JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
47, 2, pp. 307-320, Warsaw 2009

LINEAR CHARACTERISTICS OF THE SLOSHING PHENOMENON FOR THE PURPOSE OF ON-BOARD SHIP'S STABILITY ASSESSMENT

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The paper presents results of experimental research and numerical simulations of the sloshing phenomenon. The research was focused on computation of the heeling moment affecting stability of a vessel. The proposed linearisation enables application of the results to the assessment of the ship's stability. The dependence of the heeling moment upon localisation of a tank in the ship's hull is analysed. The heeling moment obtained in the course of the research was compared to the moment computed in accordance with the Intact Stability Code requirements. The study may be a contribution to the more sophisticated estimation of the ship's stability than it is achieved nowadays.

Key words: sloshing, free surface of liquids, ship's stability

1. Introduction

Dynamic behaviour of a vessel at sea is greatly affected by the dynamics of moving masses carried onboard. The cargo securing procedures ensure avoiding motion of a loose cargo, but liquids contained in partly filled tanks cannot be avoided at all. The modelling of the interaction taking place between water sloshing inside ship's tanks and the tank's structure is very important for the sake of safety of the sea transportation system, human life and environment. The effects of sloshing should be taken into consideration not only during strength calculations but in the course of the vessel's sea keeping prediction and transverse stability assessment as well.

The accuracy of the ship's transverse stability assessment is an important factor in the vessel's exploitation process. The ship's loading condition of insufficient stability may induce list, strong heel and even capsizing. On the other hand, excessive stability causes high values of mass forces acting on cargo and machinery due to strong accelerations. Therefore, any scientific efforts towards better evaluation of the ship's stability are worthy undertaking. The influence of the sloshing phenomenon on the ship's stability is one of the issues to be considered.

2. Ship's stability assessment

The stability of a ship is defined as a feature of counteracting against external heeling forces and moments, which should enable a vessel to realize main tasks in the course of its exploitation (Dudziak, 1988). A complete description of the stability may be obtained by solving equations of the ship's motion. The most convenient attitude towards this task is to assume that there is a symmetrical mass distribution and steady values of moments of mass inertia. The equations of motion, in relation to the center of gravity G, take form of six differential equations (Dudziak, 1988). The solving of such generally formulated equations of motion is impossible at the present state of the art. By neglecting couplings for the sake of simplicity, the ship's rolling is often analysed by a single degree-of-freedom system (Senjanovic *et al.*, 1997). The governing differential equation of motion, as the result of the equilibrium of moments, is:

$$I_4\ddot{\varphi} + D_4(\dot{\varphi}) + R_4(\varphi) = M_4(t) \tag{2.1}$$

where

- I_4 – moment of inertia of a ship and added masses $[\mathrm{kg\,m^2}]$,
- D_4 – damping moment $[\mathrm{N\,m}]$,
- R_4 – restoring moment $[\mathrm{N\,m}]$,
- M_4 – excitation moment $[\mathrm{N\,m}]$,
- φ – the angle of heel [rad].

The resultant excitation moment M(t) consists of as many components as many influencing factors swing the ship. The main components are waves and wind. Anyway, when the analysis of the ship's rolling comprises the effects of liquid sloshing in partly filled tanks, then the moment of the liquid-ship interaction has to be included as a component of $M(t)$ in a/m formula (2.1).

In spite of the simplified attitude towards the ship's rolling, the solution to equation (2.1) is still too complicated to be used in the course of on-board stability calculations made by cargo officers during their everyday practice.

Calculation of the vessel's stability and evaluation made on-board nowadays, is not based on the ship's equations of motion, but on the stability criteria published by the ship's classification society. These criteria are mainly based on A749(18) Resolution of International Maritime Organization. The resolution and its later amendments are known as the Intact Stability Code (IMO, 2002).

