

## Research Paper

## Acoustic Hysteresis in Flows with Different Kinds of Relaxation and Attenuation

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Graphs in the thermodynamic plane acoustic pressure versus excess acoustic density representing acoustic hysteresis, are considered as indicators of relaxation processes, equilibrium parameters of a flow, and kinds of wave exciters. Some flows with deviation from adiabaticity are examined: the Newtonian flow of a thermoconducting gas, the flow of a gas with vibrational relaxation, the flow of liquid electrolyte with a chemical reaction, and the Bingham plastic flow. The total range of characteristic frequencies of a harmonic exciter is taken into account. The impulsive sound is considered as well. The peculiarities of hysteretic behaviour are discussed in dependence with the kind and degree of deviation from adiabaticity. Examples of acoustically active flows are discussed.

**Keywords:** relaxation; acoustic hysteresis; acoustical activity.



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## 1. Introduction

Relaxation of thermodynamic processes is a physical reality. The relaxation processes are excited by an incident acoustic wave. Each process is characterised by some relaxation time and strength and takes energy from the acoustic disturbances. The relaxation processes are not the only reason for sound to be attenuated but they are not sufficiently understood. The mechanisms of relaxation are mostly of two kinds, energy relaxation and volume relaxation, and they both contribute to the nonequilibrium portion of a medium entropy. The second law of thermodynamics leads to an equation which governs acoustic pressure in the wave processes. While the energy relaxation mechanism is discussed in a literature extensively, the volume relaxation is less studied. It was first discussed by Leonard HALL (1948). The basic theory was mainly developed by HERZFELD and RISE (1928). The flows are usually characterized by one of several dominant mechanisms of relaxation. In particular, a vibrational relaxation of diatomic molecules is the dominant mechanism for air. Among the most important kinds, we may mention the Newtonian attenuation, thermal conduction, relaxation of the vibrational degrees of a molecule free-

dom, and relaxation due to a chemical reaction. The Newtonian attenuation often does not match the experimental data of attenuation in tissues (PARKER, 1983; DUCK, 1998). It is actually a high-frequency limit of the Maxwell relaxation (HAMILTON, BLACKSTOCK, 1998). Molecular collisions in a gas are always accompanied by variations of translational, vibrational, rotational, and electronic energy of the collisioning patterns. Ionisation and chemical reaction may take place in a gas (OSIPOV, UVAROV, 1992). Many multiple processes can take place in complex fluids. There may be a large amount of different relaxation processes with their own intrinsic relaxation time which have impact on the wave process. The comprehensive review concerning various mechanisms of absorption in gases may be found in many studies, from old to new (MANDELSHTAM, LEONTOWICH, 1937; HERTZFELD, LITOWITZ, 1959; EIGEN, DE MAYER, 1963; MAKAROV, OCHMANN, 1996; PIERCE, 1981). The theories including multiple relaxation processes are developed. The arbitrary number of discrete relaxation processes with continuous distribution of relaxation times was reviewed by VILENSKY *et al.* (2012).

Since there appears the time heterogeneity parameter, wave perturbations reveal a frequency depen-

dent behaviour. This concerns phase velocity  $v_{ph}$  and attenuation of a wave  $\alpha$ . A very simplified explanation of features of relaxation process relies on the links of the complex amplitudes of perturbations of pressure and density in a wave,  $\tilde{P}$  and  $\tilde{R}$ . We can treat perturbation of pressure and density as a real part of the complex amplitude times  $\exp(i\omega t - ikx)$  ( $p' = \tilde{P} \exp(i\omega t - ikx)$ ,  $\rho' = \tilde{R} \exp(i\omega t - ikx)$ ). The relaxation is described by some frequency dependent function  $K_{\text{relax}}$ , so that  $(1 + K_{\text{relax}})\tilde{P} = c^2\tilde{R}$  ( $c$  denotes the speed of sound at a very low frequency). Hence, the links between perturbations of thermodynamic variables depend in general on frequency (RUDENKO, SOLUYAN, 1977; HEDBERG, RUDENKO, 2011; LEBLE, PERELOMOVA, 2018), and the dispersion relation has the form:

$$\omega = ck \left( 1 - \frac{1}{2} K_{\text{relax}} \right), \quad (1)$$

$$\omega = v_{ph}k - i\alpha v_{ph}, \quad \alpha = \frac{\omega}{2c} \text{Im}(K_{\text{relax}}), \quad (2)$$

$$v_{ph} = c - \frac{c}{2} \text{Re}(K_{\text{relax}}),$$

where  $\omega$  is the characteristic frequency of perturbations,  $k$  is the wave number (PIERCE, 2021).

A relaxation process reveals itself by means of a hysteretic (that is, time-dependent) behaviour. In turn, the hysteretic behaviour specifies thermodynamic processes in a definite way. This concerns different kinds of relaxation processes with various relaxation times and strengths of relaxation. HEDBERG and RUDENKO (2011) were first who attracted attention to the acoustic hysteresis in nonlinear flows with various relaxation mechanisms. The general approach is to study the wave features as a response to excitation. This concerns phase velocity and attenuation at some frequency of a harmonic exciter (PIERCE, MAST, 2021). The main idea of this study is to attract attention to hysteretic curves as indicators of equilibrium parameters of a medium, relaxation process and the kind of wave excitation. The hysteretic behaviour specifies not only harmonic excitation but any other including the impulsive one which is a more common case. The links between the acoustic pressure and density are determined by thermodynamic relaxation and are integro-differential in general. We consider hysteretic curves in the plane magnitude of acoustic pressure versus magnitude of acoustic perturbation in density  $P \leftrightarrow R$ . Other relations of thermodynamic variables, for example, links between acoustic pressure and velocity, also reveal hysteretic behaviour. The curves do not only point at relaxation processes but also at the kind of excitation in a medium with the known thermodynamic properties. The hysteretic curves point impulsive, periodic, or other type of excitation. They determine an irreversible loss in the sound energy which leads to the enlargement of the background temperature. This variation in temperature is integral and may

