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Prevention of resonance oscillations in gear mechanisms using non-circular gears

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One of the main disadvantages of gear mechanisms is the occurrence of noise and vibrations. This study investigated the applicability of non-circular gears for preventing resonance oscillations in gear mechanisms. The influence of a small deviation of the gear centrodes from the nominal circles on kinematic and oscillatory characteristics was analysed. It was shown that a larger deviation results in a smaller resonance amplitude due to mesh frequency variability and simultaneously in higher additional dynamic loads on the mechanism. The shape of the gear centrodes was determined which provides a relatively small resonance amplitude with minimum additional dynamic loads. A mechanical device was developed to enable cutting of slightly non-circular gears on a hobbing machine without numerical control.

Keywords: non-circular gear, gear pair, variable gear ratio, gear noise, gear vibrations, resonance amplitude

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1. Introduction

Development of reliable, durable and noiseless transmission mechanisms is an important engineering problem. One of the ways of their enhancement is the application of non-circular gears. Mechanisms with non-circular gears are utilised in technical systems in which a mechanical control of speed variation during the working cycle is required. Examples of such systems include mechanical presses [1], textile industry machines [2], high-power starters, hydraulic engines, pumps, flow meters, clocks, and automatic toys. The most commonly encountered shape of non-circular gears is ellipse [3–8], whereas gears with a more complex shape are used to reach special transmission characteristics [9, 10]. Non-circular gears have generally involute profile teeth. To increase the transmission load capacity, one also uses Wildhaber-Novikov helical gears with circular arc teeth [11–13].

The studies investigating non-circular gears are usually aimed at resolving the following problems:

- determination of the gear centrodes that provide the required kinematic properties, for instance: smooth variation of the gear ratio [14, 15], cyclic speed variation [16], motion synchronisation [4], reduction of the working cycle duration [1], arbitrary variation of the gear ratio [17, 18], prescribed relationship between the input and output angular displacements [19], intermittent motion [20];
- determination of the gear centrodes that allow to reduce dynamic loads on the mechanism [21–23];
- synthesis of the gear tooth profile with optimum strength properties [8];
- gear design automation and enhancement of the gear cutting technology [5–7, 11, 12, 24–26].

The above literature review shows that non-circular gears were investigated with respect to the kinematic and dynamic behaviour, strength properties, and cutting technology. However, there are still unexplored aspects of the application of non-circular gears. One of them is the influence of the gear non-circularity on the occurrence and intensity of noise and vibrations in gear mechanisms.

The main reason for gear noise is considered to be the transmission error [27] defined by Welbourn [28] as ‘the difference between the actual position of the output gear and the position it would occupy if the gear drive were perfectly conjugate’. The causes of the transmission error are deformations of the gears (contact areas in the gear mesh, gear teeth, gear blanks), deformations in the mechanism (shafts, bearings, casing), geometrical errors (gear teeth profiles, gear centrodes, position of the gear carrying shafts, position of the bearings in the casing), gear teeth wear, etc. Because of the transmission error, the gear teeth come into mesh with a dynamic impact [29], generating an excitation force that acts on the mechanism with the mesh frequency and its harmonics [30]. If the mesh frequency or any of its harmonics coincides with one of the natural frequencies of the mechanism, resonance effects will occur, including additional dynamic loads on the mechanism [31] and noise emissions [32].

The aim of the present study was to theoretically investigate the applicability of non-circular gears for preventing resonance oscillations in gear mechanisms.

Notation

f_0	excitation amplitude	$\varepsilon_{1,2}$	gear angular acceleration
i	gear ratio, $i = \omega_2/\omega_1$	$\varepsilon_{\max}$	maximum absolute value of ε_2, $\varepsilon_{\max} = \max\lvert\varepsilon_2\rvert$
j	number of the gear ratio variation cycles per turn of the input gear	ξ	coefficient of the gear ratio variation smoothness, $\xi > 1$
k	parameter, $k = j/z_1$	π	Pi number, $\pi \approx 3.14$
r	nominal radius of the input gear centrode	τ	dimensionless time variable, $\tau = \omega_\mathrm{n} t$
$r_{1,2}$	instant radius of the gear centrode	$\varphi_{1,2}$	gear angular displacement
t	time variable	ω	dimensionless velocity of the input gear, $\omega = z_1\omega_1/\omega_\mathrm{n}$
u	gear teeth ratio, $u = z_2/z_1$	$\omega_{1,2}$	gear angular velocity
x	displacement	ω_n	angular natural frequency
$z_{1,2}$	gear teeth number	ω_e	dimensionless excitation frequency
B	maximum relative deviation of r_1, $B \ll 1$	ω_m	angular mesh frequency, $\omega_\mathrm{m} = r_1 z_1 \omega_1 / r$
X	dimensionless displacement, $X = x\omega_\mathrm{n}^2/f_0$	$\blacksquare_1$	quantity related to the input gear
δ	speed fluctuation coefficient	$\blacksquare_2$	quantity related to the output gear

