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A MOR Algorithm Based on the Immittance Zero and Pole Eigenvectors for Fast FEM Simulations of Two-Port Microwave Structures

Grzegorz Fotyga, Damian Szypulski, Adam Lamecki, Piotr Sypek, Michal Rewienski, Valentín de la Rubia and Michal Mrozowski

***Abstract*—The aim of this paper is to present a novel model order reduction (MOR) algorithm for fast finite-element (FEM) frequency-domain simulations of microwave two-port structures. The projection basis used to construct the reduced-order model (ROM) comprises two sets: singular vectors and regular vectors. The first set is composed of the eigenvectors associated with the poles of the FEM state-space system, while the second one is made up from the eigenvectors corresponding to the zeros of the diagonal elements of the matrix-valued immittance transfer function. Importantly, just one LU factorization of the FEM system is required to construct the projection basis during the reduction process, due to the application of a new formulation based on the Schur complement. The sets of eigenvectors that are used in the basis are independent of one another, which makes the new technique better suited for parallel computing compared to previously developed methods, which are sequential in nature. The reliability and accuracy of the proposed scheme is compared to that of the standard MOR technique, namely, the reduced-basis method (RBM), and verified through the analysis of three microwave structures: an eighth-order dual-mode waveguide filter, a dielectric resonator filter, and a folded waveguide filter.**

***Index Terms*—Computer-aided engineering, design automation, finite-element method, microwave circuits, model order reduction, reduced-basis methods.**

I. Introduction

ONE of the most popular numerical techniques for simulating passive high frequency electronic devices of complex shape is the finite-element method (FEM) [1]. Unfortunately, when the goal of the simulation is to compute the network parameters, like the scattering matrix of the considered structure over a wide frequency band, FEM analysis tends to be time consuming.

The frequency domain FEM simulations can be significantly sped up using the so-called *fast-frequency sweep* (FFS) algorithms. They rely on the assumption that the transfer function of a structure can be approximated using the results of computations obtained in a selected subset of frequency points. There are two main classes of algorithms used for the aforementioned computations. For the first one, the transfer function of a structure is approximated by a rational function, using the data gathered at the frequency points that are selected adaptively [2]–[5]. However, such an approach allows one to gain insight into the electromagnetic field distribution only for the selected frequencies.

For the second class of FFS algorithms, called model order reduction (MOR) methods, it is assumed that the FEM full-order model (FOM) can be replaced by a much smaller reduced-order model (ROM), which approximates the original model with sufficient accuracy in the specified frequency band. Historically, in the electromagnetic community, two main MOR approaches have been used. The first, called the reduced-basis method (RBM) [6]–[10], is based on the fact that in most frequency-domain simulations, the distribution of the electromagnetic field does not vary significantly in the frequency band. Thus, the field distribution in the entire band can be represented as a linear combination of a few field solutions of the original FEM system of equations, computed at properly selected frequencies. This reasoning leads to the construction of a ROM, which is then used to quickly calculate the approximate transfer function of the system (see [9] for a more detailed discussion). The other MOR approach uses the concept of moment matching (MM) [11], where the values of the original and reduced transfer functions and their subsequent derivatives are matched at a single [12]–[17] or a few frequency points [18]–[20]. In this case, the field distribution can be obtained for any frequency using the reduced-order state vector and the projection basis.

Recently, a new strategy for the construction of ROMs has been proposed in [21], resulting in two new MOR algorithms: the compact reduced basis method (CRBM) [21] and the subspace-splitting moment matching model order reduction technique (SS-MM MOR) [22]. This strategy is based on the observation that the electric field distribution can be decomposed into a singular and a regular part in the system transfer function. The first one corresponds to the natural oscillating dynamics of the electric field in the computational domain, whereas the second part spans whatever else is required to approximate the electric field accurately enough in the specified frequency band. Following this observation, the projection basis used to generate a ROM can be split into a so-called singular and a so-called regular set of vectors, which

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give rise to the singular and regular part of the system transfer function, respectively. The first (singular) set is made up of the eigenvectors found in the band of analysis, by solving the generalized eigenproblem derived from the FOM. These eigenvectors correspond to the poles of the transfer function for the FEM state-space system. The second set, which spans the regular part of the frequency response, is computed using either RBM [21], or a moment-matching technique based on SAPOR (the second-order Arnoldi method for passive order reduction), as shown in [22].

In this paper, a novel MOR approach is proposed. Here, similarly as in CRBM and SS-MM MOR [21], [22], the singular part of the projection basis is composed of the eigenvectors of the FEM matrix corresponding to in-band immittance poles. However, unlike in [21], [22], the regular part is not computed from snapshots or moments, but comprises a new class of eigenvectors associated with the element zeros of the matrix-valued transfer function (depending on the FEM formulation used, it can be impedance or admittance). Thus, the proposed technique is called the zero-pole model order reduction (ZP-MOR) method. It combines the most beneficial features of the state-of-the-art MOR methods used in computational electromagnetics and offers new attractive features, namely:

- It can be applied to wideband simulations (like algorithms [6], [8], [19], [21], [22]).
- Typically, it requires only one LU factorization of the FEM system matrix to construct ROM (as algorithms [14], [22] do), which reduces the cost of extracting the ROM.
- It generates compact ROMs (as algorithms [6], [8], [21] do), which translates into low-cost frequency sweeps.
- It is inherently parallel by definition since the sets of eigenvectors for poles and zeros are independent of each other.
- It produces ROMs that do not suffer from spurious in-band poles that manifest themselves in spikes in the reduced order model frequency response [23], [24], as it will be later discussed in detail.

It should be noted that, to the best of our knowledge, no single MOR method has been developed [6]–[9], [14]–[17], [19]–[22], [25] that includes all the features highlighted above. Moreover, the computation of the projection basis in prior-art model order reduction algorithms is a sequential process; the next vector can be computed only after the computation of the previous vectors has been completed. In the proposed approach, this problem is not present, as the eigenproblems for immittance poles and zeros can be solved in parallel.
Achieving the above is made possible by:

- replacing snapshots or moments with the electromagnetic field at immittance zeros, which allows one to accurately represent the regular part of the transfer function over the entire frequency band, while keeping the size of the reduced basis very compact, and
- applying the Schur complement to solve the required eigenproblems for zeros using a single system matrix factorization.

In the following sections, a detailed description of the ZP-MOR construction procedure and numerical validation of the reliability and accuracy of ZP-MOR for three complex microwave structures is provided.