be measured over a period of a signal or after a wave passes away. The thermodynamic processes in a fluid may be non-equilibrium. This may lead to an acoustically active flow with the energy taken from the background into the energy of the wave motion. The hysteretic curves are unusual in this case. The bypass direction of curves in the plane  $P \leftrightarrow R$  is opposite compared to the equilibrium case. The curves may be used in evaluation of the degree of disequilibrium of a flow. We consider the links between the acoustic pressure and perturbation in acoustic density in a wave for any particular case of a flow. The relations describe a different behaviour in the domains when the acoustic pressure grows and gets smaller. This is the reason for the hysteretic behaviour. The hysteretic behaviour gives a deeper understanding of the physical peculiarities of different irreversible processes accompanying propagation of moderate and intense acoustic waves. The mechanisms of relaxation and absorption in liquids in the context of wave dynamics, are still an unresolved issue (LIEBERMANN, 1948). The composition of a liquid has the key role in a variety of relaxation processes. The processes in the fresh water and sea water are different. Typically, several kinds of relaxation take place in a liquid. Sound absorption in sea water is dominated by chemical relaxations with contribution of magnesium sulfate at high frequencies of oscillations. The intermediate-frequency relaxation occurs due to magnesium and carbonic acid (YEAGER, FISHER, 1973; MELLEN *et al.*, 1979). The frequency-dependent relaxation and attenuation of sound in fluids is of importance in many technical and medical applications, especially in medical imaging and tissue specification (NYBORG, 1978; PARKER, 1983).

We consider acoustic hysteresis associating with relaxation in gases and liquids, and specify differences and applications of hysteretic behaviour in any case. The unusual scenario may take place in acoustically active flows. The hysteresis in the flows of Bingham plastics is also mentioned in Sec. 4. The main idea is to point at the applicability of the hysteretic curves in detection of the relaxation process and the kind of its exciter.

## 2. Relations between acoustic pressure and perturbation in density in various flows

The linear relations between perturbations of thermodynamic quantities (pressure  $p$ , density  $\rho$ , components of velocity  $\mathbf{v}$ , and probably other variables reflecting relaxation) follow from the dispersion relations which characterise thermodynamic processes in a flow. In fact, the relations determine the mode of a flow together with the corresponding dispersion relation. Links of perturbations specifying the acoustic mode are reflected by the hysteretic curves in the plane magni-

tude of acoustic pressure versus magnitude of acoustic perturbation in density. These links and curves indicate damping and relaxation properties of a flow and may be useful in identifying their patterns. As examples, some kinds of fluid flows are examined and compared to each other:

- 1) a fluid with the thermal conduction,
- 2) a gas with vibrational relaxation of oscillatory degrees of a molecule freedom,
- 3) an electrolyte with a chemical reaction.

In all cases, we consider a one-dimensional planar flow along axis  $x$ , only one wave mode propagating in the positive direction of axis  $x$ , and weak deviation from adiabaticity over the characteristic period of perturbations. The background parameters in all flows are constant and subscribed by zero. The bulk flows are absent. In evaluation of dispersion relations, all perturbations of a flow ( $f'$ ) are thought as a sum of planar waves:

$$f'(x, t) = \int \tilde{f}(k) \exp(i\omega(k)t - ikx) dk. \quad (3)$$

### 2.1. Thermoconducting fluid

The thermodynamic processes in the Newtonian fluids may be considered as a limiting case of the Maxwellian viscous model of relaxation with the viscous stress tensor  $\sigma$  depending on the shear rate as follows:

$$\sigma \sim \int_{-\infty}^t \exp\left(\frac{t' - t}{\omega\tau}\right) \frac{\partial v}{\partial x} dt'. \quad (4)$$

In the high-frequency regime when the characteristic frequency of perturbations is much higher than the inverse time of relaxation, the Maxwellian fluid behaves as a classic Newtonian fluid. The acoustic dispersion relation in a Newtonian flow with the thermal conduction takes the form:

$$\omega = ck + \frac{ib}{2}k^2, \quad (5)$$

where  $c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$  designates the speed of an infinitely-small magnitude sound in equilibrium and  $b$  is the total attenuation which consists of parts relating to the shear viscosity, bulk viscosity, and thermal conduction (LANDAU, LIFSHITZ, 1987; RUDENKO, SOLUYAN, 1977). It turns out that the leading-order link between the acoustic pressure and excess acoustic density depends exclusively on the thermal conduction  $\chi$ , but not on the shear and bulk viscosity contributing to the total attenuation  $b$ :

$$\begin{aligned} p' &= c^2 \rho' - \left(\frac{1}{C_V} - \frac{1}{C_P}\right) \frac{\chi c}{\rho_0} \frac{\partial \rho'}{\partial x} \\ &\approx c^2 \rho' + \left(\frac{1}{C_V} - \frac{1}{C_P}\right) \frac{\chi}{\rho_0} \frac{\partial \rho'}{\partial t}, \end{aligned} \quad (6)$$

where  $C_P$  and  $C_V$  designate the heat capacities under constant pressure and constant volume per unit mass, respectively. Velocity of a gas is connected to perturbation in density in the following way:

$$\mathbf{v} = \frac{c}{\rho_0} \rho' + \frac{b}{2c\rho_0^2} \frac{\partial \rho'}{\partial t}. \quad (7)$$

Hence the curves in the plane velocity versus perturbation in density also reveal hysteretic character which is conditioned by the total attenuation, the difference in the behaviour of  $p'(\rho')$  and  $\mathbf{v}(\rho')$  points the thermal conduction coefficient and mechanical viscosity. HEDBERG and RUDENKO (2011) considered Eq. (6) with the link depending on the total viscosity but not on the thermal conduction individually.