2. Definition of the gear centrodes and the gear ratio

We consider a meshed pair of non-circular gears. The instant radii r_1 and r_2 of the input and output gear centrodes, respectively, are defined by the following functions [13]:

$$r_1 = r\left(1 + \frac{B\sqrt{\xi^2 - 1}\ \sin(j\varphi_1)}{\xi + \cos(j\varphi_1)}\right);$$
$$r_2 = r\left(u - \frac{B\sqrt{\xi^2 - 1}\ \sin(j\varphi_1)}{\xi + \cos(j\varphi_1)}\right) \tag{1}$$

The gear ratio i has the form

$$i = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} = \frac{\xi + \cos(j\varphi_1) + B\sqrt{\xi^2 - 1}\ \sin(j\varphi_1)}{u\ (\xi + \cos(j\varphi_1)) - B\sqrt{\xi^2 - 1}\ \sin(j\varphi_1)} \approx \frac{1}{u} + \frac{B(1+u)\sqrt{\xi^2 - 1}\ \sin(j\varphi_1)}{u^2(\xi + \cos(j\varphi_1))} \tag{2}$$

Here φ_1 is the input gear angular displacement; r is the nominal radius of the input gear centrode; u is the gear teeth ratio, $u = z_2/z_1$; z_1 and z_2 are the teeth numbers of the input and output gears, respectively; ω_1 and ω_2 are the angular velocities of the input and output gears, respectively; B is the maximum relative deviation of r_1, $B \ll 1$; ξ is the coefficient of the gear ratio variation smoothness, $\xi > 1$; j is the number of the gear ratio variation cycles per turn of the input gear, $j \in \{1, 2, 3, \ldots\}$. The output gear angular displacement φ_2 is determined by integrating Eq.(2):

$$\varphi_2 = \int_0^{\varphi_1} i\, d\varphi_1 \approx \frac{\varphi_1}{u} - \frac{B(1+u)\sqrt{\xi^2 - 1}}{ju^2}\ \ln\left(\frac{\xi + \cos(j\varphi_1)}{\xi + 1}\right)$$

Figure 1 plots the gear centrodes and the gear ratio i for various values of j. In order to emphasise the curvature of the gear centrodes, B is set to a relatively large value.

