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Mathematical model to assess energy consumption using water inflow-drainage system of iron-ore mines in terms of a stochastic process

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Abstract

Purpose is to develop a unified mathematical model to assess energy efficiency of a water inflow-drainage process as the real variant of stochastic method for water pumping from underground workings of iron-ore mines.

Methods. The research process was based upon the methods of probability theory as well as stochastic modelling methods. The stochastic function integration has been reduced to summation of its ordinates and further transition to a proper boundary.

Findings. A mathematical model of a water inflow-drainage system as a stochastic process has been developed in terms of input parameters of a standard operating iron-ore mine. The abovementioned has made it possible to assess realistically, substantiate, and obtain possibilities for a specific production facility as well as for generalization of the results involving determination of stochastic characteristics of drainage process.

Originality. For the first time, a mathematical model of drainage from underground levels of iron-ore mines has been developed as a stochastic process. The process characteristics have been identified relying upon randomness of a water pumping technique. In contrast to the available settings, the new model parameters characterize their dispersion. Possibility to obtain complete characteristics of energy consumption has been obtained: for drainage; for water accumulation volume in underground water collectors; for water pumping from the specified mine depths over the specific period as random processes. A number of drainage features have been analyzed and differentiated being determined with the help of nor-mal law of water accumulation velocity in the underground water collectors in iron-ore mines.

Practical implications. In terms of operating iron-ore mine, a generalized drainage mathematical model has been developed as a stochastic process using statistical data concerning water accumulation velocity in the underground water collectors. It has been proved that if the ordinates of water accumulation velocity in the underground water collectors obey the normal distribution law then it is expedient to characterize drainage as a stochastic process. The developed methods, studying drainage as a stochastic process, help expand the research boundaries involving other auxiliary operations performed during underground mining of iron ore raw materials.

Keywords: *efficiency, methodology, model, mine, drainage, stochastic process*

1. Introduction

Capitalization of relations in Ukraine needs detailed approach to mining inclusive of such a strategically important and primary for GDP filling mineral type as iron ore (IO). In the context of current mining techniques, such integral components as auxiliary operations are quite important. However, these activities, being defined as auxiliary, help provide conditions required for performance of mining enterprises as well as for safety of personnel. The status of attention to the operation type is supported by domestic studies and especially by the world ones. Significant amount from the cohort of the known publications belongs to such a system-forming composition type in the class of auxiliary operations as the main drainage systems removing water, accumulating naturally in the workings of mines, quarries, and open pits, to the specific surface collectors. Papers [1]-[4] represent viewpoints by foreign researchers concerning both possibility and expediency to restructure water pumping systems of mining enterprises from a consumer-regulator status to a status of consumer-regulator-generator of electricity.

While developing the abovementioned tendency, papers [5]-[8] make arguments to reformat drainage systems of mines and quarries in the pumped storage power stations. To analyze correctness of the ideas and taking into consideration complexity (and, more precisely, even impossibility) of the field studies under mining conditions, numerous scientific sources have applied model studies as a research tool [9], [10].

It is logical that digital support of the research process is important under the current conditions of the procedure study as a complex. The current methods to compute drainage characteristics as a process are of insufficient accuracy since the calculations are performed somewhat simplified, i.e. in

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terms of average values ignoring stochastic nature of the procedures. Stochasticity consideration by means of mathematical modelling of drainage using IT helps improve calculation of its characteristics which is basically required moment while developing an algorithm of functioning of the automated control system (ACS) for the process. For greater clarity, the paper considers mathematical modelling of drainage in terms of one of operating mines in Kryvyi Rih iron ore basin.

1.1. The research tasks

To find the abovementioned purpose, following tasks have been set for coverage:

– to analyze input parameters in terms of specific indices of operating enterprises to develop the basic model assessing parameters influencing modes of functioning of the main mine drainage systems;

– to develop a multifactor mathematical drainage model as a stochastic process assessing its features to form managerial algorithm of energy efficient control of the procedure in the context of underground levels of iron-ore mines.

1.2. The research topicality

Electric power is almost 95% of the total energy consumption by enterprises engaged in underground IO mining [11]. In turn, more than 90% of the abovementioned EP amounts are those consumed by so-called stationary mine facilities [12], [13]. Among the system of the facilities, drainage systems, providing removal (pumpage) of naturally accumulated water from the underground water collectors of the enterprises to the specific surface reservoirs, consume more than 30% of the quantity. The systems of water pumping from underground mine levels include local and main drainage facilities (systems) (MDFs) [14]. According to drainage technique, water is pumped out from local facilities, and gets to the MDFs from which it moves gradually from lower levels to upper ones. Finally, it is removed to the specific surface reservoirs. In this context, the total depth, associated with water height, being pumped out from the underground water collectors of iron-ore mines, is almost 1500 m (Table 1). Average length of the water pipelines is 8-9 km. Electric capacity of drive MDF motors excess significantly corresponding indices of local facilities. For example, the total electric capacity of drive MDF motors in one iron-ore mine is 17-18 GW [14].

