

# ADAPTIVE IDENTIFICATION OF UNDERWATER ACOUSTIC CHANNEL WITH A MIX OF STATIC AND TIME-VARYING PARAMETERS

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## ABSTRACT

We consider the problem of identification of communication channels with a mix of static and time-varying parameters. Such scenarios are typical, among others, in underwater acoustics. In this paper, we further develop adaptive algorithms built on the local basis function (LBF) principle resulting in excellent performance when identifying time-varying systems. The main drawback of an LBF algorithm is its high complexity. The subsequently proposed fast LBF (fLBF) algorithms, based on the preestimation principle, allow a significant reduction in the complexity for recursively computable basis functions, such as the complex exponentials. We propose a debiased fLBF algorithm which exploits the fact that only a part of the system parameters are time-varying. We also propose an adaptive technique to identify whether a particular tap is static or time-varying.

**Index Terms**— Identification of time-varying systems, local basis function approach, underwater acoustic channels

## 1. INTRODUCTION

There are applications that require accurate estimation of parameters of time-varying linear systems with only a part of the parameters being time-varying. Such applications are typical in underwater acoustics (UWA) and include UWA communications, navigation, and sonar applications, which deal with estimation of the UWA channel, often modelled as a time-varying linear system [1–3]. The UWA channel is characterised by multipath propagation and often is described as a finite impulse response (FIR) filter, whose parameters vary with time due to the Doppler effect caused by the moving transmitter, receiver and/or the sea surface [4]. The Doppler effect is known to be different for different multipaths [5].

If the transmitter and receiver are static, then multipaths propagated through the water and not interacting with the

moving sea surface will be almost static, whereas multipaths reflected from the sea surface will be time-varying. This scenario is typical for full-duplex (FD) UWA communications [6, 7]. In FD UWA communications, there is a strong near-end self-interference (SI) from the transmit to receive antenna within the same transceiver. To allow the FD operation, i.e., reliable detection of a weak signal from a far-end transmitter, the near-end SI channel should be accurately estimated, the SI signal recovered and subtracted from the received signal. In this way the SI can be cancelled. The SI channel may include reflections from the sea surface; even if the SI sea-surface signal components are of a low power (tens of decibels lower than the signal components due to the line-of-sight propagation), they still need to be estimated with a high precision to achieve a level of SI cancellation required for the FD operation. Therefore, the channel estimation should deal with a mix of static and time-varying parameters. A similar problem arises in a continuous-wave sonar, where instead of traditional pulse transmission, a pseudo-noise wideband signal is transmitted [8, 9].

In this paper, we will show how the mixed-mode property of UWA channels can be exploited to improve the identification performance.

## 2. PROBLEM STATEMENT

Many communication channels (terrestrial, underwater) can be well approximated by a time-varying FIR model of the form [4, 10]

$$y(t) = \sum_{i=1}^n \theta_i^*(t) u(t-i+1) + e(t) = \boldsymbol{\theta}^H(t) \boldsymbol{\varphi}(t) + e(t) \quad (1)$$

where  $t = \dots, -1, 0, 1, \dots$  denotes discrete (normalized) time,  $y(t)$  the complex-valued system output (a received signal in a communication system),  $\boldsymbol{\varphi}(t) = [u(t), \dots, u(t-n+1)]^T$  the regression vector made up of past samples of the complex-valued (transmitted) signal  $u(t)$ ,  $\boldsymbol{\theta}(t) = [\theta_1(t), \dots, \theta_n(t)]^T$  is the vector of channel taps, and  $e(t)$  denotes a measurement noise. The sequence  $\{\theta_i(t)\}$  can be

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interpreted as a time-varying impulse response of the channel to be estimated.

We will assume that:  $\{u(t)\}$  is a sequence of zero-mean independent and identically distributed circular random variables with variance  $\sigma_u^2$ ,  $\{e(t)\}$  is a zero-mean circular white noise, independent of  $\{u(t)\}$ , with variance  $\sigma_e^2$ , and  $\{\theta(t)\}$  is a sequence independent of  $\{u(t)\}$  and  $\{e(t)\}$ .