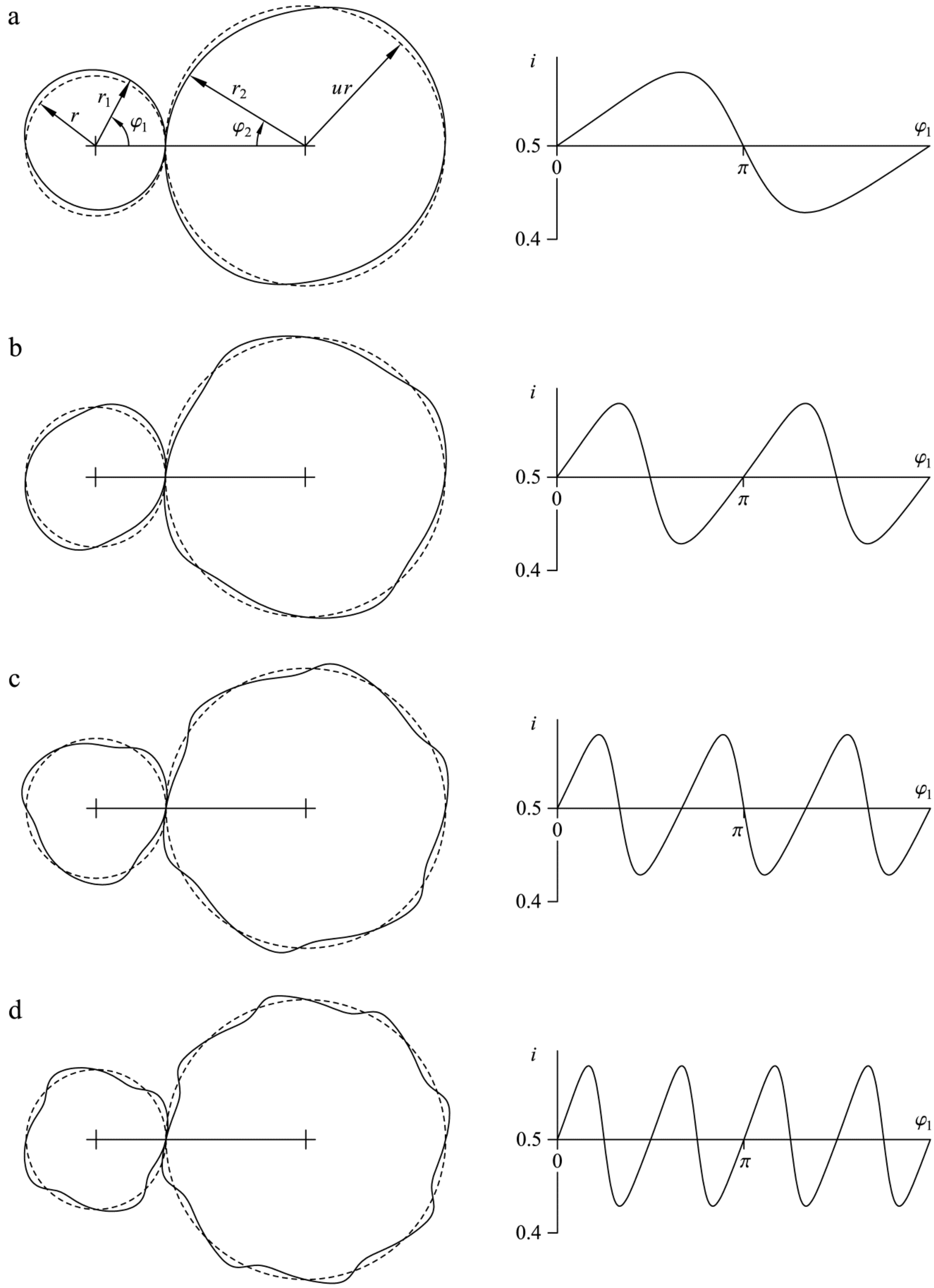

Fig.1. Gear centrodes and gear ratio i at $u = \xi = 2$ and $B = 0.1$:
(a) $j = 1$; (b) $j = 2$; (c) $j = 3$; (d) $j = 4$ (2-column image)

The coefficient ξ affects the shape of the gear ratio function, as illustrated in Fig.2. At ξ close to 1, i changes abruptly (see the curve $\xi = 1.1$). With increasing ξ, the variation of i becomes smoother. At large values of ξ, i is almost a sinusoid (see the curve $\xi = 10$).

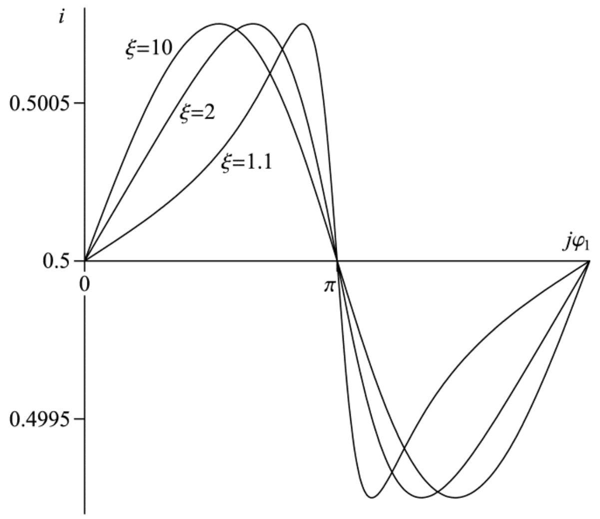

Fig.2. Influence of the coefficient ξ on the gear ratio i at u = 2 and B = 0.001 (1-column image)