II. FEM FORMULATION

Let us consider a source-free computational domain Ω bounded by perfect electric conductor (PEC) walls and cross-sections of n_p waveguide ports denoted by S_E and S_W^k, respectively, for $k = 1, \dots n_p$ (in the discussion below, we assume $n_p = 2$). The distribution of the electric field $\vec{E}$ in Ω is described by the frequency-domain boundary value problem (BVP) [18], [26]:

$$\begin{aligned} \nabla \times ({\mu_r}^{-1} \nabla \times \vec{E}) - k_0^2 \epsilon_r \vec{E} = 0 \qquad & \text{in } \Omega, \\ \vec{E} \times \hat{n} = 0 \qquad & \text{on } S_E, \\ \hat{n} \times [(\nabla \times \vec{E}) \times \hat{n}] + j k_0 \eta_0 \vec{h}_k = 0 \qquad & \text{on } S_W^k, \end{aligned} \tag{1}$$

where k_0 is the wavenumber, j is the imaginary unit, $\hat{n}$ is the outward unit vector, ϵ_r and μ_r are the relative permittivity and permeability, respectively, η_0 is the characteristic impedance of free space, and $\vec{h}_k$ is the frequency-dependent normalized pattern of the tangential magnetic field at the k-th port.

Applying FEM discretization [1] with n degrees of freedom to the weak form of (1) leads to the following n-dimensional linear system of equations:

$$\begin{aligned} (\mathbf{\Gamma} + s^2 \mathbf{C}) \mathbf{E}(s) = s \mathbf{B} \mathbf{i}, \\ \mathbf{u} = \mathbf{B}^T \mathbf{E}(s). \end{aligned} \tag{2}$$

where $\mathbf{\Gamma}$ and $\mathbf{C} \in \mathbb{C}^{n \times n}$ are FEM system matrices, $\mathbf{B} \in \mathbb{C}^{n \times 2}$ is the excitation matrix, $\mathbf{E}(s) \in \mathbb{C}^{n \times 2}$ is the unknown vector with elements, dependent on frequency, being the amplitudes of the basis functions which determine the electric field. $\mathbf{u}$, $\mathbf{i} \in \mathbb{C}^{2 \times 2}$ contain the amplitudes of voltage and current waves, respectively, and $s = j k_0$ is the normalized complex frequency. Note that we have assumed the number of waveguide ports $n_p = 2$.

The goal of the simulation is to compute the scattering parameters and the field distribution for the analyzed structure, for a sequence of frequency points: $s \in \{s_1, s_2, \dots s_{n_F}\}$. To this end, one has to first compute the impedance matrix:

$$\mathbf{Z}(s) = \mathbf{B}^T (\mathbf{\Gamma} + s^2 \mathbf{C})^{-1} s \mathbf{B}. \tag{3}$$

at each frequency point s_i. Next, the scattering parameters $\mathbf{S}(s)$ can easily be found using the formula:

$$\mathbf{S}(s) = 2(\mathbf{I}_D + \mathbf{Z}(s)^{-1})^{-1} - \mathbf{I}_D, \tag{4}$$

where $\mathbf{I}_D$ is the identity matrix and $\mathbf{Z}(s), \mathbf{S}(s) \in \mathbb{C}^{2 \times 2}$. Alternatively, an analogous formulation can be derived for the admittance matrix.

The above frequency sweep is very time consuming when the size of the problem n and/or the number of considered frequency points n_F is large, due to the high numerical cost of factorizing the matrices $(\mathbf{\Gamma} + s_i^2 \mathbf{C})$ at each frequency point s_i.

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III. MOR-BASED FAST FREQUENCY SWEEP

To lower the cost associated with repetitive matrix factorizations, the original state-space system (2) is approximated with a reduced-order model (ROM):

$$\begin{aligned}(\mathbf{\Gamma}_R + s^2\mathbf{C}_R)\mathbf{E}_R(s) &= s\mathbf{B}_R\mathbf{i},\\ \mathbf{u} &= \mathbf{B}_R^T\mathbf{E}_R(s),\end{aligned} \tag{5}$$

giving rise to the corresponding transfer function:

$$\mathbf{Z}_R(s) = \mathbf{B}_R^T(\mathbf{\Gamma}_R + s^2\mathbf{C}_R)^{-1}s\mathbf{B}_R. \tag{6}$$

This reduced order model is then used to compute the approximate scattering matrix:

$$\mathbf{S}_R(s) = 2(\mathbf{I}_D + \mathbf{Z}_R(s)^{-1})^{-1} - \mathbf{I}_D. \tag{7}$$

Reduced-order matrices in (5) and (6) are obtained using Galerkin projection: $\mathbf{\Gamma}_R = \mathbf{V}^T\mathbf{\Gamma}\mathbf{V} \in \mathbb{C}^{r\times r}$, $\mathbf{C}_R = \mathbf{V}^T\mathbf{C}\mathbf{V} \in \mathbb{C}^{r\times r}$, $\mathbf{B}_R = \mathbf{V}^T\mathbf{B} \in \mathbb{C}^{r\times 2}$, where r is the dimension of the reduced-order space, and $\mathbf{V} \in \mathbb{C}^{n\times r}$ is the *so-called* projection basis. The frequency sweep using the ROM is much faster, since $r \ll n$. In the MOR-based sweep, an approximate distribution of the electromagnetic field can be recovered at each frequency $s \in \{s_1, s_2, \dots s_{n_F}\}$ using the expression:

$$\mathbf{E}(s) \approx \mathbf{V}\mathbf{E}_R(s). \tag{8}$$

The choice of projection basis $\mathbf{V}$ differentiates the MOR techniques mentioned in the previous section. For RBM [6]–[10], the projection basis consists of field distributions (snapshots) $\mathbf{E}(s)$, which are solutions of (2) at frequencies $\hat{s}_1, \hat{s}_2, ..., \hat{s}_{n_R}$:

$$\mathbf{V} = [\mathbf{E}(\hat{s}_1), \mathbf{E}(\hat{s}_2), ..., \mathbf{E}(\hat{s}_{n_R})]. \tag{9}$$

The frequencies $\hat{s}_1, \hat{s}_2, ..., \hat{s}_{n_R}$, where $n_R \ll n_F$, are selected using a greedy algorithm, where the next snapshot is computed at the frequency for which the current reduced model exhibits the largest estimated error [6], [7], [9], [27]. This approach allows one to achieve the desired accuracy of the model in the frequency band of interest, yet it involves factorizing the full-order model (FOM) system matrix at each frequency point, which may lead to a high computational cost in extracting the ROM.