The LBF principle is based on the assumption that in a local analysis interval  $T_k(t) = [t - k, t + k]$  of length  $K = 2k + 1$ , centered at  $t$ , system parameters can be expressed as linear combinations of a certain number of linearly independent complex-valued functions of time  $f_1(j), \dots, f_m(j)$ ,  $j \in I_k = [-k, k]$ , further referred to as basis functions. In this paper, we adopt the complex exponential basis set of the form (see [10–12] for a physical justification of such a choice)

$$\{f_1(j), \dots, f_m(j), j \in I_k\} = \left\{ \frac{1}{\sqrt{K}} e^{ij\omega_1}, \dots, \frac{1}{\sqrt{K}} e^{ij\omega_m}, j \in I_k \right\}, \quad (2)$$

where  $i = \sqrt{-1}$ ,  $\omega_1 = 0$ ,  $m = 2m_0 + 1$ , and

$$\omega_{2l} = -\frac{2\pi l}{K}, \quad \omega_{2l+1} = \frac{2\pi l}{K}, \quad l = 1, \dots, m_0.$$

It is straightforward to check that the basis (2) is orthonormal, i.e.,  $\sum_{j=-k}^k \mathbf{f}(j) \mathbf{f}^H(j) = \mathbf{I}_m$  where  $\mathbf{f}(j) = [f_1(j), \dots, f_m(j)]^T$  and  $\mathbf{I}_m$  denotes the  $m \times m$  identity matrix. We will denote  $f_0 = f_1(j) = 1/\sqrt{K}$ ,  $j \in I_k$ .

Unlike [13–16], we will assume that only some of the estimated parameters  $\theta_i(t)$  in (1) vary in the local analysis interval  $T_k(t)$ , while the remaining parameters are constant.

Denote by  $S$  the set indicating, within  $\Omega = \{1, \dots, n\}$ , positions of time-invariant taps, and by  $\bar{S} = \Omega - S$  the set of time-varying taps. Furthermore, let  $n_S = \text{card}\{S\}$ ,  $n_{\bar{S}} = \text{card}\{\bar{S}\}$ , then  $n_S + n_{\bar{S}} = n$ , and denote  $\ell = n_S + mn_{\bar{S}}$ .

In the sequel we will adopt the following *mixed-mode* model of local parameter variation within the interval  $T_k(t)$ :

$$\theta_i(t+j) = \begin{cases} f_0 a_{i1}(t) & \text{if } i \in S \\ \sum_{l=1}^m f_l^*(j) a_{il}(t) & \text{if } i \in \bar{S} \end{cases} \quad (3)$$

$j \in I_k, i = 1, \dots, n.$

In agreement with the local estimation paradigm, estimation of parameter trajectories, based on the hypermodel (3), will be carried out independently for each location of the analysis interval  $T_k(t)$ , i.e., it will be performed in the sliding window manner. Therefore, even though system hyperparameters (expansion coefficients)  $a_{il}$  are assumed to be constant in the interval  $[t - k, t + k]$ , their values are allowed to change along with the position of the analysis window. For this reason they are written down as functions of  $t$ .

The hypermodel (3) can be expressed in a more compact form

$$\theta(t+j) = \mathbf{F}(j) \boldsymbol{\alpha}(t), \quad j \in I_k, \quad (4)$$

where  $\boldsymbol{\alpha}(t) = [\alpha_1^T(t), \dots, \alpha_n^T(t)]^T$  is an  $\ell$ -dimensional vector of hyperparameters,

$$\alpha_i(t) = \begin{cases} a_{i1}(t) & \text{if } i \in S \\ [a_{i1}(t), \dots, a_{im}(t)]^T & \text{if } i \in \bar{S} \end{cases}$$

and  $\mathbf{F}(j) = \text{bl diag}\{\mathbf{F}_1(j), \dots, \mathbf{F}_n(j)\}$  denotes the  $n \times \ell$  matrix, where

$$\mathbf{F}_i(j) = \begin{cases} f_0 & \text{if } i \in S \\ \mathbf{f}^H(j) & \text{if } i \in \bar{S} \end{cases}$$

Using (4), the system model (1) in the local analysis interval  $T_k(t)$  can be written in the form

$$y(t+j) = \boldsymbol{\alpha}^H(t) \boldsymbol{\psi}(t, j) + e(t+j), \quad j \in I_k, \quad (5)$$

where  $\boldsymbol{\psi}(t, j) = \mathbf{F}^H(j) \boldsymbol{\varphi}(t+j)$  denotes the generalized regression vector.