The criteria quality the shape of the righting arm curve, especially the minimum value of initial metacentric height, the maximum righting arm and the angle of this maximum, the area under the righting arm curve calculated within the prescribed range of the angle of heel. In addition, the weather criterion, as an attempt at dynamic stability calculation, is to ensure the sufficient stability of a ship to withstand severe wind gusts during rolling. But the weather criterion is a very simple model of ship's dynamic behaviour where the static stability curve is used. Anyway, the weather criterion is the only one that is partly based on the model of the heeling phenomenon and not only on statistical data, while the other criteria are based on the statistics of historical disasters only (Francescutto, 2002).

According to the IMO regulations, the righting arm curve should be corrected due to the effect of free surfaces of liquids in tanks. The correction may be done by any of the three accepted methods (IMO, 2002):

1) correction based on the actual moment of fluid transfer calculated for each angle of heel;
2) correction based on the moment of inertia of tank's horizontal projection;
3) correction based on summation of M_{sf} values for all tanks taken into consideration, where the moment M_{sf} should be obtained form the simplified formula given in the Intact Stability Code.

All of the three mentioned above methods of calculation of free surface correction consider the static attitude towards the sloshing phenomenon only. They do not consider the localisation of a tank within the hull of a ship and localisation of the rolling axis. The only advantage of current compulsory corrections is the simplicity of calculation.

3. Study of pressure distribution in a moving tank

The liquid sloshing phenomenon is the result of motion of a partly filled tank. As the tank moves, it supplies the energy to induce and sustain fluid motion (Akyildiz and Unal, 2005). Both the liquid motion and its effects are called sloshing. The interaction between the ship's and tank's structure and water sloshing inside the tank consists in steady transmission of energy. As the ship rolls, the walls of the partly filled tank induce movement of water. Then, the water presses against the opposite tank's wall and returns the energy to the ship, taking simultaneously the next portion and enabling the counter-direction movement. The mass and energy are conserved within the cycle. The rolling characteristic of a vessel at sea is affected by the movement of liquids in tanks (Kim, 2002).

An experimental research on the sloshing phenomenon was carried out at the Ship Operation Department of Gdynia Maritime University. It enabled one to measure the dynamic pressure distribution on the side wall of the model tank and in its upper corner (Krata, 2006). The experimental investigation on the pressure distribution due to sloshing required the arousing of the sloshing phenomenon. After that, the dynamic pressure time history in selected spots was measured and recorded. To achieve this, the test apparatus was designed and built (Krata, 2006).

The main part of the apparatus is a tank. It is equipped with pressure transducers and an inclinometer. The tank is forced to oscillate which excites water movement inside it. The dimensions of the model tank are breadth – 1.040 m, length – 0.380 m, depth – 0.505 m. The tank was hanged on a shaft by bearings and forced to oscillate by a drive mechanism. The drive mechanism is based on an electric motor, transmission reducing revolution velocity and a crank mechanism. The view of the testing apparatus and localisation of dynamic pressure sensors is shown in Fig. 1.

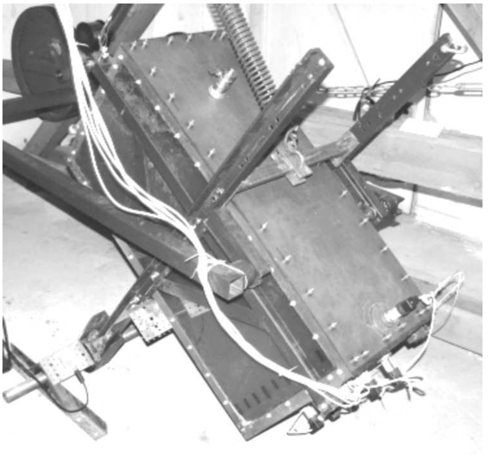

Fig. 1. Experimental setup

Assuming plane tank oscillations and negligible water viscosity, one observes a two-dimensional character of the considered liquid flow inside the tank

(Warmowska and Jankowski, 2005). It allows one to equip the tank with one set of pressure transducers only, fixed in the middle line of the tank. The pressure transducers are equally spaced along the vertical wall of the tank and in the roof of the tank close to the upper corner.