To address this computational cost issue associated with RBM methods, moment-matching MOR algorithms are often used. These methods construct a projection basis $\mathbf{V}$, which ensures that the values of the transfer functions for the FOM (3) and the ROM (6) and their subsequent derivatives are matched at a single [13], [14] or at a few frequency points [18]–[20]. The advantage of this approach is that the ROM construction process requires fewer FOM system matrix factorizations, compared to the RBM method, but at the cost of a larger projection basis.

As observed in [21], [22], a significant contribution to the field distribution comes from the so-called singular part of the system transfer function, associated with the pole frequencies. Therefore, obtaining a good accuracy of the reduced-order models around the poles of the system has been one of the key issues. However, traditional RBM or moment-matching MOR methods do not explicitly ensure that the electromagnetic field at pole frequencies can be accurately represented by the ROMs, and consequently additional snapshots need to be computed, or additional moments need to be matched, which increases the size of the reduced models, and accounts for extra computational cost. To address this issue, it has been proposed in CRBM and SS-MM MOR ([21], [22], respectively) to use a projection basis that includes the following two sets of vectors:

$$\mathbf{V} = [\mathbf{V}_E, \mathbf{V}_M]. \tag{10}$$

where $\mathbf{V}_E$ corresponds to the electric field related to the singular part of the system transfer function. These are the eigenmodes of the device under analysis and, as a result, $\mathbf{V}_E$ is composed of the eigenvectors of the following generalized eigenproblem derived from the FOM (2):

$$\mathbf{\Gamma}\mathbf{V}_E = -\mathbf{C}\mathbf{V}_E\mathbf{\Lambda}_E. \tag{11}$$

where the diagonal matrix $\mathbf{\Lambda}_E$ contains the eigenvalues corresponding to the square of the resonant frequencies of the system in the band of interest. These resonances are the poles of the transfer function $\mathbf{Z}(s)$, whereas the vectors in $\mathbf{V}_E$ are modal fields at the system resonances. The second set of vectors: $\mathbf{V}_M$ is composed of field contributions related to the regular part of the system transfer function, and is computed using RBM in [21] or a moment-matching technique (SAPOR [14]) in [22].

Note that the process of constructing the projection basis is essentially sequential. In RBM-like techniques, a subsequent snapshot is added at the frequency point that is determined from the *a posteriori* error estimator, evaluated based on the ROM constructed from previous samples. In moment-matching techniques, in order to achieve a numerically stable algorithm, new moments have to be orthogonal to the previous moments, so the moments cannot be computed concurrently. This is a drawback of prior-art MOR techniques in the context of the fact that computer hardware more and more evolves into parallel architectures (clusters or multicore/manycore systems).

IV. EIGENVECTORS AS PROJECTION BASIS

In [21], [22]), it was shown that using a projection basis that includes eigenvectors of the FEM matrix associated with the FOM poles brings accuracy benefits and produces very compact models. Furthermore, the singular part of the ROMs themselves can be effectively generated, thanks to robust and efficient eigenvalue solvers, which have also been developed for cluster architectures [28], [29]. However, the approximation of the field contribution to the regular part of the system transfer function in the cited methods was still based on RBM snapshots (in CRBM [21]) and/or transfer function moments (in SS-MM MOR [22]), with no special attention drawn to ensure the accuracy of the reduced model around the frequencies corresponding to the impedance zeros. Following these observations, it has been proposed in this work to consider in the projection basis not only the eigenvectors associated with poles, but also those corresponding to zeros of impedance (or admittance) elements, which could allow one

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to more compactly and accurately represent the electric field contributing to the regular part of the system transfer function. As a matter of fact, this new approach eventually exploits the efficiencies associated with generating the basis vectors by solving eigenproblems, instead of computing snapshots or moments of the system transfer function.

The rationale behind including fields associated with zeros in the projection basis stems from the pole-zero representation of the transfer function for Linear Time Invariant (LTI) systems [30]. The matrix entries in the matrix-valued transfer function $\mathbf{Z}(s)$ defined in (3) can be represented as rational functions:

$$z_{ij}(s) = \frac{N_{ij}(s)}{D(s)} = K_{ij} \frac{(s - z_1^{ij})(s - z_2^{ij}) \dots (s - z_{n+1}^{ij})}{(s - p_1)(s - p_2) \dots (s - p_n)} \tag{12}$$

where $N_{ij}(s)$ and $D(s)$ are the numerator and denominator polynomials, respectively, and K_{ij} is a constant. The roots of $N_{ij}(s)$, denoted by z_k^{ij}, are the zeros of the impedance matrix element $z_{ij}(s)$, whereas the roots of $D(s)$, are the system poles p_k. The set of poles is common for all elements in the matrix-valued transfer function: $z_{ij}(s)$. If one assumes that the problem analyzed is passive, lossless, reciprocal, and has two ports, the set of input/output zeros ($N_{ij}(s)$ for $i = j$) and the set of poles of the system $D(s)$ are sufficient to represent all entries in the matrix $\mathbf{Z}(s)$ in the specified frequency band, since the transmission zeros ($N_{ij}(s)$ for $i \neq j$) for the two-port structures can be computed using the Feldtkeller equation [31].

As shown in Section III, the poles p_k can be found by solving the generalized eigenproblem (11). (More precisely, the poles are the square roots of the eigenvalues, located on the diagonal of $\mathbf{\Lambda}_E$.) In order to compute the zeros of the diagonal component of the transfer function associated with ith-port ($z_{ii}(s)$, for $i \in \{1, 2\}$), and the corresponding electromagnetic field, one considers system (2) restricted to a single excitation vector $\mathbf{b}_i$, which means that the excitation is applied only to the i-th port:

$$\begin{aligned} (\mathbf{\Gamma} + s^2\mathbf{C})\mathbf{e}_i &= s\mathbf{b}_i i_i, \\ u_i &= \mathbf{b}_i^T \mathbf{e}_i. \end{aligned} \tag{13}$$

where $\mathbf{e}_i$ is the vector of unknowns, and u_i, i_i are the amplitudes of the voltage and current waves, respectively. By definition of a zero: $u_i = 0$. This means that the excitation applied to the i-th port results in the amplitude of voltage wave equal to zero at that port. By grouping the corresponding components from (13), the system transforms into the following nonsymmetric, but skew-Hamiltonian generalized eigenproblem:

$$\begin{bmatrix} \mathbf{\Gamma} & -\mathbf{b}_i \\ \mathbf{b}_i^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_M^i \\ i_i \end{bmatrix} = - \begin{bmatrix} \mathbf{C} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_M^i \\ i_i \end{bmatrix} \mathbf{\Lambda}_M^i \tag{14}$$

where the diagonal of $\mathbf{\Lambda}_M^i$ contains the eigenvalues which are squares of the zeros of $z_{ii}(s)$ and finally $\mathbf{V}_M^i$[1] are the eigenvectors associated with the zeros of $z_{ii}(s)$. Once the zeros of $z_{ii}(s)$ and the poles have been found, the remaining elements of the matrix-valued transfer function ($z_{ij}(s)$ for $i \neq j$) can be calculated using the Feldtkeller equation [31].