3. Analysis of the impacts of the gear non-circularity

3.1. Output gear motion non-uniformity

The variation of the gear ratio i due to Eq.(2) leads to a non-uniform motion of the output gear. The non-uniformity of motion can be characterised by the speed fluctuation coefficient δ equal to the ratio of the difference between the maximum and minimum speeds of an element considered to its average speed. Assuming a constant angular velocity ω_1 for the input gear, we obtain that

$$\delta = \frac{\max \omega_2 - \min \omega_2}{\omega_1/u} = \frac{\max i - \min i}{1/u} \tag{3}$$

The following estimates of Eq.(2)

$$\max i \approx \frac{1}{u} + \frac{B(1+u)}{u^2}; \qquad \min i \approx \frac{1}{u} - \frac{B(1+u)}{u^2}$$

allow expressing Eq.(3) in the form

$$\delta \approx \frac{2B(1+u)}{u} \tag{4}$$

The coefficient δ is thus proportional to B.

3.2. Output gear acceleration

The non-uniform motion of the output gear causes additional dynamic loads on the mechanism. These loads are proportional to the output gear angular acceleration ε_2. Taking into account that $\varepsilon_1 = d\omega_1/dt = 0$, we derive the following expression:

$$\begin{aligned}\varepsilon_2 &= \frac{d\omega_2}{dt} = \omega_1^2 \frac{di}{d\varphi_1} \\ &= \frac{Bj\omega_1^2(1+u)\sqrt{\xi^2-1}(1+\xi\cos(j\varphi_1))}{\left(u(\xi+\cos(j\varphi_1)) - B\sqrt{\xi^2-1}\,\sin(j\varphi_1)\right)^2} \approx \frac{Bj\omega_1^2(1+u)\sqrt{\xi^2-1}(1+\xi\cos(j\varphi_1))}{u^2(\xi+\cos(j\varphi_1))^2}\end{aligned} \tag{5}$$

Based on the equality

$$\max\left|\frac{1+\xi\cos(j\varphi_1)}{(\xi+\cos(j\varphi_1))^2}\right| = \frac{1}{\xi-1}, \qquad \xi > 1$$

the maximum absolute value of Eq.(5) can be estimated as

$$\varepsilon_{\max} = \max|\varepsilon_2| \approx \frac{Bj\omega_1^2(1+u)\sqrt{\xi+1}}{u^2\sqrt{\xi-1}} \tag{6}$$

whence it follows that $\varepsilon_{\max}$ is proportional to Bj and decreases with ξ.

3.3. Oscillation amplitude

Teeth of circular gears (B = 0) come into mesh with angular frequency equal to $z_1\omega_1$. For a non-circular gear (B > 0), the teeth number per unit angular displacement is proportional to the instant radius of the centrode. Consequently, the non-circular gear pair under consideration has angular mesh frequency ω_{m} which changes in accordance with the equation below:

$$\omega_{\mathrm{m}} = \frac{r_1 z_1 \omega_1}{r} \tag{7}$$

We represent the mechanism connected to the gear pair in the form of a system with a single degree of freedom, x, and a natural frequency ω_{n} [33–36]. Depending on the type of the mechanism and its schematisation [37], x can be the radial / angular displacement of one of the gears due to bending / torsion of the carrying shaft, or a relative displacement of the gears, or a displacement of another element. We also assume that the excitation force, acting on the system due to the transmission error, changes sinusoidally with the angular frequency ω_{m} [38]. Then, with account of Eq.(7), the motion equation is defined as follows:

$$\frac{d^2x}{dt^2} + \omega_{\mathrm{n}}^2 x = f_0 \sin\left(\frac{r_1 z_1 \omega_1}{r} t\right) \tag{8}$$

where f_0 is the excitation amplitude with the dimension of acceleration. The dimensionless time variable $\tau = \omega_{\mathrm{n}} t$, displacement $X = x\omega_{\mathrm{n}}^2/f_0$, input gear velocity $\omega = z_1\omega_1/\omega_{\mathrm{n}}$, and parameter $k = j/z_1$ allow to rewrite Eq.(8) in the form

$$\frac{d^2X}{d\tau^2} + X = \sin(\omega_{\mathrm{e}}\tau) \tag{9}$$

where the dimensionless excitation frequency

$$\omega_{\mathrm{e}} = \left(1 + \frac{B\sqrt{\xi^2-1}\,\sin(k\omega\tau)}{\xi + \cos(k\omega\tau)}\right)\omega \tag{10}$$

At B = 0 it is true that $\omega_{\mathrm{e}} = \omega$ and Eq.(9) represents a classical equation of undamped forced oscillations. If, in addition, ω = 1, i.e. the gear mesh frequency is equal to the natural frequency, resonance oscillations of X will occur, with the amplitude increasing theoretically indefinitely [39].