The oscillating motion which induces sloshing is described accurately enough by a harmonic function. The experimental research on the pressure distribution due to sloshing was performed for a variety of external excitation parameters. The period of the oscillation varied from $T = 2.6\,\mathrm{s}$ to $T = 6.5\,\mathrm{s}$. The lever os, as the distance between the center of the tank and the rotary motion axis, was changed from $os = -0.718\,\mathrm{m}$ to $os = 0.718\,\mathrm{m}$. The positive value of os describes the tank localisation beneath the shaft (modelling the rolling axis) and the negative value of os describes the localisation above the shaft. The amplitude of the tank rotary motion during the tests was assumed to be 40°. It reflects heavy sea conditions and enables one to draw conclusions for the worst possible condition at sea (Francescutto and Contento, 1999). The tank filling level was assumed to be 30%, 60% and 90%.

The pressure signal, measured by the transducers, consists of two components. One of them is called non-impulsive dynamic pressure and the other one impulsive pressure, or impact pressure (Akyildiz and Unal, 2005). The non-impulsive dynamic pressure is slowly varying. It is a result of global motion of the liquid in the tank and it affects the transverse stability of the ship.

All signals received from the sensors were verified and the measuring instruments were calibrated. The gain coefficient and shift coefficient were determined for every pressure sensor and the inclinometer. The calibration procedure allowed one to deem the experimental measurements to be correct and reliable (Krata, 2006).

The analog signals received from the sensors were sampled and transformed into discrete digital signals by a 12-bit A/D card and then they were recorded in text format files. The maximum working frequency of the measuring device was 1000 Hz. Thus, the aliasing distortions of the measured signal were avoided, because the measuring instruments were much faster than the required Nyquist rate for the sloshing phenomenon (Zieliński, 2002). A further digital signal processing was carried out. The main operation was low pass filtering for high frequency noise reduction. The filtering enabled one to decompose the recorded digital signal and emerged the non-impulsive dynamic pressure component (Zieliński, 2002).

The pressure distributions obtained in the course of experimental investigation were supplemented by the results of numerical simulations. The simulations of the sloshing phenomenon were performed by the computer program

"Tank" by M. Warmowska, used for the estimation of the dynamic pressure distribution. The sloshing problem was described by the two-dimensional model. It was also assumed that the liquid is inviscid, incompressible, and of constant density. As the flow of the liquid was assumed to be irrotational, the potential theory was used to solve the sloshing problem (Jankowski and Warmowska, 1997).

The numerical simulation of the sloshing phenomenon was performed for oscillation and tank's geometry corresponding with suitable geometric parameters of the experimental investigation. The height of the water level varied from 30% to 90% of the maximum tank's height. The program allowed one to compute the time history of dynamic pressures in ninety points around the tank's model. The control points were situated along vertical walls, the bottom and the tank's roof. The correctness of the simulation results was verified experimentally (Krata, 2006).

4. Linearisation of the heeling moment due to sloshing

The pressure distribution on the walls of the tank was obtained in the course of experimental tests and numerical simulations. The results of the research enable one to compute the heeling moment due to sloshing. The heeling moment $\boldsymbol{M}$ [N m] was calculated according to the following formula

$$\boldsymbol{M} = \int\limits_S \boldsymbol{r} \times \boldsymbol{n} p \, ds \tag{4.1}$$

where

- S – surface of the tank walls [m^2],
- $\boldsymbol{r}$ – position vector of the considered point on the tank wall [m],
- $\boldsymbol{n}$ – normal vector [–],
- p – local pressure on the tank wall [Pa].