### 3. LBF AND FAST LBF ESTIMATORS

In this section we will assume that the support sets  $S$  and  $\bar{S}$  are known. Later, in Section 4, we will propose an adaptive estimation scheme capable of identifying the support.

The LBF estimator has the form [13]

$$\hat{\boldsymbol{\alpha}}^{\text{LBF}}(t) = \arg \min_{\boldsymbol{\alpha}} \sum_{j=-k}^k |y(t+j) - \boldsymbol{\alpha}^H \boldsymbol{\psi}(t, j)|^2 \quad (6)$$

$$\hat{\boldsymbol{\theta}}^{\text{LBF}}(t) = \mathbf{F}_0 \hat{\boldsymbol{\alpha}}^{\text{LBF}}(t),$$

where  $\mathbf{F}_0 = \mathbf{F}(0)$ .

As shown in [14], under the assumptions made above, the LBF estimates  $\hat{\boldsymbol{\alpha}}^{\text{LBF}}(t)$  and  $\hat{\boldsymbol{\theta}}^{\text{LBF}}(t)$  can be approximated by the fLBF estimates

$$\hat{\boldsymbol{\alpha}}^{\text{fLBF}}(t) = \arg \min_{\boldsymbol{\alpha}} \sum_{j=-k}^k \|\tilde{\boldsymbol{\theta}}(t+j) - \mathbf{F}(j) \boldsymbol{\alpha}\|^2 \quad (7)$$

$$= \sum_{j=-k}^k \mathbf{F}^H(j) \tilde{\boldsymbol{\theta}}(t+j),$$

$$\hat{\boldsymbol{\theta}}^{\text{fLBF}}(t) = \mathbf{F}_0 \hat{\boldsymbol{\alpha}}^{\text{fLBF}}(t),$$

where  $\|\mathbf{x}\|^2 = \mathbf{x}^H \mathbf{x}$ , and  $\{\tilde{\boldsymbol{\theta}}(t)\}$  denotes a sequence of preestimated system parameters. The fLBF estimates can be obtained in a significantly more computationally efficient way than the LBF estimates, thus the word ‘fast’.

Fast LBF estimators can be rewritten in a decomposed form as follows. For  $i \in S$ , we have

$$\hat{\alpha}_i^{\text{fLBF}}(t) = f_0 \sum_{j=-k}^k \tilde{\theta}_i(t+j), \quad (8)$$

$$\hat{\boldsymbol{\theta}}_i^{\text{fLBF}}(t) = f_0 \hat{\alpha}_i^{\text{fLBF}}(t) = \frac{1}{K} \sum_{j=-k}^k \tilde{\theta}_i(t+j),$$



and, for  $i \in \bar{S}$ ,

$$\begin{aligned}\hat{\alpha}_i^{\text{flBF}}(t) &= \sum_{j=-k}^k \tilde{\theta}_i(t+j)\mathbf{f}(j), \\ \hat{\theta}_i^{\text{flBF}}(t) &= \mathbf{f}_0^H \hat{\alpha}_i^{\text{flBF}}(t) = \sum_{j=-k}^k h(j)\tilde{\theta}_i(t+j),\end{aligned}\quad (9)$$

where  $\mathbf{f}_0 = \mathbf{f}(0)$  and  $h(j) = \mathbf{f}_0^H \mathbf{f}(j)$ ,  $j \in I_k$ , denotes the impulse response of an FIR filter associated with the LBF estimator.