Table 1. Underground levels and mine pumps

Mines	Level (m) / the number of pumps
Kryvorizka (Kryvyi Rih)	500/7; 940/7; 1240/6; 1465/6
Pokrovska (Kryvyi Rih)	437/4; 965/4; 1115/4; 1265/4
Kozatska (Kryvyi Rih)	472/5; 792/4; 1190/4; 1350/3
Ternivska (Kryvyi Rih)	527/5; 1050/4; 1200/4; 1350/4
Frunze (Kryvyi Rih)	410/3; 910/3; 1060/4; 1135/2
Yuvileina (Kryvyi Rih)	480/5; 940/5; 1340/5
Artem (Kryvyi Rih)	475/6; 865/8; 1045/5; 1135/9
Hihant-Hlyboka (closed down) (Kryvyi Rih)	380/3; 540/5; 630/2; 710/2; 800/3
Ekspluatatsiina (Dniprorudne)	400/10; 480/7; 640/8; 840/5; 940/3; 1040/3

It is interesting fact that water, being pumped out from the underground levels, is quite mineralized; thus, desalinization process is required to use it in the corresponding cycle. The desalinization procedure is quite complex in terms of its technique and energy consumption while being a separate case study, which is not the research purpose.

No matter how trivial it may seem, it is also logical that achievement of the expected level of positive in this field is solving the components characterizing parameters and assessing behaviour of the target function, i.e. a process of water removal from underground levels to the surface as a water inflow function.

In their ultimate solution, the studies, known by the authors of scientific search, are a research option with the tries to substantiate scientifically a development process of both local and complex alternatives of measures to improve energy efficiency of mine MDFs [4]-[6], [15]-[18]. Nevertheless, according to the purposes of the abovementioned studies, they did not consider the problems declared as basic ones in the paper.

2. Methods

Underground water drainage from mines is quite a complex process from the viewpoint of its operation technique as well as from the viewpoint of its technical facilities as power and electromechanical structure with a potentially required control system. Mine drainage schemes may be either individual (i.e. water is pumped out from one mine) or unified (i.e. water from several mines is accumulated in underground collector of one of the mines to be pumped out to the specific surface water reservoir [19], [20].

A structure (scheme) for underground water removal from mine levels are determined by depth of IO mining technique as well as by water inflow amount throughout the day. The latter indicator is identified with the help of water inflow ratio (Fig. 1) characterizing intensity of water inflow amount in the mine. As Figure 1 shows, the indicator varies depending upon a mine.

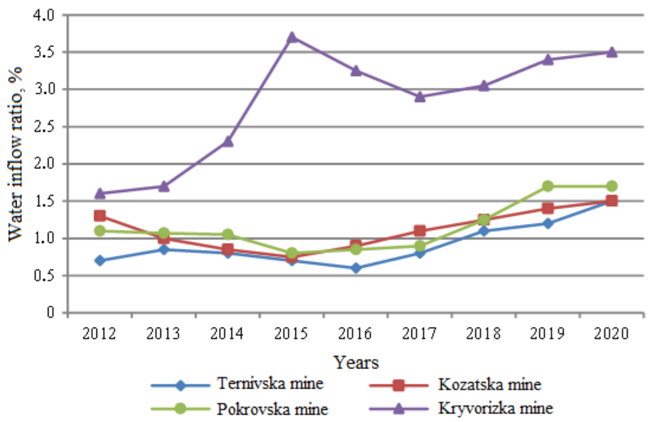

Figure 1. Daily ratios of water inflow in a number of iron-ore mines of Kryvyi Rih iron ore basin

However, the authors believe that ideology of the research structure development, which results are stated in the paper, should consider the velocity of water amount accumulation in the underground collectors as the basic indicator (Fig. 2).

It is logical that in its ultimate alternative, development of a managerial algorithm to control the system identify its operational efficiency on the whole and electric power efficiency in particular [21]-[23]. Achievement of the goal (i.e. efficient control of the process) should involve each or almost each factor influencing the process. The postulate reaching in the most optimal option is possible through simulation of a water pumping-out process.

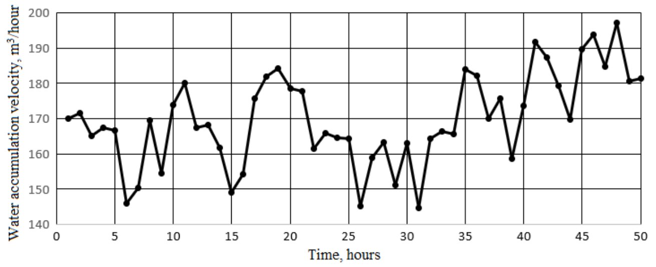

Figure 2. Water accumulation velocities in the underground collector of a standard iron-ore mine (Kryvyi Rih)

Under such conditions, a system-forming value has tracking in the digital form of technological processes connected with water inflow-drainage; it can be done expediently and efficiently using mathematical modelling [24]. For the purpose, consider logistics of the mathematical model development for a drainage system. It seems expedient to involve the known law of energy conservation. Elementary work has to be performed to lift water from underground collectors of mine levels to the surface [25]-[27].

$$da = g \cdot h \cdot dm \,, \tag{1}$$

where:

dm – water mass element, kg;

g – gravitation acceleration, m/s^2;

h – depth from which water pumping takes place, m.