[1]Note that vectors $\mathbf{e}_i$ in (13) and $\mathbf{V}_M^i$ in (14) are equivalent, however $\mathbf{V}_M^i$ is used to preserve the notation from (10).

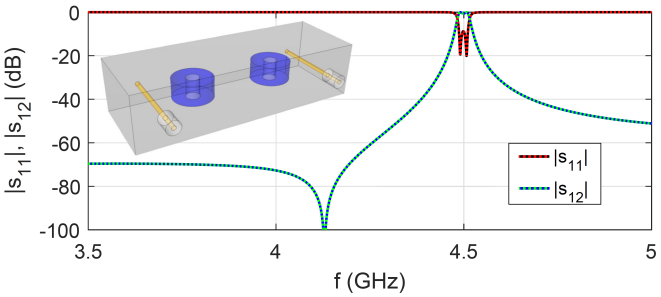

Fig. 1. Dielectric resonator filter: geometry and scattering characteristics (computed using FOM). All dimensions of the filter can be found in [32].

If one is interested in finding the elements of the impedance matrix (12) in a particular frequency band, it is typically enough to consider the poles and zeros located inside and, optionally, near the band. An approximation of (12) is obtained by removing the factors associated with the zeros and poles located far enough from the band of interest from the numerator and denominator, respectively, of the rational function (12). The accuracy of the approximation is determined by the distance between the band of interest and the sets of omitted poles and zeros. The omitted poles and zeros closest to the band have the most impact on the accuracy of the approximation, and can be used to estimate the approximation error.

To illustrate how the approximation of the impedance matrix transfer function based on the in-band poles and zeros works in practice, we have analyzed a dielectric resonator filter, the geometry and scattering parameters of which are shown in Fig. 1. Impedance poles and zeros for the diagonal elements located inside the frequency band of interest were computed by solving the eigenproblems described above, and then used to construct rational functions approximating (12). Figures 2 and 3 show the impedance parameters $|z_{11}(s)|$ and $|z_{21}(s)|$ computed using the full FEM model (3) and the approximate rational functions. It can be seen that these two sets of characteristics match with good accuracy. This illustrates that the zero-pole-based approximation can be used to efficiently compute the characteristics of microwave structures.

Yet, the goal of this work is also to efficiently find the distributions of the electromagnetic field in the frequency band of interest, and not just the Z or S parameters. To this end, one needs to construct a reduced-order model in the form of (5) and to do this, the projection subspace has to be constructed.

The results of the test described above show that the set of in-band poles and zeros is sufficient to retrieve the transfer function with high accuracy. This observation suggests that a good projection basis to construct a ROM could be composed of two sets of vectors: $\mathbf{V}_E$ and $\mathbf{V}_M$ (similarly as in [21] and [22]).

- The column vectors $[\mathbf{v}_{E1}, \mathbf{v}_{E2}, \dots, \mathbf{v}_{Ep}]$ in matrix $\mathbf{V}_E$, where p is the number of in-band poles, represent the electric field distribution associated with the poles of

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the transfer function, i.e., the eigenmodes; thus, they correspond to the natural oscillating dynamics of the field.

- Matrix $\mathbf{V}_M$ is composed of block column submatrices $[\mathbf{V}_M^1, \mathbf{V}_M^2]$ in a two-port device. Each submatrix $\mathbf{V}_M^i$ contains column vectors $[\mathbf{v}_{M1}^i, \mathbf{v}_{M2}^i, \ldots, \mathbf{v}_{Mm_i}^i]$ that correspond to the regular part of the electric field distribution, associated with the zeros of the diagonal entries of the matrix-valued transfer function $z_{ii}(s)$. Here m_i is the number of in-band zeros for $z_{ii}(s)$.

The first set of vectors ($\mathbf{V}_E$) is computed using the generalized eigenproblem defined in (11), while the second set ($\mathbf{V}_M$) is computed using (14). The eigenvectors are subsequently used to construct the projection basis:

$$\mathbf{V} = [\mathbf{V}_E, \mathbf{V}_M^1, \mathbf{V}_M^2], \tag{15}$$

where the columns of $\mathbf{V}$ are subject to orthogonalization. The final step is the regularization of the reduced order model. The remaining steps of the proposed ZP-MOR approach follow the standard procedure described in Section III. The obtained ROM has the form given by (5) and can be used to perform a fast frequency sweep using (6) and (7), respectively. Moreover, the corresponding approximate field distribution of the electric field in the band of interest can be recovered using (8). It should be remarked upon the fact, that ZP-MOR is given for the impedance setting, but the foregoing derivation can be applied for the admittance formulation. In this case, the starting point is the weak formulation for the wave equation for the magnetic field, and the projection basis is spanned by the eigenvectors related to zeros and poles of the admittance matrix elements.

A. Regularization

As discussed in [23], [24], ROMs constructed in systems with poles may exhibit nonphysical spikes in the frequency response. These spikes are due to spurious poles that may appear inside the band of interest when the projection is applied to the FOM. This is an artifact of the MOR process, often overlooked in prior-art literature on MOR. Regularization removes these spurious poles, resulting in ROMs with a clean frequency response. These spurious poles can be identified and then removed using the technique proposed in [24]. However, since in ZP-MOR the location of actual poles is determined while computing the singular part of the basis, any extra in-band poles that are revealed after the projection has been applied, have to be nonphysical and therefore they can easily be detected and deflated. This procedure is described next.