### 2.2. A gas with excited vibrational degrees of a molecule

The dispersion relation for an acoustic mode in a gas with a relaxation process associating with the excited vibrational degrees of a molecule sounds as (OSIPOV, UVAROV, 1992; MOLEVICH, 2001):

$$\omega = ck - \frac{(\gamma - 1)^2 T_0 k \tau}{2c(1 + ick\tau)} \Phi, \quad (8)$$

where  $T_0$  is the equilibrium temperature of a gas and  $\Phi$  measures the degree of deviation from adiabaticity of a wave process,  $\gamma = \frac{C_P}{C_V}$  is the ratio of specific heats under constant pressure and constant volume, respectively:

$$\Phi = \frac{1}{\tau} \frac{d\varepsilon_{eq}}{dT} + \frac{\varepsilon - \varepsilon_{eq}}{\tau^2} \frac{d\tau}{T}, \quad (9)$$

where  $\varepsilon$  denotes the internal energy of a gas,  $\tau$  is the relaxation time, and the equilibrium internal energy  $\varepsilon_{eq}(T)$  in the case of the system of harmonic oscillators equals:

$$\varepsilon_{eq}(T) = \frac{\hbar\Omega}{m(\exp(\hbar\Omega/k_B T) - 1)}, \quad (10)$$

where  $m$  is the mass of a molecule,  $\hbar\Omega$  is the magnitude of the vibrational quantum, and  $k_B$  designates the Boltzmann constant. The relaxation time in the most cases is a function of temperature,  $\ln \tau \sim T^{-1/3}$  (LANDAU, LIFSHITZ, 1987). The leading-order link between the acoustic pressure and excess acoustic density takes the form (PERELOMOVA, 2017):

$$\begin{aligned} p' &= c^2 \rho' - \frac{(\gamma - 1)^2 T_0}{c} \Phi \int_x^\infty \exp\left(\frac{x - x'}{c\tau}\right) \rho' dx' \\ &\equiv c^2 \rho' - \frac{c\alpha_V}{\tau} \int_x^\infty \exp\left(\frac{x - x'}{c\tau}\right) \rho' dx', \end{aligned} \quad (11)$$

where

$$\alpha_V = \frac{(\gamma - 1)^2 T_0 \tau \Phi}{c^2} \quad (12)$$

is a dimensionless small parameter responsible for deviation from adiabaticity of a flow due to relaxation. It is negative in an acoustically active flow due to the negative  $\Phi$ . The connection linking velocity and perturbation in density takes the form:

$$\mathbf{v} = \frac{c}{\rho_0} \rho' - \frac{\alpha_V}{2\tau\rho_0} \int_x^\infty \exp\left(\frac{x-x'}{c\tau}\right) \rho' dx'. \quad (13)$$

Hence, the hysteretic curves determined by Eqs (11) and (13) reveal a different behaviour. Their comparison can give information about  $\alpha_V$ . It follows from Eqs (11) and (13) that:

$$\frac{p'}{c\rho_0} + \frac{c\rho'}{\rho_0} - 2v' = 0. \quad (14)$$

All quantities in Eq. (14) are local and easy to measure. The relation does not include an integral which reflects the time dependent behaviour of perturbations but points at the kind of thermodynamic relaxation.

### 2.3. An electrolyte

Following NACHMAN *et al.* (1990), one relaxation process with the characteristic time of relaxation  $\tau$  for the vibrational energy in an electrolyte is considered. The entropy is a sum of an equilibrium part  $s^e$  and of an irreversible part associating with one relaxation process (EIGEN, TAMM, 1962; PIERCE, 1981; NACHMAN, 1990):

$$s(p, \rho, n) = s^e(p, \rho) + \frac{C_P \kappa}{\beta_P T} \Delta\xi, \quad (15)$$

where

$$\Delta\xi = \frac{n - n^e(p, T)}{\partial n^e(p, T) / \partial p}, \quad (16)$$

where  $n$  designates the number of molecules per unit mass,  $\kappa$  denotes the contribution of the dissolved molecules to the isothermal compressibility (reciprocal of the bulk modulus), and  $\beta_P$  is the isobaric volumetric thermal expansion coefficient:

$$\beta_P = -\frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial T} \right)_p. \quad (17)$$

$\Delta\xi$  satisfies the relaxation equation:

$$\left( \frac{\partial}{\partial t} + \frac{1}{\tau} \right) \Delta\xi = -\frac{\partial p'}{\partial t}. \quad (18)$$