Due to the two-dimensional character of the considered flow in the tank, the heeling moment is a vector of the perpendicular direction to the plane of tank's motion. As the transverse stability of a ship is considered, the heeling moment has one component only, as follows

$$\boldsymbol{M} = [M_x, M_y, M_z] = [M_x, 0, 0] \tag{4.2}$$

where M_x, M_y, M_z are the spatial components of $\boldsymbol{M}$ vector, determined in relation to the x, y and z axis, respectively, in the reference system fixed to the vessel.

As the direction of the heeling moment is fixed and steady in the time domain, the heeling moment due to sloshing may be described by the M_x component. The resultant moment obtained from formula (4.1) represents one time-step only. Computation of the heeling moment should be performed for at least one period of roll. Thus, the pressures have to be investigated for at least one period of the ship's roll as if in fact they were obtained for a longer time comprising few rolling periods. An example of the heeling moment changes is shown in Fig. 2.

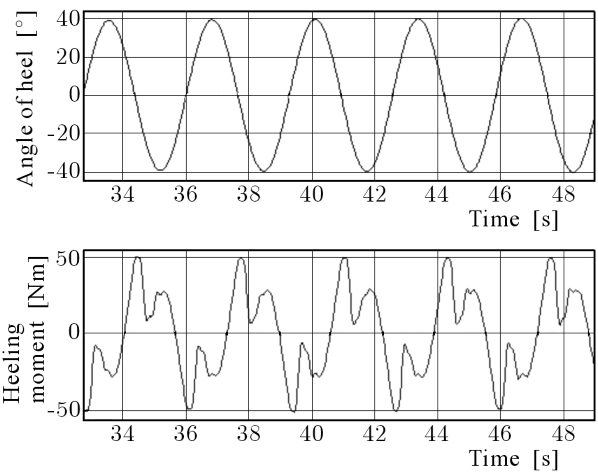

Fig. 2. Time history of the heeling moment due to sloshing for the filling level 30% and $os = 0.359\,\mathrm{m}$

The time domain presentation of the computation results can be useful when the ship's rolling is computed according to formula (2.1). In such a case, the heeling moment due to sloshing is one of the components of the total heeling moment swinging a vessel at sea. However, the assessment of her stability is not based on the equations of motion, but on the static stability curve (IMO, 2002). The curve presents the righting arm GZ versus the angle of heel, and the righting arm is reduced by the statically computed free surface correction. Therefore, the most convenient way to present the results of the heeling moment calculation due to dynamic sloshing of a liquid in a partly filled tank, is to plot a graph of the angle of heel domain. An example of such a graph is presented in Fig. 3. The heeling moment was time-domain calculated, but it is plotted as a function of the angle of heel.

The interpretation of the results of the heeling moment is much easier in the domain of the angle of heel. The main disadvantage of the graph shown in Fig. 3 is the hysteresis resulting from wave type phenomena taking place inside

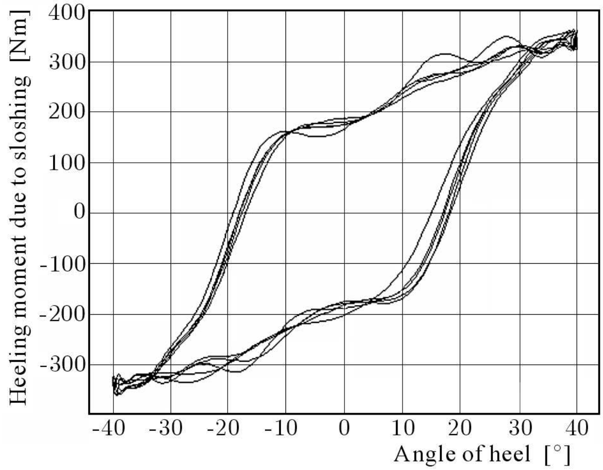

Fig. 3. Heeling moment vs. angle of hed for the filling level 30% and $os = -0.718\,\mathrm{m}$