The projection basis has to be orthogonal in order to provide a well-conditioned system of equations (5). The standard procedure is to compute the singular value decomposition of $\mathbf{V}$ or orthogonalize the columns of $\mathbf{V}$ using the modified Gram-Schmidt procedure. As a result, we get:

$$\mathbf{V}^H\mathbf{V} = \mathbf{I_D}.$$

However, for the generalized eigenvalue problems that define the projection basis, the eigenvectors are C-orthogonal by definition. This orthogonality is ensured for each set of eigenvectors separately ($\mathbf{V}_E, \mathbf{V}_M^1$, or $\mathbf{V}_M^2$). It is beneficial to orthogonalize the final set of vectors to ensure that the C-orthogonality property holds for all vectors

$$\mathbf{V}^H\mathbf{C}\mathbf{V} = \mathbf{I_D}.$$

To this end, matrix $\mathbf{C}$ has to be taken into account in the modified Gram-Schmidt process. Note that it is sufficient to C-orthogonalize each vector $\mathbf{v}_M^i$ against $\mathbf{V}_E$ since, as noted above, vectors in $\mathbf{V}_E$ are eigenvectors of (11) and therefore are C-orthogonal. Once done, it becomes possible to completely decouple the singular and regular parts of ROM (16) [22].

$$\begin{aligned} \mathbf{\Gamma}_R &= \begin{bmatrix} \mathbf{\Lambda}_E & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}^Z \end{bmatrix}, \\ \mathbf{C}_R &= \begin{bmatrix} \mathbf{I_D} & \mathbf{0} \\ \mathbf{0} & \mathbf{I_D} \end{bmatrix}. \end{aligned} \tag{16}$$

Matrix $\mathbf{\Lambda}_E$ is a diagonal matrix with the in-band eigenvalues of the FOM (eigenvalues of (11)), so it is related to the true poles. Any spurious pole has to be due to the in-band eigenvalues of matrix $\mathbf{\Gamma}^Z$. Regularization is the process of detection and removal of spurious poles. It is performed by means of the eigendecomposition of the matrix $\mathbf{\Gamma}^Z$. This eigendecomposition reveals the location of additional poles. Next, the eigenvalues related to the in-band poles in this diagonalized submatrix associated with the regular part are set to zero, thus effectively removing them from the response of the constructed ROM.

V. Efficient Computation of the Projection Basis

To generate the projection basis (15) for a two-port device, one has to solve three eigenproblems. This process can be time consuming, particularly when the size of the FEM

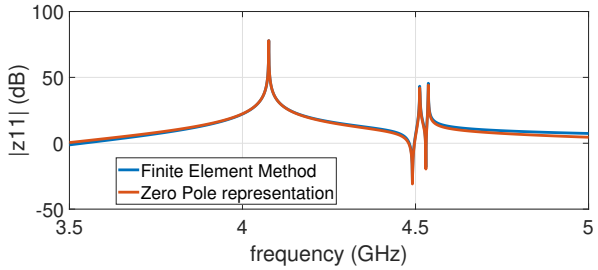

Fig. 2. Impedance characteristic $|z_{11}|$ of the dielectric resonator filter computed using the full FEM model and the zero-pole representation (12).

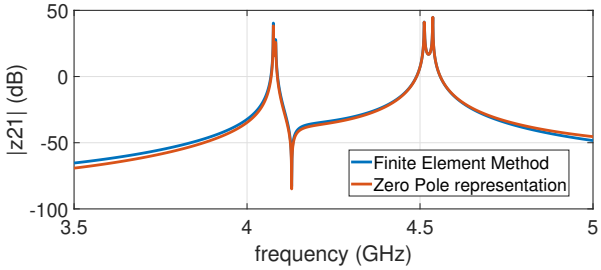

Fig. 3. Impedance characteristic $|z_{21}|$ of the dielectric resonator filter computed using the full FEM model and zero-pole representation (12).

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system is large. In this section, we would like to describe the method which allows one to generate the projection basis with reduced numerical effort. In general, eigenvectors are computed using the Arnoldi iteration with a shift-and-invert preconditioner [33]. In order to explain how it can be used for the computation of the projection basis, let us consider the generalized eigenvalue problem (11) with a single eigenpair $(\lambda_E, \mathbf{v}_E)$:

$$(\mathbf{\Gamma} + \lambda_E \mathbf{C})\mathbf{v}_E = 0. \tag{17}$$

To compute the eigenpair associated with the considered frequency band, we first perform the spectral transformation, which pushes these eigenvalues to the end of the eigenspectrum. To this end, (17) is transformed into the standard eigenproblem:

$$\mathbf{T}\mathbf{v}_E = \gamma_E \mathbf{v}_E, \tag{18}$$

where $\gamma_E = (\lambda_E - \sigma)^{-1}$ and $\sigma \neq \lambda_E$ is a frequency shift, usually corresponding to the central frequency of the considered band. Matrix $\mathbf{T}$ is defined as follows:

$$\mathbf{T} = (\mathbf{\Gamma} + \sigma\mathbf{C})^{-1}\mathbf{C}. \tag{19}$$

In effect, the eigenvalues of (19) that are the largest in magnitude correspond to the eigenvalues of (17) which are the closest to the specified σ (central frequency of the band). Then, the Arnoldi process [33] is applied to compute the eigenpair: γ_E and $\mathbf{v}_E$. The most time-consuming step is the LU factorization of the matrix $\mathbf{A} = (\mathbf{\Gamma} + \sigma\mathbf{C})$. However, usually (in standard microwave engineering simulations) only one factorization is required to compute all eigenpairs from the considered frequency band, giving rise to the component $\mathbf{V}_E$ in the basis (15).