Equilibrium quantities are supplied by the upper index  $e$ . A principal result by NACHMAN (1990) is the generalised frequency domain compressibility expressed in his work in Eq. (37) as:

$$\frac{1}{c^2 \rho_0} - \kappa \frac{i\omega}{i\omega - 1/\tau}. \quad (19)$$

The compressibility at zero frequency equals  $\frac{1}{c^2 \rho_0}$  and at the infinite frequency it takes the form  $\frac{1}{c^2 \rho_0} - \kappa$ . At the presence of relaxation, the compressibility at zero frequency is always larger than that at the infinite frequency. These thermodynamic premises lead to the dispersion relation:

$$\omega = ck + i\kappa \frac{c^4 k^2 \rho_0 \tau}{2(1 + i\kappa c\tau)} \equiv ck + i\alpha_E \frac{c^2 k^2 \tau}{2(1 + i\kappa c\tau)}. \quad (20)$$

Equation (20) introduces a dimensionless parameter responsible for the small impact of relaxation:

$$\alpha_E = c^2 \rho_0 \kappa \ll 1. \quad (21)$$

The dispersion relation (20) determines links between the Fourier transforms of thermodynamic perturbations in the sound mode, and the correspondent links in  $x, t$  space. In particular, the relation between the acoustic pressure and excess density in the sound mode takes the form:

$$p' = c^2 \rho' - \alpha_E c^2 \int_x^\infty \exp\left(\frac{x-x'}{c\tau}\right) \frac{\partial \rho'}{\partial x'} dx'. \quad (22)$$

Velocity relates to the perturbation of density as (PERELOMOVA, 2015):

$$\mathbf{v} = \frac{c}{\rho_0} \rho' - \frac{\alpha_E c}{2\rho_0} \int_x^\infty \exp\left(\frac{x-x'}{c\tau}\right) \frac{\partial \rho'}{\partial x'} dx'. \quad (23)$$

The hysteretic curves determined by Eqs (22) and (23) also reveal different hysteretic behaviour which may ensure evaluations of  $\alpha_E$ . In spite of different mechanisms of relaxation, the equality (14) is valid in this case as well.

## 3. Hysteretic behaviour

Relations between the acoustic pressure and excess acoustic density (6), (11), (22) are specified different hysteretic graphs in the thermodynamic plane. The hysteresis is due to a different thermodynamic behaviour in the domains when  $p'$  enlarges in time, and in the domains when  $p'$  gets smaller. The hysteretic curves depend on the kinds of thermodynamic relaxation and excitation. A similar hysteretic behaviour is revealed by curves  $\mathbf{v}(\rho')$  for all cases of relaxation. In this section, we do not consider nonlinear effects which bring corrections to the links of thermodynamic variables and distort the form of hysteretic curves.

### 3.1. Harmonic variations of pressure

We consider dimensionless variations in pressure in the form:

$$P \equiv \frac{p'}{M c_0^2 \rho_0} = \sin(\omega t - kx), \quad (24)$$

which is the leading-order approximation solution to a linear wave equation without relaxation and attenuation of any kind (the Mach number  $M$  measures the magnitude of perturbations; it represents the ratio of magnitude of velocity to the local speed of sound), and dimensionless perturbation of density:

$$R = \frac{\rho'}{M\rho_0}. \quad (25)$$

Making use of Eqs (6), (11), (22), the hysteretic curves at  $x = 0$  are determined in the parametric forms as follows ( $\theta = \omega t$  designates the dimensionless time):

a) thermoconducting

$$P = \sin(\theta),$$

$$R = \sin(\theta) - \alpha_T \cos(\theta),$$

$$\alpha_T = \left( \frac{1}{C_V} - \frac{1}{C_P} \right) \frac{\chi\omega}{\rho_0 c^2},$$

b) vibrational

$$P = \sin(\theta),$$

$$R = \sin(\theta) - \alpha_V \frac{\omega\tau \cos(\theta) - \sin(\theta)}{1 + \omega^2\tau^2},$$

c) electrolyte

$$P = \sin(\theta),$$

$$R = \sin(\theta) - \alpha_E \frac{\omega\tau \cos(\theta) + (\omega\tau)^2 \sin(\theta)}{1 + \omega^2\tau^2}.$$

Figure 1 shows hysteresis curves at  $\omega\tau = 1$  for all kinds of relaxation and attenuation considered in Subsecs 2.1, 2.2, and 2.3. All curves start at the negative  $R$  at zero  $P$ . The “vertical” (the difference between maximum and minimum magnitudes of  $R$  at zero  $P$ ) and “horizontal” (the difference between maximum and minimum magnitudes of  $P$  at zero  $R$ ) widths of curves are equal but vary with  $\omega\tau$ :

a) thermoconducting  $2\alpha_T$ ,

b) vibrational  $\frac{2|\alpha_V|\omega\tau}{1 + (\omega\tau)^2}$ ,

c) electrolyte  $\frac{2\alpha_E\omega\tau}{1 + (\omega\tau)^2}$ .

The case of vibrational relaxation is special since  $\alpha_V$  may take zero and negative (in acoustically active flow) values. Negative  $\alpha_V$  specifies the clockwise direction of the hysteretic curve. The amplitude of  $R$  varies with  $\alpha$  and  $\omega\tau$ :

$$R_{T,A} \approx 1, \quad R_{V,A} = 1 + \frac{\alpha_V}{1 + (\omega\tau)^2}, \quad (26)$$

$$R_{E,A} \approx 1 - \frac{\alpha_E(\omega\tau)^2}{1 + (\omega\tau)^2},$$

where  $R_{V,A}$  and  $R_{E,A}$  constantly but differently decrease as functions of  $\omega\tau$ .