the moving tank. This disadvantage can be eliminated by linearisation. As the main task of the research is a more reliable stability assessment with regard to the sloshing phenomenon, the linearisation should refer to the ship's stability criteria, especially the weather criterion. The area under the GZ curve is regulated within the weather criterion, which represents the work of the heeling moment due to wind gusts when the ship rolls. Hence the linearisation of the examined heeling moment should be based on the work of the moment as well. The linearisation method applied to the heeling moment due to sloshing of liquids is based on the formula

$$\int_0^{\varphi_{40}} M_{(\varphi)}\; d\varphi + \int_{\varphi_{40}}^{0} M_{(\varphi)}\; d\varphi = 2\int_0^{\varphi_{40}} M_l\; d\varphi \tag{4.3}$$

where

M – heeling moment due to sloshing [N m],
M_l – resultant linear heeling moment due to sloshing [N m],
φ – the angle of ship's heel [rad],
φ_{40} – the angle of heel equal $40°$ [rad].

An example of the linear heeling moment due to sloshing is presented in Fig. 4.

The linear function of the heeling moment can be determined by fixing two in-line points having the coordinates (φ, M). One of them is the point $(0, 0)$ and the other one the point $(40°, M_{l40})$. Therefore, the complete description of the linear heeling moment may be done by one scalar only. This is value M_{l40}

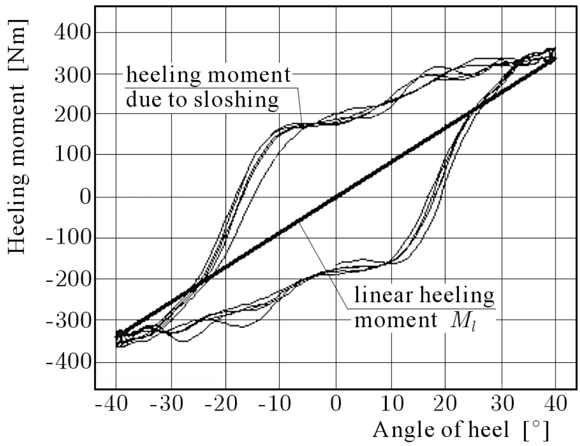

Fig. 4. Linear heeling moment for the filling level 30% and $os = -0.718\,\mathrm{m}$

of the linear heeling moment due to sloshing for the angle of heel equal 40° and obtained from formula (4.3).

5. Characteristics of the linear heeling moment due to sloshing

The linear heeling moment due to sloshing taking place in partly filled tanks in ships depends on some parameters. One of the most important seems to be the localisation of the tank with respect to the vessel's rolling axis. The comparison of the linear heeling moment due to sloshing and the quasi-static moment of free surface correction may be a point of interest as well.

The moment of heeling of a ship, as a consequence of the liquid carried in any partly filled tank, has two components. One of them is the moment M_m of the liquid weight and the other is the heeling moment M_T due to transfer of the fluid calculated for each angle of heel. The total heeling moment due to liquids contained in the vessel's tanks can be described from the static point of view by the formula

$$M_{stat} = M_m + M_T \tag{5.1}$$

where

M_m – static heeling moment due to weight of "frozen" liquid in a tank [N m],

M_T – static heeling moment of the transfer of the liquid's center of gravity [N m].

The moment M_m is taken into consideration while calculating the ship's center of gravity, and it assumes the liquid to be "frozen" at the angle of heel equal to $0°$. The other component M_T concerns the shift of the liquid's center of gravity only, and can be calculated in accordance with the Intact Stability Code requirements (IMO, 2002). Both a/m moments are functions of sine of the angle of heel. Anyway, the resultant heeling moment can be compared with the linear heeling moment due to sloshing for angles of heel where the sine function may be approximated accurately enough by the linear function. The reasonable range of such a linear approximation is about $40°$, which is shown in Fig. 5.