In order to compute the electromagnetic field at the impedance zeros ($[\mathbf{V}_M^1, \mathbf{V}_M^2]$) associated with the eigenvalues from the considered frequency band, one can follow the same procedure for poles, however, this time matrix $\mathbf{T}$ (derived from (14)) has a more complex, nonsymmetric form:

$$\mathbf{T} = \left(\begin{bmatrix} \mathbf{\Gamma} & -\mathbf{b}_i \\ \mathbf{b}_i^T & 0 \end{bmatrix} + \sigma \begin{bmatrix} \mathbf{C} & 0 \\ 0 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{C} & 0 \\ 0 & 0 \end{bmatrix}. \tag{20}$$

Using the shift-and-invert method directly in this case would require factorizing the nonsymmetric matrix twice (for each port), which would significantly increase the computational effort. To speed up the computations, we propose applying the Schur complement [34] technique, which leads to the formulation which requires just one factorization of a symmetric matrix. Substituting $\mathbf{A} = \mathbf{\Gamma} + \sigma\mathbf{C}$, the inversion of the matrix from (20) can be computed as follows:

$$\begin{bmatrix} \mathbf{A} & -\mathbf{b}_i \\ \mathbf{b}_i^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} - \boldsymbol{\beta}(\mathbf{b}_i^T\boldsymbol{\beta})^{-1}\mathbf{b}_i^T\mathbf{A}^{-1} & \boldsymbol{\beta}(\mathbf{b}_i^T\boldsymbol{\beta})^{-1} \\ -(\mathbf{b}_i^T\boldsymbol{\beta})^{-1}\mathbf{b}_i^T\mathbf{A}^{-1} & (\mathbf{b}_i^T\boldsymbol{\beta})^{-1} \end{bmatrix}, \tag{21}$$

where $\boldsymbol{\beta} = \mathbf{A}^{-1}\mathbf{b}_i$. As can be seen, the whole process requires just one LU factorization of a full FEM system matrix $\mathbf{A}$, which is used both in (19) and (21) to compute poles and zeros, together with the associated electromagnetic field distribution,

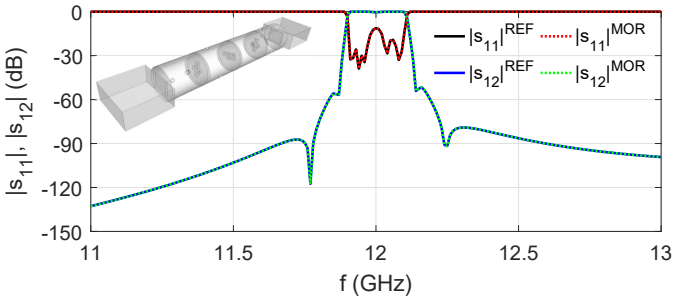

Fig. 4. Eighth-order dual-mode waveguide filter structure and scattering parameters. Dimensions can be found in [8].

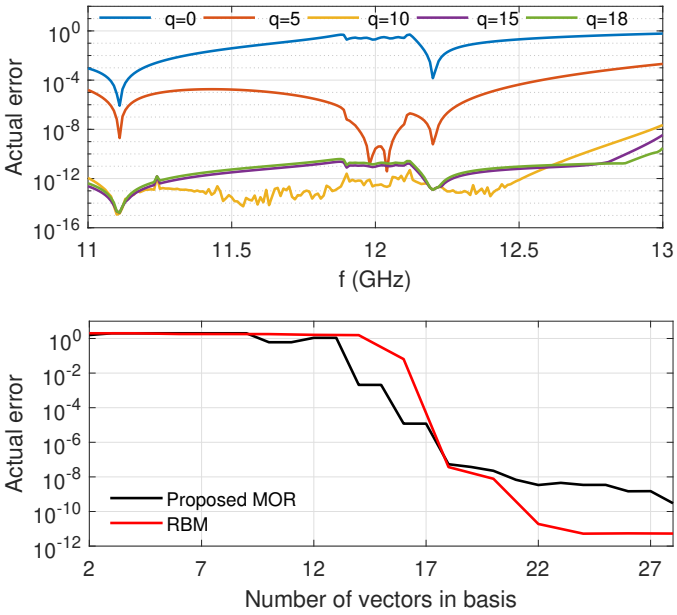

Fig. 5. Eighth order dual-mode filter. Actual reduction-error as a function of ROM size, where q is the number of FOM eigenvectors associated with impedance zeros in the projection basis. The number of poles in the projection basis is 10.

respectively. It is worth mentioning that the LU factorization of the FEM system matrix is the most time-consuming stage of the proposed reduction algorithm.

The remaining steps of the reduction process proceed in the same way as described in formulas (5)-(7).

VI. Numerical Tests

In order to validate the accuracy of the presented ZP-MOR approach, we have analyzed three microwave structures: an eighth-order dual-mode waveguide filter, a dielectric resonator filter, and a folded waveguide filter. All numerical tests were performed on an Intel Core i5-7400 with 32 GB RAM workstation. The code has been written in Matlab.

The first numerical example focuses on the eighth-order dual-mode waveguide filter [8], fed by two WR-90 waveguide ports, assuming a single-mode excitation. The purpose of the simulation was to compute the filter scattering parameters at 201 equidistantly distributed points in the 11–13 GHz band, using the standard FEM formulation [36], where the FEM system of equations had 198878 degrees of freedom. The

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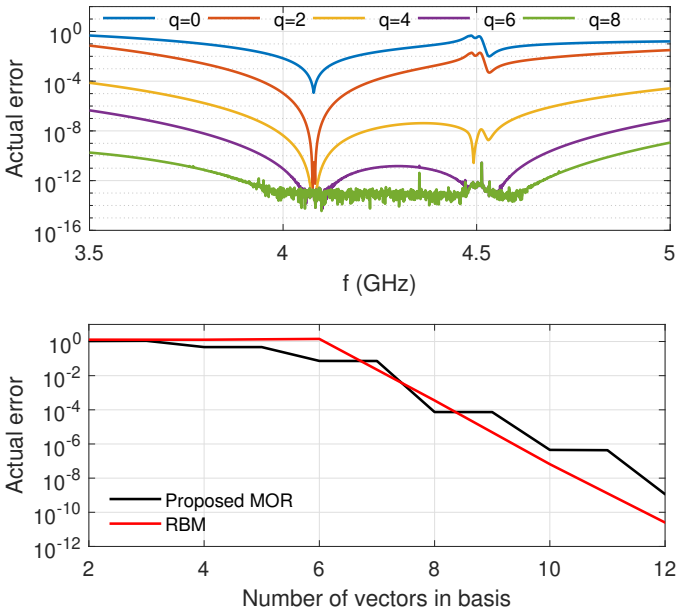

Fig. 6. Dielectric resonator filter. Actual reduction-error as a function of ROM size, where q is the number of FOM eigenvectors associated with impedance zeros in the projection basis. The number of poles in the projection basis is 4.