In the case B > 0, ω_{e} is a variable quantity. At ω = 1, according to Eq.(10), the equality $\omega_{\mathrm{e}} = 1$ holds only at the values of τ which are multiples of π/k. Fig.3 shows a numerical solution of Eq.(9) with zero initial conditions in the interval of τ from 0 to 10^5. The solution was obtained using a Runge-Kutta type Rosenbrock method (MATLAB). It is seen that there is no unbounded growth of the amplitude. The amplitude that corresponds to the final moment of simulation is denoted by A.

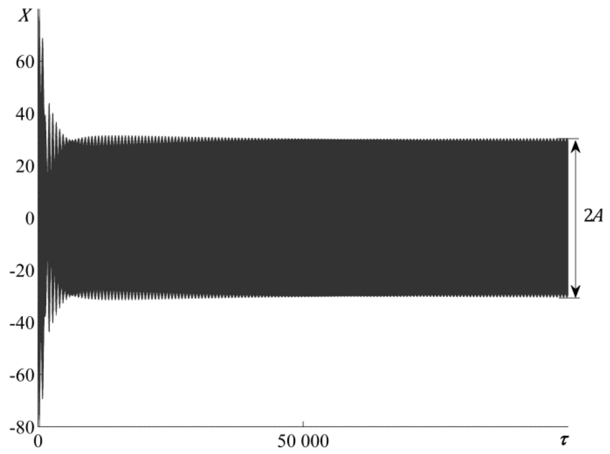

Fig.3. Dimensionless displacement X vs. time τ at $\omega = 1$, $B = 0.01$, $\xi = 2$, $k = 1/10$ (1-column image)

Figure 4 presents the relationship between A and ω for various values of B. The dashed line depicts the transmissibility function for an undamped forced oscillator [39]. As follows from the obtained result, A takes a substantially larger value in the vicinity of the point $\omega = 1$.

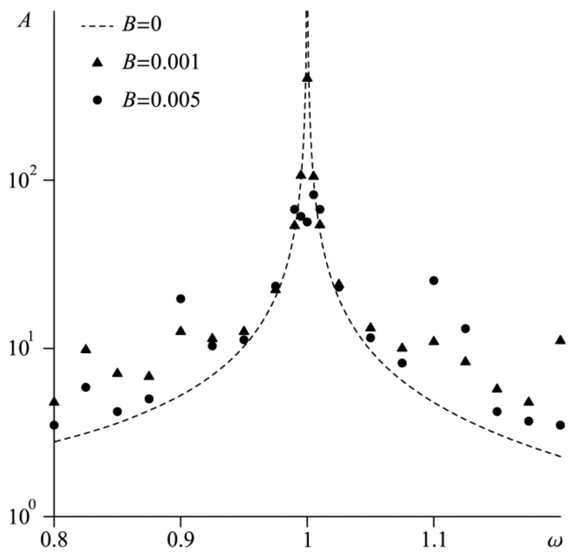

Fig.4. Dimensionless amplitude A vs. input gear velocity ω at $\xi = 2$ and $k = 1/10$ (1-column image)

Of interest is the dependence of A on the parameters B, ξ and k for the case of resonance. Fig.5 shows the relationship between A and B at various values of k. One can see that A decreases by an order of magnitude with B increasing from 0.001 to 0.01. This is explained by that ω_e deviates stronger from 1 with an increase in B. A variation of k in the range between $1/100$ and $1/5$, which corresponds to the ranges of $20 \leq z_1 \leq 100$ and $1 \leq j \leq 4$, leads to a several tens of percent change in A. Fig.6 shows the relationship between A and B, this time with respect to ξ. With an increase in ξ, A substantially decreases.