The preestimates, proposed in [17] and further developed in [14, 18], can be obtained by ‘inverse filtering’ of estimates yielded by the short memory exponentially weighted least squares (EWLS) algorithm [19, 20]

$$\hat{\theta}^{\text{EWLS}}(t) = \arg \min_{\theta} \sum_{j=1}^t \lambda^{t-j} |y(j) - \theta^H \varphi(j)|^2 \quad (10)$$

where  $\lambda$ ,  $0 < \lambda < 1$ , is a forgetting factor. The recommended value of  $\lambda$  is:  $\lambda = \max\{0.9, 1 - 2/n\}$  [16].

For large values of  $t$ , when the effective width of the exponential window  $M(t) = \sum_{i=1}^t \lambda^{t-i}$  reaches its steady state value  $M_\infty = 1/(1 - \lambda)$ , inverse filtering has the form

$$\tilde{\theta}(t) = \frac{1}{1 - \lambda} \left[ \hat{\theta}^{\text{EWLS}}(t) - \lambda \hat{\theta}^{\text{EWLS}}(t-1) \right]. \quad (11)$$

The nonasymptotic version of (11) can be found in [14]. As shown there, when the sequence  $\{\varphi(t)\}$  is (locally) stationary with exponentially decaying autocorrelation function, the preestimates are approximately unbiased, i.e.  $E[\tilde{\theta}(t)] \cong \theta(t)$  where the expectation is over  $\{e(t)\}$  and  $\{\varphi(t)\}$ . Under the assumptions made, the preestimation noise  $\mathbf{z}(t) = \tilde{\theta}(t) - \theta(t)$  is approximately white with a high variance.

#### 4. ADAPTIVE TIME-INVARIANCE TEST

A clear advantage of the preestimation approach is that it allows the system dynamics to be ‘X-rayed’ prior to its formal identification. We will use this property to adaptively decide, at each time instant  $t$ , which parameters can be regarded as time-invariant and which are time-varying.

To assess existence of a trend in the sequence of preestimates (which justifies choosing  $m > 1$ ), one can use the classical approach based on counting the number of sign changes amongst residuals [21]. When the system parameter  $\theta_i(t)$  is constant in the analysis interval  $T_k(t)$ , the residual noise defined as

$$\varepsilon_i(t+j|t) = \tilde{\theta}_i(t+j) - \bar{\theta}_i(t), \quad j \in I_k, \quad (12)$$

where

$$\bar{\theta}_i(t) = \frac{1}{K} \sum_{j=-k}^k \tilde{\theta}_i(t+j),$$

is approximately equal to the preestimation noise  $z_i(t) = \tilde{\theta}_i(t+j) - \theta_i(t)$ , which is zero-mean and white.

Consider the real part of the residual noise:

$$\varepsilon_i^{\text{R}}(t+j|t) = \text{Re}\{\varepsilon_i(t+j|t)\}.$$

and denote by  $q_i^{\text{R}}(t)$  the number of sign changes of  $\varepsilon_i^{\text{R}}(\cdot|t)$  observed in the analysis interval  $T_k(t)$ . By  $q_i^{\text{I}}(t)$  we will denote the analogous count for  $\varepsilon_i^{\text{I}}(t+j|t) = \text{Im}\{\varepsilon_i(t+j|t)\}$ .

For a real-valued white noise sequence, the sign change can be observed on average every second sample. Hence, when the number of sign changes is smaller than some threshold, one has to assume that the parameter trajectory is not constant inside the analysis window.

Consider the following null hypothesis:

$\mathcal{H}_0^{\text{R}}(t)$ :  $\{\varepsilon_i^{\text{R}}(t+j|t), j \in I_k\}$  is a sequence of independent random variables obeying the condition

$$P(\varepsilon_i^{\text{R}}(t+j|t) > 0) = P(\varepsilon_i^{\text{R}}(t+j|t) \leq 0), \quad \forall j \in I_k,$$

where  $P(\cdot)$  is a probability. Note that this hypothesis is true when the sequence  $\{\varepsilon_i^{\text{R}}(\cdot|t)\}$  is uncorrelated, zero-mean and Gaussian, but the requirements are in fact much weaker.