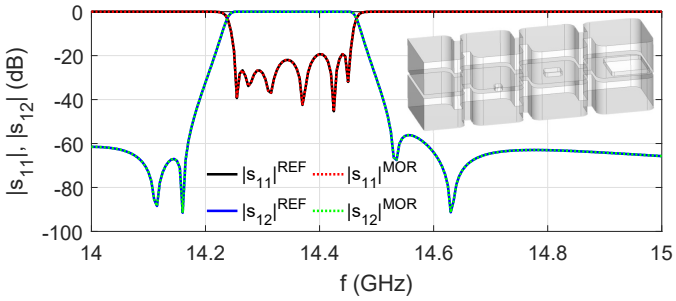

Fig. 7. Folded waveguide filter: geometry and scattering parameters. Dimensions can be found in [35].

resulting scattering characteristics and the geometry of the structure are shown in Fig. 4.

The same structure was then analyzed using the proposed MOR method. To this end, we computed the eigenvectors of the eigenproblems (11) and (14), respectively. Note that only one LU factorization of the FEM system matrix is carried out during this computation. Also, it should be noted that only in-band poles and zeros (that is, the poles and zeros which lie in the 11 to 13 GHz band) were considered. The computed eigenvectors have been used to construct the projection basis (15), and finally to construct the ROM in the form of (5). Fig. 4 shows the scattering parameters of the structure computed using the full FEM model and the reduced order model, respectively, and it can be seen that the characteristics are indistinguishable. To compare these results more precisely, we introduce the following actual error definition:

$$E^{ACT}(s) = \|\mathbf{S}(s) - \mathbf{S}_R(s)\|_2, \tag{22}$$

where the scattering parameters $\mathbf{S}(s)$ and $\mathbf{S}_R(s)$ were computed using (4) and (7), respectively. The actual error plots for five cases are shown in Fig. 5. In each of these cases the

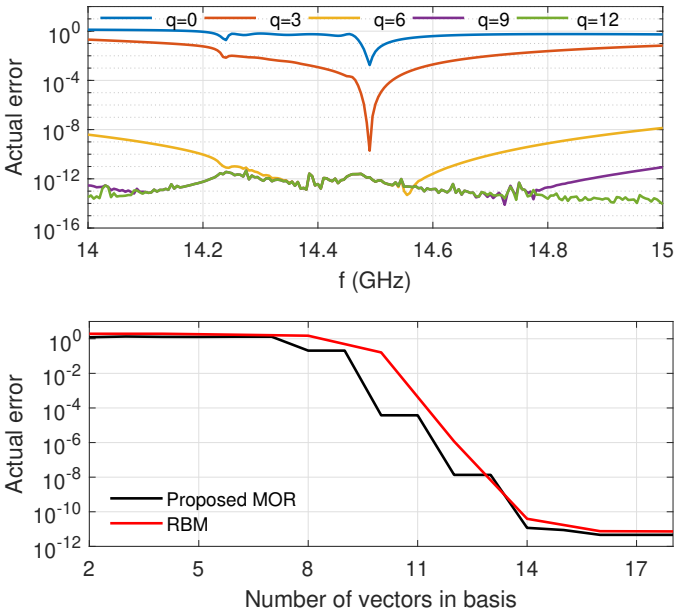

Fig. 8. Folded waveguide filter. Actual reduction-error as a function of ROM size, where q is the number of FOM eigenvectors associated with impedance zeros in the projection basis. The number of poles in the projection basis is 6.

projection basis is composed of all 10 in-band pole eigenvectors and $q \in \{0, 5, 10, 15, 18\}$ eigenvectors associated with the impedance matrix element zeros. The number of all in-band input/output zeros of the FOM is 9. As a result, $q = 18$ corresponds to all eigenvectors associated with the zeros in the diagonal entries of the matrix-valued transfer function since this is a two-port device. Finally, the total number of vectors in the projection basis for all in-band zeros and poles is 28. It can be seen that the basis composed of all relevant in-band eigenvectors for both zeros and poles guarantees that the actual error is below 10^{-8}, whereas the basis which includes only the in-band poles (i.e., $q = 0$) leads to a very poor reduced model with the maximum accurate error above 0.1. The quality of the model increases rapidly as more and more eigenvectors for impedance zeros are added (q increases). Thus, the proposed technique significantly increases the ROM accuracy. The second plot shows the maximum in-band error level as a function of the number of vectors in the basis. In the initial phase of building the reduced model, the maximum reduction error across the band is at a similar high level. This is due to the fact that the initial basis vectors reduce the error only in selected parts of the frequency band, while in the rest of the band the error is still at a high level. The same plot also shows the results for the RBM method. It can be seen that the quality of the ROM generated using the proposed MOR is slightly better compared to the ROM generated using RBM (which generates the almost optimum-size projection basis). Note that in the proposed approach only one LU factorization of the FEM system matrix is carried out, whereas this is not the case for RBM, where multiple LU factorizations are needed.

In the second numerical test, we considered a dielectric resonator filter [32], which was analyzed in Section IV. Here, the goal of the simulation was to compute the scattering

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parameters at 2001 equidistantly distributed points in the 3.5-5 GHz band. Firstly, a full-order FEM model with 198878 variables has been constructed. Next, we constructed an ROM using the proposed MOR scheme. To this end, all relevant in-band eigenvectors have been computed, resulting in the ROM of size $r = 10$ (six in-band eigenvectors for zeros and four for in-band poles, respectively). The actual error plots can be seen in Fig. 6. Similar to the first numerical test, the ROM is characterized with high accuracy with respect to the FOM (the actual error is below 10^{-6} in the entire considered frequency band).

The last test deals with a folded waveguide filter shown in Fig. 7. This filter is analyzed using a FOM with 864958 degrees of freedom in the 14-15 GHz band. As can be seen, the reduced model of size $r = 18$ (twelve eigenvectors for zeros and six for poles) provides an accuracy below 10^{-11}, which is detailed in Fig. 8.