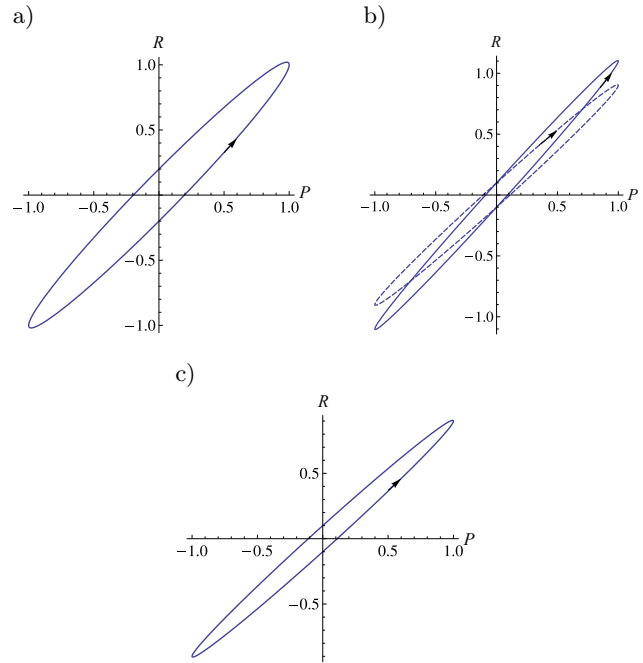


Fig. 1. Hysteretic curves reflecting different mechanisms of attenuation. The case of harmonic acoustic pressure at an exciter,  $x = 0$ .  $\alpha_T = \alpha_E = 0.2$ ,  $\alpha_V = 0.2$  (normal line),  $\alpha_V = -0.2$  (dashed line),  $\omega\tau = 1$ .

### 3.2. Impulsive variations of pressure

Let the variation in pressure takes the Gaussian form

$$P = \exp(-(\omega t - kx)^2). \quad (27)$$

Making use of (2.6), (2.11), (2.22) we arrive at the hysteretic curves at  $x = 0$  in the parametric forms:

a) thermoconducting

$$P = \exp(-\theta^2),$$

$$R = \exp(-\theta^2) + 2\alpha_T\theta \exp(-\theta^2),$$

b) vibrational

$$P = \exp(-\theta^2),$$

$$R = \exp(-\theta^2) + \frac{\alpha_V\sqrt{\pi}}{2\omega\tau} \exp\left(\frac{1 - 4\omega\tau\theta}{4\omega^2\tau^2}\right) \cdot \left(1 - \operatorname{erf}\left(\frac{1}{2\omega\tau} - \theta\right)\right),$$

c) electrolyte

$$P = \exp(-\theta^2),$$

$$R = \exp(-\theta^2) - \frac{\alpha_E}{2\omega\tau} \exp\left(-\theta\left(\theta + \frac{1}{\omega\tau}\right)\right) \cdot \left(2 \exp\left(\frac{\theta}{\omega\tau}\right)\omega\tau + \sqrt{\pi} \exp\left(\theta^2 + \frac{1}{4\omega^2\tau^2}\right) \cdot \left(1 - \operatorname{erf}\left(\frac{1}{2\omega\tau} - \theta\right)\right)\right).$$

Figure 2 shows hysteretic curves as an impulse evolves from  $-\infty$  till  $\infty$  in time. The starting and residual values of  $R$  in all cases equal zero. In the case of non-equilibrium vibrational relaxation,  $\alpha_V < 0$  corresponds to unusual direction of the loop traversal. The correspondent curve reveal smaller amplitudes  $R$  for the equal magnitudes  $P$ .

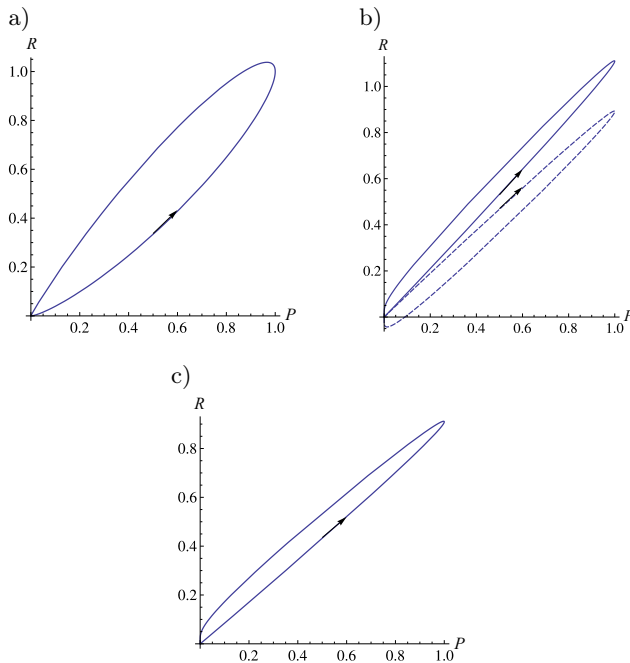


Fig. 2. Hysteretic curves reflecting different mechanisms of attenuation. The case of acoustic pressure in the form of a Gaussian impulse:  $\alpha_T = \alpha_E = 0.2$ ,  $\alpha_V = 0.2$  (normal line),  $\alpha_V = -0.2$  (dashed line),  $\omega\tau = 1$ .