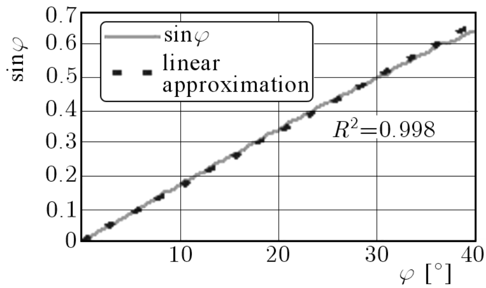

Fig. 5. Linear approximation of the sine function

As the sine function is almost linear up to the angle of heel $40°$, the static heeling moment M_{stat} computed according to the Intact Stability Code requirements may be compared to the linear heeling moment M_l obtained in the research program. Both moments M_{stat} and M_l have the zero values for the zero angle of heel, so their comparison may be done as the comparison of their values for the angle of heel equal $40°$.

The graphs showing the heeling moments are prepared as non-dimensional. The excitation period T is referred to the first harmonic natural sloshing period of the liquid in the model tank T_w. The scope of T/T_w ratios reflects a wide variety of characteristics that can take place on board of ships in different loading conditions. The distance os between the center of the moving tank and the axis of rotary motion is referred to the breadth of the tank b_z. And the values of the heeling moments are referred to the moment $M_{f.s.40}$ of the static free surface correction computed according to the Intact Stability Code requirements. The characteristics of the linear heeling moment due to sloshing for different localisations of the tank are shown in Figs. 6, 7 and 8.

The analysis of graphs of the heeling moment obtained for different localisations of the partly filled tank reveals a distinguishable relationship between them. The values of the linear heeling moment M_{l40} due to sloshing at the

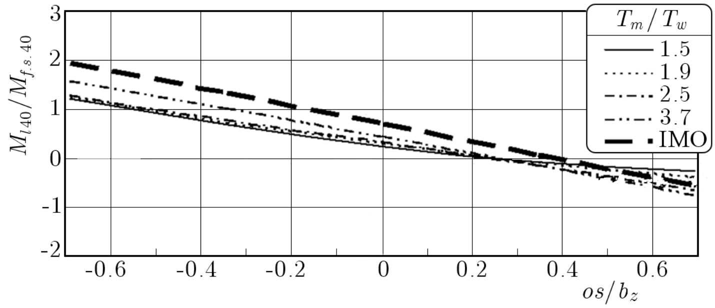

Fig. 6. Non-dimensional linear heeling moment at the angle of heel 40° for the filling level 30% and different localisation of the partly filled tank

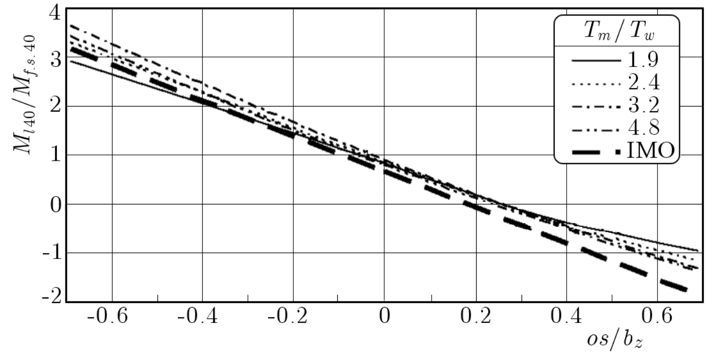

Fig. 7. Non-dimensional linear heeling moment at the angle of heel 40° for the filling level 60% and different localisation of the partly filled tank