If the null hypothesis is true,  $q_i^{\text{R}}(t)$  is a discrete random variable with an ‘almost binomial’ distribution (as remarked by Geary [21], since the sum of residuals in the interval  $T_k(t)$  is - by construction - zero, the value  $q_i^{\text{R}}(t) = 0$  is inadmissible) characterized by the probability of success 0.5:

$$P(q_i^{\text{R}}(t) = q | \mathcal{H}_0^{\text{R}}(t)) = \frac{(K-1)!}{(2^{K-1} - 1)q!(K-1-q)!}. \quad (13)$$

Furthermore, for any  $q_0 \in [1, K-1]$  it holds that

$$\begin{aligned}P(q_i^{\text{R}}(t) \leq q_0 | \mathcal{H}_0^{\text{R}}(t)) \\ = \sum_{q=1}^{q_0} \frac{(K-1)!}{(2^{K-1} - 1)q!(K-1-q)!} = \eta_0.\end{aligned}\quad (14)$$

The sign statistic  $q_i^{\text{R}}(t)$  can be used to verify the null hypothesis for a given probability of Type I error  $\eta_0$ :

$$\begin{cases} \text{accept } \mathcal{H}_0^{\text{R}}(t) & \text{if } q_i^{\text{R}}(t) > q_0 \\ \text{reject } \mathcal{H}_0^{\text{R}}(t) & \text{if } q_i^{\text{R}}(t) \leq q_0 \end{cases} \quad (15)$$

The exemplary thresholds evaluated for the significance level  $\eta_0 = 0.01$  are  $q_0 = 83, 176$  and  $366$  for  $K = 2k + 1 = 201, 401$  and  $801$ , respectively. For  $\eta_0 = 0.01$  and  $K \in [200, 1000]$ , a tight approximation of the threshold is given by  $q_0 = \lfloor 0.95k - 14 \rfloor$  where  $\lfloor \cdot \rfloor$  denotes the floor function.

The same analysis can be carried out for the imaginary components of the residuals  $\varepsilon_i^{\text{I}}(\cdot|t)$ . Combining both inferences, one arrives at the following decision rule, which can be used to determine whether a given parameter  $\theta_i(t)$  should be regarded as constant or time-varying in the analysis interval  $T_k(t)$ :

$$\begin{cases} i(t) \in S(t) & \text{if } q_i^{\text{R}}(t) > q_0 \text{ and } q_i^{\text{I}}(t) > q_0 \\ i(t) \in \bar{S}(t) & \text{otherwise} \end{cases} \quad (16)$$

## 5. DEBIASING

It was observed that the estimated parameter trajectory, obtained using the fLBF approach, lags behind the true parameter trajectory, and that the size of this delay depends on the forgetting factor  $\lambda$  used in the preestimation stage. This effect is evidently caused by the estimation delay feature [19] of the EWLS algorithm used to generate preestimates.

We propose a simple adaptive scheme for minimization of the time shift between  $\hat{\theta}^{\text{fLBF}}(t)$  and  $\theta(t)$ . The search will be carried out around  $\Delta$ , where  $\Delta = \text{int}[\lambda/1 - \lambda]$  denotes the nominal (low-frequency) delay introduced by the EWLS algorithm [19] (the nominal delay is only an approximation of the true delay);  $\text{int}[x]$  denotes the integer closest to  $x$ .

Let

$$\varepsilon_\delta(t) = y(t) - [\hat{\theta}^{\text{fLBF}}(t + \delta)]^H \varphi(t) \quad (17)$$

and  $D = [\Delta - \delta_0, \Delta + \delta_0]$ . Define the exponentially weighted sum of squares of  $\varepsilon_\delta(t)$  evaluated recursively for every  $t$  and every  $\delta \in D$

$$J(t, \delta) = \lambda_0 J(t - 1, \delta) + |\varepsilon_\delta(t)|^2, \quad 0 < \lambda_0 < 1.$$

The (approximately) debiased fLBF estimates can be obtained using the formula

$$\hat{\theta}^{\text{dfLBF}}(t) = \hat{\theta}^{\text{fLBF}}(t + d(t)), \quad (18)$$