#### 4. Conclusions

The hysteretic curves are manifestation of irreversible thermodynamic processes, in particular, relaxation processes. They are useful in identification not only of individual but multiple relaxation processes. As usual, all of them have small impact on a wave process and are considered as linear and independent. Hence the attenuation coefficients and links of wave perturbations are corrected by the sum of terms responsible for any kind of attenuation. For processes with continuously distributed times of relaxation, the weighting function  $G(\tau)$  is introduced (PIERCE, 2021) which is used in integrals over the whole range of physically meaningful times of relaxation. The rates of individual relaxation processes vary and form a hierarchy from short (establishment of the equilibrium between translational degrees of molecules) till large (relaxation of chemical processes) ones. The non-equilibrium distribution of energy has impact on adiabatic compressibility and by means of that on the sound speed. The relaxation processes and all that follows (dispersion relations, sound speed, attenuation coefficients, relations

between specific perturbations, hysteretic curves) are dependent on a characteristic frequency of the exciter. In this study, we consider acoustic hysteretic behaviour of flows with different individual mechanisms of non-adiabaticity:

- 1) the Newtonian ones with the thermal conduction;
- 2) flows where the vibrational degrees of molecules are excited;
- 3) flows of electrolytes with a chemical reaction.

The total range of sound frequencies is considered. The case of vibrational relaxation has much in common with the relaxation due to a chemical reaction in a gas. There is an evident analogy in the dispersion relations and links between the acoustic pressure and excess acoustic density (LEBLE, PERELOMOVA, 2018; PERELOMOVA, 2013; 2019). The dispersion relation for the wave propagating in the positive direction of axis  $x$  and corresponding link of thermodynamic perturbations in a gas with a chemical reaction are very similar to Eqs (8) and (11):

$$\omega = ck + \frac{ck\tau Q_0(\gamma - 1)(Q_\rho + (\gamma - 1)Q_T)}{2\gamma T_0(1 + ick\tau)},$$

$$p'_1 = c^2 \rho'_1 + \frac{Q_0 C_V (\gamma - 1)^2 (Q_\rho + (\gamma - 1)Q_T)}{c} \quad (28)$$

$$\cdot \int_x^\infty dx_1 \exp\left(\frac{x - x_1}{c\tau_c}\right) \rho'_1,$$

where  $\tau$  is the characteristic duration of a chemical reaction,  $Q$  is the heat produced in a gas per one molecule due to a chemical reaction, and the dimensionless quantities  $Q_T$ ,  $Q_\rho$ ,  $Q_Y$  are expressed in terms of partial derivatives of the heat produced in a chemical reaction,  $Q$ :

$$Q_T = \frac{T_0}{Q_0} \left( \frac{\partial Q}{\partial T} \right)_{T_0, \rho_0, Y_0},$$

$$Q_\rho = \frac{\rho_0}{Q_0} \left( \frac{\partial Q}{\partial \rho} \right)_{T_0, \rho_0, Y_0}, \quad (29)$$

$$Q_Y = \frac{Y_0}{Q_0} \left( \frac{\partial Q}{\partial Y} \right)_{T_0, \rho_0, Y_0}$$

at equilibrium temperature  $T_0$ , density  $\rho_0$  and the mass fraction of reagent  $Y_0$ .  $C_V$ ,  $\gamma$ , and  $c$  designate “frozen” values, corresponding to processes which take place at infinitely high frequencies. The hysteresis occurs unusually in an acoustically active flow, that is, if

$$Q_\rho + (\gamma - 1)Q_T > 0. \quad (30)$$

In general, relaxation processes such as molecular relaxations, energy exchanges between translational and internal degrees of molecules freedom and chemical reactions, determine the form of the kernel in an integral relation of acoustic pressure and perturbation in

the acoustic density. This kernel is responsible for the frequency dependent absorption and may be reconstructed basing on experimental data. Acoustic hysteresis may be readily detected experimentally. It may be useful in evaluations of different relaxation mechanisms in a medium by means of observing excitation at different frequencies during the nondestructive testing of materials. The width of a hysteretic curve and the magnitude of excess acoustic density (26) in harmonic excitation may be the indicators. Often, some relaxation mechanisms contribute to a hysteresis. This leads to a sum of integrals in the link  $R(P)$  with different kernels. Hence, the hysteretic curves may be useful in the detection of individual relaxation mechanisms and comparative analysis of the relaxation parameters. The bypass direction of a hysteretic curve is usually counter clockwise (clockwise in acoustically active flows). That ensures negative (positive in acoustically active flows) work done by a liquid element along the pass  $\int p dV = -\frac{1}{\rho^2} \int p d\rho < 0$  and hence positive (negative) variation in the internal energy in the quasi-adiabatic processes:

$$dU = - \int p dV = \frac{1}{\rho^2} \int p d\rho, \quad (31)$$

which reflects an irreversible transform of acoustic energy into that of the chaotic motion. The dimensionless variation of the internal energy over the period  $dU$  takes the form in the case of harmonic excitation:

- a) thermoconducting  $\pi\alpha_T$ ,
- b) vibrational  $\frac{\pi\alpha_V\omega\tau}{1 + (\omega\tau)^2}$ ,
- c) electrolyte  $\frac{\pi\alpha_E\omega\tau}{1 + (\omega\tau)^2}$ .

The macroscopic sound energy increases in acoustically active media taking energy from the background. This leads to a cooling of a medium (MOLEVICH, 2011; LEBLE, PERELOMOVA, 2018).

The hysteretic behaviour also specifies flows of fluids which are at the boundary between liquids and solids. A stand-alone example is a Bingham plastic, a liquid rigid for shear stress less than some critical value  $P_c$ . The leading-order relation between the acoustic pressure and excess acoustic density in the Bingham flows is:

$$\begin{aligned} R &= (1 - F)P, & \text{if } \frac{\partial P}{\partial T} > 0, \\ R &= (1 + F)P, & \text{if } \frac{\partial P}{\partial T} < 0, \end{aligned} \quad (32)$$

where  $F = \frac{P_c \beta_P}{K \rho_0^2 C_V c^2}$  and  $K = \frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial p} \right)_T$  (PERELOMOVA, 2011a; 2011b). The hysteretic curves in the case of harmonic and impulsive excitors are shown in Fig. 3 for the exemplary value  $F = 0.1$ .