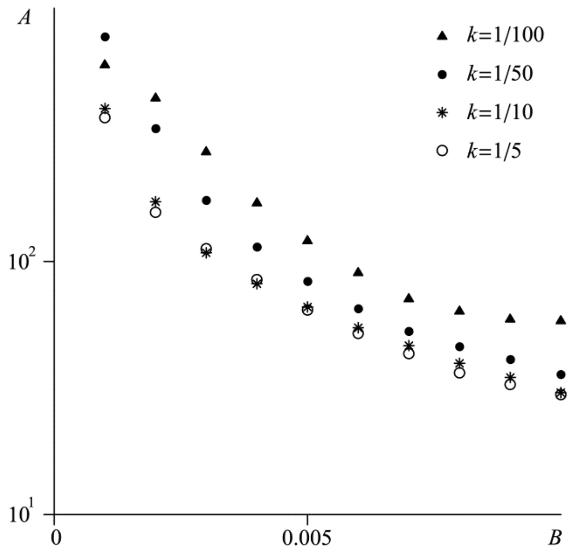

Fig.5. Dependence of the dimensionless amplitude A on B and k at ω = 1 and ξ = 2 (1-column image)

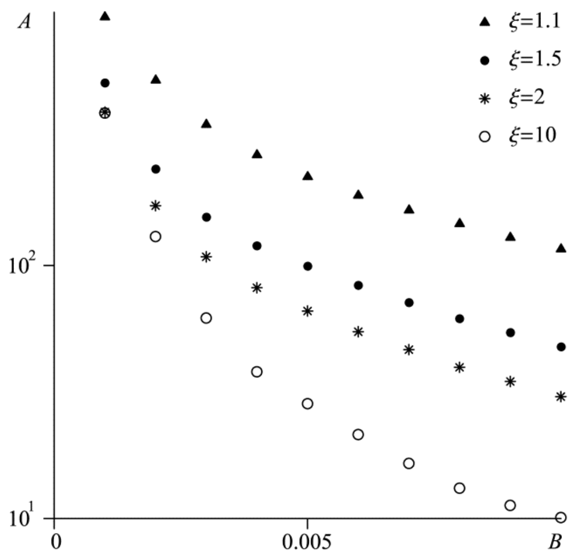

Fig.6. Dependence of the dimensionless amplitude A on B and ξ at ω = 1 and $k = 1/10$ (1-column image)

4. Efficient shapes of the noncircular gears

Analysis of Eq.(6) and the results presented in Figs.5 and 6 shows that an increase in ξ results in a simultaneous decrease in $\varepsilon_{\max}$ and A, i.e. in a reduction of the additional dynamic loads and resonance amplitude. On the other hand, a decrease in j results in a proportional decrease in $\varepsilon_{\max}$ but has no significant effect on A. Note that the minimum value of j at which both input and output gears are balanced is 2 (see Fig.1). From Eqs.(1) and (2) at $\xi \to \infty$ and j = 2, we obtain the gear centrodes with sinusoidal deviation

$$r_1 = r(1 + B\sin(2\varphi_1));$$
$$r_2 = r(u - B\sin(2\varphi_1))$$

which ensure a relatively small resonance amplitude with minimum additional dynamic loads.

Since a larger B implies a larger $\varepsilon_{\max}$ and a smaller A, the choice of B is a compromise between the level of the additional dynamic loads and the intensity of the resonance oscillations. Besides, B must be small enough so that the speed fluctuation coefficient δ, calculated from

Eq.(4), would not exceed the maximum allowable value which is between 0.001 and 0.2 depending on the mechanism type [40].

5. Manufacture of slightly non-circular gears

Manufacture of non-circular gears presents certain difficulties due to a complex gear shape. For cutting gears with slightly non-circular centrodes, we developed the device presented schematically in Fig.7. The gear blank is mounted on a floating shaft together with a master form and a master form gearwheel. The master form, being pressed by two press rollers against a support roller, governs the position of the floating shaft. The contour of the master form and the centrode of the master form gearwheel satisfy Eq.(1) and are identically oriented. The master form gearwheel is driven by a driving gearwheel which has the same number of teeth and centrode length. Thereby, the device provides rotation of the gear blank and its simultaneous translation perpendicularly to the hob axis. A detailed description of the device can be found in [13].