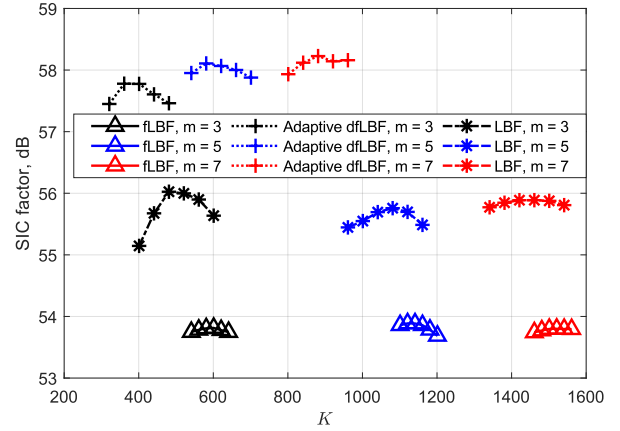
where  $d(t) = \arg \min_{\delta \in D} J(t, \delta)$ .

## 6. EXPERIMENTAL RESULTS

In the real experiment the true impulse response is not available. We therefore investigate the SI cancellation (SIC) performance measured using the SIC factor, which shows how much the signal-to-interference ratio (SIR) at the output of the SI canceller is reduced compared to the SIR at the input of the canceller; the methodology of measuring the SIC factor is described in [6].

The FD experiment was conducted in a lake of depth 8 m. The distance between the transmitter and receiver, both positioned at a depth of 4 m, is 7 cm. In the experiment, binary-shift keying (BPSK) symbols are transmitted with a rate of 1000 symbols/s at the carrier frequency 32 kHz; a root raised cosine filter with a roll-off factor of 0.2 is used for the pulse shaping [22]. The received signal after analog-to-digital conversion is down shifted in frequency, low-pass filtered and down sampled to the sampling rate 1 kHz. These samples are applied to the adaptive filter as the desired signal. The same operation is performed on the analogue signal applied to the transmit antenna [23]; these samples are used as the regressor in the adaptive filter.

In the experiment, the self-interference to noise ratio is 60.4 dB and the number of taps is  $n = 80$ . The SIC factor is computed over a 10-s interval after the convergence of



**Fig. 1.** The performance of the compared algorithms in cancelling the self-interference in the real UWA FD experiment.

the adaptive filter (see more details on the procedure in [6]). When applying the EWLS algorithm to the experimental data, the highest SIC factor is 50.9 dB.

Fig. 1 shows the SIC factor achieved, for different values of  $m$ , by the original state-of-the-art LBF and fLBF algorithms, and by the proposed adaptive dfLBF algorithm with online parameter mode selection. Since in the real experiment the support set  $S$  is unknown and possibly time-varying, in the first two algorithms all estimated parameters are regarded as time-dependent. The design parameters were set to  $\lambda = 0.975$ ,  $\lambda_0 = 0.98$  and  $\delta_0 = 30$ .

The original fLBF algorithm provides a SIC factor of 53.8 dB, 53.9 dB, and 53.8 dB for  $m = 3, 5, 7$ , respectively. The analogous values for the LBF algorithm and the adaptive dfLBF algorithm are 56.0 dB, 55.8 dB, 55.9 dB and 57.8 dB, 58.1 dB, 58.2 dB, respectively. Note that adaptive selection of time-varying taps yields performance improvement of 1.8 dB–2.3 dB against the LBF algorithm and 4.0 dB–4.4 dB against the fLBF algorithm. Finally, when compared against the EWLS algorithm, the dfLBF algorithm increases the SIC factor by 6.9 dB–7.3 dB. Thus, we can conclude that the proposed technique allows significant improvement in the SI cancellation performance in this lake experiment.

## 7. CONCLUSION AND RELATION TO PRIOR WORK

The paper extends results presented in [14] and [6]. To the best of our knowledge this is the first attempt to apply the local basis function approach to identification of systems (channels) with mixed static and time-varying parameters. Experimental results confirm that the proposed debiased fLBF algorithm with online parameter mode selection yields improvement over the current state-of-the-art.

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