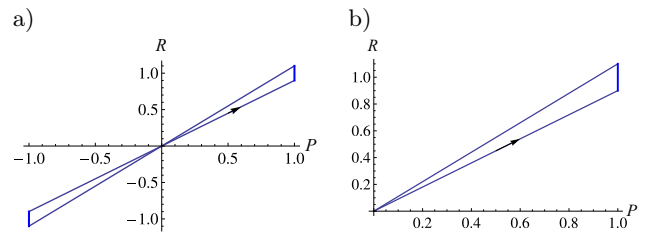


Fig. 3. Hysteretic curves in a flow of a Bingham plastic. The case of acoustic pressure in the form of harmonic wave (24) (one period of oscillations (a)) and a Gaussian impulse (27) (evolution in time from  $-\infty$  till  $\infty$  (b)),  $F = 0.1$ .

The dimensionless variation of the internal energy  $\int P dR$  over a period in the case of harmonic excitation of a Bingham plastic equals  $2F$ . It equals  $F$  in the case of an impulsive excitation. The example of flows of the Maxwell liquids also reveals relaxation. The links between acoustic pressure and perturbation of acoustic density does not contain an integral term but includes only a term proportional to the thermal conduction (PERELOMOVA, 2008). Hence, the hysteretic behaviour is the same that in the case of the Newtonian flows with the thermal conduction.

We do not take into account nonlinearity in the links of thermodynamic perturbations. The quadratic nonlinearity deforms an elliptic diagram into a crescent with the downcast ends (HEDBERG, RUDENKO, 2011; PERELOMOVA, 2020). Hysteretic processes due to the pure nonlinear losses at the front of the shock wave were considered in detail by HEDBERG and RUDENKO (2011). Also, the nonlinearity of a flow and some kind of damping (including relaxation of thermodynamic processes) are the necessary conditions for interaction of modes and excitation of the secondary entropy mode in the field of intense sound. The entropy mode contributes to the hysteretic behaviour in the plane total excess pressure  $\Leftrightarrow$  total perturbation of density (since it is specified by the non-zero perturbation in density), but does not contribute to the acoustic hysteresis. An excess density associating with the entropy mode is usually negative (this corresponds to the positive excess temperature due to the transform of the macroscopic energy into the microscopic one) but may be positive in the acoustically active flows. This determines two kinds of behaviour of the hysteretic curves in the plane perturbation of pressure  $\Leftrightarrow$  total perturbation of density. In particular, harmonic variation in pressure yields a shift of total excess density (negative of positive) over every period which enlarges with the number of periods. The peculiarities introduced by an acoustic heating (cooling) in the thermodynamic relations and hysteretic curves have been discussed by PERELOMOVA (2013) regarding the flows of non-equilibrium gas and a gas with an exothermic chemical reaction. A fairly complex case of magnetoacoustic hysteresis in flows of a magnetised gas which incorporates

deviation from adiabaticity due to thermal conduction and some heating-cooling function, is considered by PERELOMOVA (2020). This case is of a special interest due to different scenarios of hysteresis necessary to balance the thermal conduction and the heating-cooling function and due to dependence of the hysteretic behaviour on the ratio of magnetic and hydrostatic pressures and the angle between the equilibrium magnetic field and the wave vector.

The results are formulated in general. The dimensionless parameters  $\alpha_T$ ,  $\alpha_V$ ,  $\alpha_E$  may vary in different flows. The most interesting case is a flow in the vibrationally relaxing gas which may be acoustically active. This corresponds to the negative  $\alpha_V$ . Evaluations for the typical laser mixture  $\text{CO}_2 : \text{N}_2 : \text{He} = 1 : 2 : 3$  at normal conditions  $p_0 = 101\,325\text{ Pa}$ ,  $T = 300\text{ K}$ , give  $\tau = 5 \cdot 10^{-5}\text{ s}$  and the threshold pumping intensity  $1.5 \cdot 10^6\text{ W} \cdot \text{m}^{-3}$ . The pumping intensity  $5.3 \cdot 10^8\text{ W} \cdot \text{m}^{-3}$  corresponds to  $\alpha_V = -0.07$ . The absolute value of  $\alpha_V$  increases when the pumping intensity grows.