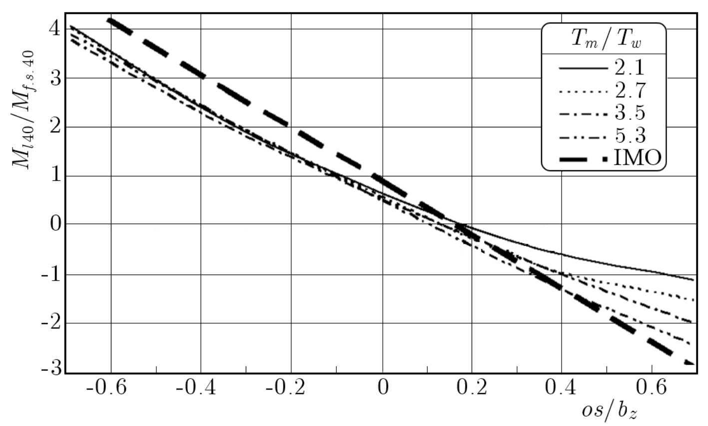

Fig. 8. Non-dimensional linear heeling moment at the angle of heel 40° for the filling level 90% and different localisation of the partly filled tank

investigated angle of heel 40° referred to the roll axis localisation have characteristics very close to the linear ones. Their linear correlation coefficient R^2 exceeds 0.95. The value of the static heeling moment for 40° angle of heel, the computation of which was based on the free surface correction, is linear as well. But this correction does not consider the dynamics of liquid sloshing in the tanks. A considerable difference is noted between the results of investigations and simple static calculations required to be done in the course of the ship's stability assessment on board of a vessel.

6. Conclusions

The research of the influence of the sloshing phenomenon on the heeling moment is based on a more sophisticated method than that used nowadays according to the Intact Stability Code. It takes into consideration the dynamics of the sloshing liquids, and the only disadvantage is the hysteresis. This problem was solved by linearisation performed according to proposed formula (4.3). The obtained linear heeling moments represent the equivalent work of the heeling moment, which is the key issue in the weather criterion of stability.

Motion of liquids in partly filled ship's tanks affects her stability and, therefore, it is to be taken into account in the course of the stability assessment in accordance with the IMO recommendations. The research presented in the paper points out that the very simplified methods recommended by IMO could be improved and reach better accuracy to meet modern requirements of ships exploitation. The analysis of the heeling moment characteristics reveals their almost linear dependence on the localisation of the tank, although the obtained lines significantly vary from the IMO recommended ones. As a result of the discrepancies, the stability of a vessel may be less or more affected by the liquid sloshing in partly filled tanks. This may be dangerous to the vessel or – on the other hand – restrict its ability to carry cargo. Both situations could be avoided by the use of more accurate dynamic free surface corrections.

The results of the research and the proposed method can be the basis for further investigation on the new formula of free surface correction. As long as the prescriptive attitude towards the stability criteria will be in force, the dynamic free surface correction could be used. It should consider the localisation of partly filled tanks referred to the axis of roll. This would be a step ahead towards the increase in accuracy of the ship's stability estimation, which should allow one to improve the safety of the ship.

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Linearyzacja w opisie zjawiska sloshingu na potrzeby eksploatacyjnej oceny stateczności statku

Streszczenie

Artykuł prezentuje wyniki badań eksperymentalnych i symulacji numerycznych ruchu cieczy w niepełnych zbiornikach statku. Istotą badań było wyznaczenie momentu przechylającego statek wskutek ruchu cieczy w zbiornikach, co wpływa na stateczność poprzeczną ststku. Zaproponowana metoda linearyzacji momentu umożliwia

implementację wyników badań przedmiotowego zjawiska do praktyki oceny stateczności poprzecznej statku.

Szczególną uwagę zwrócono na zależność pomiędzy momentem przechylającym statek a lokalizacją niepełnego zbiornika względem osi kołysań bocznych. Zarazem wyznaczany moment przechylający został porównany do quasi-statycznego momentu wyliczonego zgodnie z zaleceniami Kodeksu ISC. Przeprowadzone badania mogą stanowić element poprawy wiarygodności oceny stateczności statku dokonywanej rutynowo przez oficerów ładunkowych.

Manuscript received July 10, 2008; accepted for print November 18, 2008