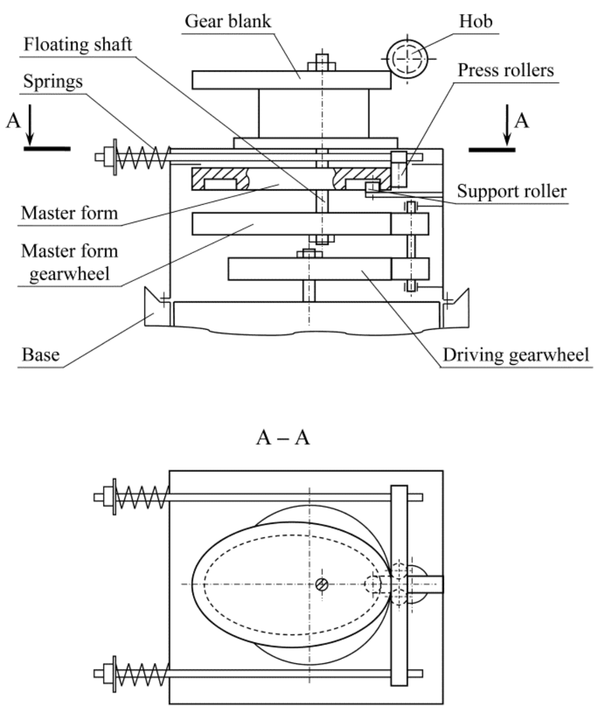

Fig.7. Schematic of the experimental device (the shape of the master form contour is simplified and elongated for clarity) (1-column image)

The device was installed on a semi-automated hobbing machine 5K32 adjusted for manufacturing Wildhaber-Novikov helical gears with circular arc teeth. Fig.8 shows a photograph of the experimental set-up. Two single-thread hobs code-named СИЗ 9330-537 and СИЗ 9330-532 were available which allow cutting convex teeth on the input gear and concave circular tooth profiles on the output gear, respectively. The rotation of the driving gearwheel of the device was synchronised with the motion of the hob. The settings of the hobbing machine and the parameters of the cutting mode were identical to those set for cutting the corresponding circular gears ($B = 0$) in the absence of the device.

An experimental pair of non-circular gears manufactured using the set-up is shown in Fig.9. The material of the gears was steel EN 37Cr4. Table 1 presents the parameters of the gears. Measurements confirmed that the gear centrodes satisfy Eq.(1). Thus, the device enables cutting of gears with slightly non-circular centrodes without using numerical control machines.

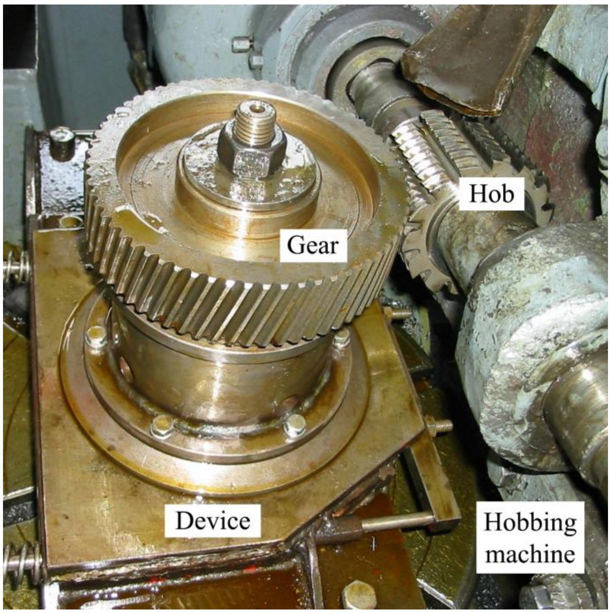

Fig.8. Photograph of the set-up to manufacture slightly non-circular gears (1-column image, colour online only)

Fig.9. Photograph of the experimental non-circular gears (1-column image, colour online only)

Table 1. Parameters of the experimental non-circular gears

Parameter	Input gear	Output gear
Gear teeth ratio u	2	
Normal module, mm	3	
Centre distance, mm	150	
Helix angle, °	16.0911	
Deviation B	0.01	
Coefficient ξ	2	
Number j	4	
Gear teeth numbers z_1 and z_2	32	64
Gear centrode nominal radii r and ur, mm	50	100
Face width, mm	46	44

5. Conclusions

The influence of a small deviation of the gear centrodes from the nominal circles on kinematic and oscillatory characteristics of the gear mechanism was theoretically investigated. Simulations showed that a larger deviation results in a smaller resonance amplitude due to mesh frequency variability and simultaneously in higher additional dynamic loads on the mechanism. It was found that gear centrodes with sinusoidal deviation provide a relatively small resonance amplitude with minimum additional dynamic loads. A mechanical device was developed to enable cutting of slightly non-circular gears on a hobbing machine without numerical control. The findings of this study suggest that slightly non-circular gears can be potentially used for preventing resonance oscillations in gear mechanisms and are manufacturable.

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