### References

1. DUCK F.A., BAKER A.C., STARRITT H.C. (1998), *Ultrasound in Medicine*, Institute of Physics Publishing, Bristol–Philadelphia.
2. EIGEN M., TAMM K. (1962), Sound absorption in electrolyte solutions as a result of chemical relaxation I. Relaxation theory of multilevel dissociation [in German: Schallabsorption in Elektrolytlösungen als Folge chemischer Relaxation I. Relaxationstheorie der mehrstufigen Dissoziation], *Zeitschrift für Elektrochemie, Berichte der Bunsengesellschaft für physikalische Chemie*, **66**(2): 93–107.
3. EIGEN M., DE MAYER L. (1963), Relaxation methods, [in:] *Techniques of Organic Chemistry*, Freiss S.L., Lewis E.S., Weissberger A. [Eds], Interscience Publishers, New York.
4. HALL L. (1948), The origin of ultrasonic absorption in water, *Physical Review*, **73**(7): 775–781, doi: 10.1103/PhysRev.73.775.
5. HAMILTON M., IL'INSKII Yu., ZABOLOTSKAYA E. (1998), Dispersion, [in:] *Nonlinear Acoustics*, Hamilton M., Blackstock D. [Eds], Academic Press.
6. HEDBERG C.M., RUDENKO O.V. (2011), Dissipative and hysteresis loops as images of irreversible processes in nonlinear acoustic fields, *Journal of Applied Physics*, **110**(5): 053503, doi: 10.1063/1.3631800.
7. HERTZFELD K.F., LITOWITZ T.A. (1959), *Absorption and Dispersion of Ultrasonic Waves*, Academic Press, New York.
8. HERZFELD K.F., RISE F.O. (1928), Dispersion and absorption of high frequency sound waves, *Physical Review*, **31**(4): 691–695, doi: 10.1103/PhysRev.31.691.
9. LANDAU L.D., LIFSHITZ E.M. (1987), *Fluid Mechanics*, 2nd ed., Pergamon, New York.
10. LEBLE S., PERELOMOVA A. (2018), *The Dynamical Projectors Method: Hydro and Electrodynamics*, CRC Press.
11. MAKAROV S., OCHMANN M. (1996), Nonlinear and thermoviscous phenomena in acoustics, Part I, *Acta Acustica united with Acustica*, **82**(4): 579–606.
12. MANDELSHTAM L.I., LEONTOWICH M.A. (1937), To the theory of sound absorption in liquids, *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, **7**(3): 438.
13. MELLEEN R.H., SIMMONS V.P., BROWNING D.G. (1979), Sound absorption in sea water: a third chemical relaxation, *The Journal of the Acoustical Society of America*, **65**(4): 923–925, doi: 10.1121/1.382595.
14. MOLEVICH N.E. (2001), Sound amplification in inhomogeneous flows of nonequilibrium gas, *Acoustical Physics*, **47**(1): 102–105, doi: 10.1134/1.1340086.
15. NACHMAN A., SMITH J.F., WAAG R.C. (1990), An equation for acoustic propagation in inhomogeneous media with relaxation losses, *The Journal of the Acoustical Society of America*, **88**(3): 1584–1595, doi: 10.1121/1.400317.
16. NYBORG W.L. (1978), *Physical Mechanisms for Biological Effects of Ultrasound*, Rockville: Bureau of Radiological Health.
17. OSIPOV A.I., UVAROV A.V. (1992), Kinetic and gasdynamic processes in nonequilibrium molecular physics, *Soviet Physics Uspekhi*, **35**(11): 903–923.
18. PARKER K.J. (1983), Ultrasonic attenuation and absorption in liver tissue, *Ultrasound in Medicine and Biology*, **9**(4): 363–369, doi: 10.1016/0301-5629(83)90089-3.
19. PIERCE A.D. (1981), *Acoustics: An Introduction to Its Physical Principles and Applications*, McGraw-Hill, New York.
20. PIERCE A.D., MAST T.D., (2021), Acoustic propagation in a medium with spatially distributed relaxation processes and a possible explanation of a frequency power law, *Journal of Theoretical and Computational Acoustics*, **29**(2): 2150012, doi: 10.1142/S2591728521500122.
21. PERELOMOVA A. (2008), Acoustic heating in a weakly dispersive fluid flow, *Acta Acustica united with Acustica*, **94**(3): 382–387, doi: 10.3813/AAA.918045.
22. PERELOMOVA A. (2011a), Acoustic heating produced in the thermoviscous flow of a Bingham Plastic, *Central European Journal of Physics*, **9**(1): 138–145, doi: 10.2478/s11534-010-0043-7.
23. PERELOMOVA A. (2011b), Propagation of acoustic pulses in some fluids with yield stress, *Canadian Journal of Physics*, **89**(2): 219–224, doi: 10.1139/p10-121.



24. PERELOMOVA A. (2013), Hysteresis curves and loops for harmonic and impulse perturbations in some non-equilibrium gases, *Central European Journal of Physics*, **11**(11): 1541–1547, doi: 10.2478/s11534-013-0305-2.
25. PERELOMOVA A. (2015), The nonlinear effects of sound in a liquid with relaxation losses, *Canadian Journal of Physics*, **93**(11): 1391–1396, doi: 10.1139/cjp-2014-0676.
26. PERELOMOVA A. (2017), Instantaneous heating and cooling caused by periodic or aperiodic sound of any characteristic duration in a gas with vibrational relaxation, *Acta Acustica united with Acustica*, **103**(4): 607–615, doi: 10.3813/aaa.919089.
27. PERELOMOVA A. (2019), Excitation of non-wave modes by sound of arbitrary frequency in a chemically reacting gas, *Acta Acustica united with Acustica*, **105**(6): 918–927, doi: 10.3813/AAA.919373.
28. PERELOMOVA A. (2020), Hysteresis curves for some periodic and aperiodic perturbations in magnetosonic flow, *Physics of Plasmas*, **27**(10): 102101; doi: 10.1063/5.0015944.
29. RUDENKO O.V., SOLUYAN S.I. (1977), *Theoretical Foundations of Nonlinear Acoustics*, Plenum, New York.
30. VILENSKY G., TER HAAR G., SAFFARI N. (2012), A model of acoustic absorption in fluids based on a continuous distribution of relaxation times, *Wave Motion*, **49**(1): 93–108, doi: 10.1016/j.wavemoti.2011.07.005.
31. YEAGER E., FISHER F.H. (1973), Origin of the low-frequency sound absorption in sea water, *The Journal of the Acoustical Society of America*, **53**(6): 1705, doi: 10.1121/1.1913523.

