathfrak{z}}^{-1} \sum_{i_{\mathfrak{z}}=1}^{n_{\mathfrak{z}}} \sum_{t=0}^{H_{\rm E}} \sum_{i_{\rm N}=1}^{\overline{\overline{\Omega}}_{\rm N}} b\left(i_{\rm N}, t, \mathfrak{z}_{i_{\mathfrak{z}}}, \boldsymbol{x}\right), \quad (13)$$

where:

$$b\left(i_{\rm N}, t, \mathfrak{z}_{i_{\mathfrak{z}}}, \boldsymbol{x}\right) \overset{\rm def}{=} \begin{cases} a_{1,i_{\rm N}} c + b_1, \text{ for } c < \underline{c}^*_{i_{\rm N}} \\ a_{2,i_{\rm N}} c + b_2, \text{ for } \underline{c}^*_{i_{\rm N}} \le c < c^*_{i_{\rm N}} \\ a_{3,i_{\rm N}} c + b_3, \text{ for } c^*_{i_{\rm N}} \le c < \overline{c}^*_{i_{\rm N}} \\ a_{4,i_{\rm N}} c + b_4, \text{ for } c \ge \overline{c}^*_{i_{\rm N}} \end{cases} \quad (14)$$

with $c \overset{\rm def}{=} c\left(i_{\rm N}, t, \mathfrak{z}_{i_{\mathfrak{z}}}, \boldsymbol{x}\right)$, $a_{j,i_{\rm N}}$, $b_{j,i_{\rm N}}$, $\forall j \in \overline{1,4}$, being the $i_{\rm N}$th node dependent parameters, and $c^*_{i_{\rm N}}$ the user-defined (preferred) concentration of the disinfectant.

Finally, at this point the elements of (4) have been well-defined and the booster stations allocation can be performed.

D. Joint sensors and actuators placement

As it can be noticed, the previous authors works have been focused on delivering solutions aiming solely on either the water quality monitoring system design coupled with hard sensors placement or control system design coupled with booster stations placement, assuming access to solution of hydraulics in the former case or quality measurements at the DWDS nodes in the latter ones. However, taking into account presented considerations it is obvious that these issues are closely interrelated. Thus, a proposal to formulate the task of optimal joint allocation of sensors and actuators at the DWDS is given in this section. The joint optimised monitoring and booster stations placement problem is formulated in the following lines. In the most direct approach, first, the decision variables $\boldsymbol{x}$ are the joint variables of the sensor and actuator allocation tasks. Second, take subindices 'b' and 's' to distinguish the elements related to boosters and sensors, then recall the $\Omega_{\rm J\,bf}$ and $\Omega_{\rm J\,sf}$. It is to be expected that $\Omega_{\rm J\,sf} \cap \Omega_{\rm J\,bf} \neq \emptyset$. This needs to be encompassed in the allocation procedure. Considering a fact that booster station also can provide information for monitoring system as it 'produces' prescribed concentration within a given node a certain task hierarchy might be proposed. The remaining set of constraints is aggregated and treated jointly in this task so that $\Omega_{\rm FPS} = \Omega_{\rm FPS\,s} \cup \Omega_{\rm FPS\,b}$. Third, consider the assessment criteria. Again the most direct approach is to include all of the purposed cost functions in the joint task, hence, the components of $\boldsymbol{J}$, yield $\boldsymbol{J}_{\rm c} = [J_{\rm cs},\, J_{\rm cb}]^T$ and $\boldsymbol{J}_{\rm e} = \left[\boldsymbol{J}^T_{\rm es},\, \boldsymbol{J}^T_{\rm eb}\right]^T$. Finally, the above considerations allow one to formulate a joint sensor-actuator allocation and monitoring-control system design task kept with a predefined framework given, in general, by (4).

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IV. Conclusions

In this paper, a comprehensive literature review on the state-of-the-art allocation of infrastructure elements, i.e. sensors and actuators, at DWDSs has been presented. In particular, this work focuses on the previous work of the authors in this area included in a generic framework of resource allocation using multi-criteria optimisation. In addition, the authors put forward hypotheses concerning the synergistic effects of treating the two mentioned tasks in a joint manner. The initial formulation of such a task has been included in one of the sections of the article. Future works of the authors will focus on providing support for the hypothesis formulated. The related research concerns mainly attempt to solve the formulated task numerically as well as investigation of other approaches to its formulation.

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