VII. Discussion

In order to better assess the potential of the proposed ZP-MOR method and compare it to other MOR techniques, in this section, we will discuss a few aspects that affect the numerical efficiency of the model order reduction process. The comparison will be qualitative rather than quantitative. This is because the quantitative comparison in terms of runtime or memory usage would be meaningful only if the numerical implementation of each algorithm were optimized in terms of performance. The performance of numerical code for MOR depends on many factors such as the hardware used for tests, linear algebra libraries used for the solution of linear systems of equations and eigenproblems, the degree of parallelization applied at the level of code or the algorithm, and simulation parameters (required accuracy, bandwidth, size of the problem, the number of right-hand side vectors, etc.). Finding an optimal combination of test problems and tuning the numerical code for each method goes beyond the scope of this paper. We used Matlab for rapid prototyping and proving the validity of the proposed algorithm, so the code is neither efficient nor it exploits the inherently parallel nature of ZP-MOR that results from the independence between computing the eigenvectors for poles and impedance element zeros. However, the efficiency of the proposed ZP-MOR method compared to sweeps based on other MOR techniques can be assessed by qualitatively analyzing the workload related to the various steps required in each algorithm to generate a ROM. Besides ZP-MOR, in the following discussion, we shall consider four ROM techniques: two classical methods, namely, RBM [8] and SAPOR [14], as well as two techniques that employ splitting the field solutions into their singular and regular contributions to the system transfer function, that is, CRBM [21] and SS-MM MOR [22]. For all MOR algorithms compared, the most time-consuming portion of the reduced-order model generation is associated with computing LU factorizations for full-order system matrices. The RBM method requires multiple factorizations (for instance, RBM needed 14, 5, 9 LU factorizations to calculate results for test cases 1–3 above, respectively), while moment-matching-based algorithms such as SAPOR, need fewer factorizations (just one in the case of a single-point expansion, which is enough for some applications). For the methods which include pole eigenvectors in the projection bases (SS-MM MOR [22] and CRBM [21]) the number of factorizations required is typically smaller than in RBM. It can be as small as one for SS-MM MOR and a few for CRBM. In the case of the proposed ZP-MOR method, only a single factorization is performed, so in this regard the new technique is equivalent to SAPOR and SS-MM MOR. However, it should be noted that there is also an additional cost of using just one factorization in the ZP-MOR that is related to inverting the matrix with the Schur complement technique, as seen in (21).

Another noteworthy aspect of ROM generation is the ability to parallelize this process. Methods which employ greedy strategies, namely, RBM and multipoint MM algorithms, as well as numerically stable moment matching schemes, are inherently hard to parallelize since vectors constituting the projection basis have to be computed sequentially. Methods which involve solving eigenproblems may take advantage of the available efficient parallel solvers (for instance, SLEPc [29] or P_ARPACK [28] developed for clusters). The proposed ZP-MOR method offers yet another parallelization opportunity: in the proposed approach, the projection basis is found from three independent eigenproblems. These eigenproblems are most often solved by means of a single LU factorization of the system matrix. Once the factorization has been carried out, each of the eigenproblems can be solved individually in parallel. This means that the proposed ZP-MOR algorithm shows the highest potential for parallelism, not offered by prior-art methods.

Another important issue is associated with the size of the projection basis. The orthogonalization process may be time-consuming, especially when the number of columns in the MOR projection basis $\mathbf{V}$ is large (which is often the case in moment-matching based methods, such as SAPOR). Compared to SAPOR or, to some extent, also to SS-MM MOR, the proposed technique generates more compact models, thus reducing the number of times orthogonalization is performed and additionally avoiding a lengthy orthogonalization process involving a large number of vectors.

The size of the ROMs, for any reduction method applied, is obviously much smaller compared to the size of the initial model, and hence the time required to compute the approximate system response in the frequency band of interest is greatly reduced. If only a single frequency sweep is performed, then the runtime needed for ROM generation dominates the computational cost, and differences in the size of ROMs generated by different methods do not significantly impact the overall frequency-sweep cost. By the same token, in case the field distribution is needed at each frequency point, we can still use the same ROM (extracted once). However, RBM, pole-based CRBM, and the proposed ZP-MOR approach typically generate ROMs with smaller sizes, compared to moment-matching based algorithms, which yields some performance gains in this case.

It should also be pointed out that the proposed approach does not rely on the *a posteriori* error estimator [7], [9], [27] to assess the accuracy of the ROM, as well as to select subsequent

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TABLE I
QUALITATIVE PERFORMANCE COMPARISON OF DIFFERENT MOR TECHNIQUES: RBM, MOMENT-MATCHING-BASED (SAPOR), MOR WITH POLE EIGENVECTORS (CRBM/SS-MM MOR), AND THE PROPOSED METHOD (ZP-MOR). SYMBOLS '–', '-', '+', '++' ARE THE NOTES FROM THE WEAKEST TO THE BEST, RESPECTIVELY, AND (1) MEANS THAT METHOD REQUIRES JUST ONE FACTORIZATION.

	RBM	SAPOR	CRBM/ SS-MM	ZP-MOR
No. of factorizations	- -	++ (1)	-/++ (1)	++ (1)
Parallelizable	- -	-	+/+	+ +
ROM size	+ +	- -	++	+ +
Error estimation	- -	- -	-	+
Orthogonalization	+	- -	+/-	+ +

expansion points (as it is the case in RBM and CRBM). It is assumed that all in-band impedance zeros and poles guarantee high accuracy in the ROM with respect to FOM. Thus, the time needed to monitor the error behavior within the entire frequency band, performed each time the projection basis is augmented and is typically present in prior-art algorithms, is saved.

The qualitative performance comparison of MOR techniques, summarizing the above observations, is given in Table I. We used notes from the weakest to the best, in order: '–', '-', '+', '++', respectively. It can be seen that the proposed method obtained the highest notes almost in all categories.

Finally, as far as the shortcomings of the proposed technique are concerned, the inversion of a matrix using the Schur complement involves several extra solves that have to be applied when a new Krylov space vector is computed while searching the eigenvalues and eigenvectors. Moreover, at this stage, we do not have a good error bound estimate showing the impact of out-of-band poles and zeros on the accuracy of ROM. Future work may include the derivation of error bounds as well as the extension of the proposed technique to antennas and systems with conductor loss and frequency-dependent materials (lossy dielectrics with complex permittivity can be handled in the same way as proposed in [22]). A possible extension of this work can be related to using the state zero directions of the matrix-valued transfer function zeros [37] (rather than the eigenvectors associated with the zeros of its elements as it is proposed in this work).

VIII. CONCLUSIONS

This paper has presented a novel model-order reduction technique called ZP-MOR to perform fast frequency sweeps of frequency-domain finite-element method simulations of microwave structures. The projection basis used to construct the reduced-order model is composed of the eigenvectors associated with the poles and zeros of selected entries of the matrix-valued transfer function computed from the FEM equations. Special emphasis has been made in keeping the computational cost low reducing the number of LU factorizations of the FEM matrix system to one, as well as, allowing for MOR projection basis build parallelization. Numerical tests of three microwave structures have demonstrated the reliability and accuracy of the proposed technique.

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