If water amount is taken into consideration then Formula 1 is:

$$da = \rho \cdot g \cdot h \cdot du \,, \tag{2}$$

where:

ρ – water density, kg/m^3;

du – element of the water amount, m^3.

To transfer energy to kWh, apply the coefficient $\delta = 2.78 \cdot 10^{-7}$ kWh /J.

Then Formula 2 will look like:

$$da = \rho \cdot g \cdot h \cdot \delta \cdot du \,. \tag{3}$$

In turn, taking into consideration $\rho = 103$ kg/m^3 and $g = 9.8$ m/s^2 values, Formula 3 takes the form:

$$da = \beta \cdot h \cdot du \,, \tag{4}$$

where:

$\beta = \rho \cdot g \cdot \delta = 0.002724$ N/m^3.

Formula 4 helps calculate the work required to lift water amount element du to h height.

Taking into account that:

$$du = v(t)dt \,,$$

where:

$v(t)$ – water accumulation velocity at a t time moment, m^3/h, Formula 4 will look like:

$$da = \beta \cdot h \cdot v(t)dt \,. \tag{5}$$

In such a way, dependence of the consumed energy capacity to lift water amount to the surface will be defined using the formula:

$$n(t) = \beta \cdot h \cdot v(t) \,, \tag{6}$$

where:

$n(t) = \dfrac{da(t)}{dt}$ – is capacity, kW.

At the same time, it should be emphasized that under real conditions water accumulation velocity in the underground mine workings is a stochastic function since at each time moment it depends upon different previously unknown and unpredictable reasons [28]. As a result, energy consumption level, required to provide the drainage process, will be a random function as well. Hence, Formula 6 will take the form:

$$N(t) = \beta \cdot h \cdot V(t) \,, \tag{7}$$

where:

$V(t)$ – random velocity of water accumulation at t time moment, m^3/h;

$N(t)$ – random capacity of energy consumption at t time moment, kW.

Analysis of water accumulation velocity as a stochastic function makes it possible to define its numerical characteristics, i.e. average $\bar{v}(t)$ and dispersion $D(V(t))$. Hence, according to Formula 7, such numerical characteristics of energy consumption capacity as a stochastic function are obtained: average $\bar{n}(t) = \beta \cdot h \cdot \bar{v}(t)$ and dispersion:

$$D\left[N(t)\right] = \beta^2 \cdot h^2 \cdot D\left[V(t)\right]. \tag{8}$$

The results help involve stochastic nature of changes in water accumulation velocity in the underground mine collector while calculating energy consumption capacity for water pumping out from the specified depth of underground mine level. Consequently, characteristic of energy consumption capacity for drainage from the specified depth of underground mine level should involve its average value as well as dispersion.

In practice, questions arise connected with the necessity to calculate the expected amount of water accumulation in the underground mine collector for the given time period; i.e. calculation of an integral of water accumulation velocity as a stochastic function:

$$U(T) = \int_0^T V(t)dt \,, \tag{9}$$

where:

$[0; T]$ – given time interval of water accumulation, hours.

In this context, correlation stochastic function $V(t)$ is required. Specify it as [29]:

$$K_v(t', t'') \,. \tag{10}$$

Availability of integral (9) within the given area of upper boundary T involves availability of a double integral within the same area:

$$\int_0^T \int_0^T K_v(t', t'')\, dt'dt'' \,. \tag{11}$$

While assuming that condition of integral (11) existence is fulfilled, define correlation function of a stochastic Function 10:

$$K_u(t_1, t_2) = M\left[\left(U(t_1) - M\left[U(t_1)\right]\right)\left(U(t_2) - M\left[U(t_2)\right]\right)\right] =$$

$$= M\left[\int_0^{t_1}\int_0^{t_2}\left(V(t') - M\left[V(t')\right]\right)\left(V(t'') - M\left[V(t'')\right]\right)dt'dt''\right],$$

where:

$M[\bullet]$ – mathematical expectation.

While interchanging integration operation and mathematical expectation finding, we obtain:

$$K_u(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} K_v(t',t'')\,dt'dt''. \quad (12)$$

In an individual case when subintegral function in (9) is stationary, Formula 12 looks like [30]:

$$K_u(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} K_v(t''-t')\,dt'dt''. \quad (13)$$

At the same time, the subintegral function dependence upon variable difference helps avoid double integral, i.e. simplify calculations:

$$K_u(t_1,t_2) = \int_0^{t_2}(t_2-\tau)K_v(\tau)d\tau + \int_0^{t_1}(t_1-\tau)K_v(\tau)d\tau - \int_0^{t_2-t_1}(t_2-t_1-\tau)K_v(\tau)d\tau. \quad (14)$$

Among other things, if $t_1 = t_2 = T$ then Formula 14 will define dispersion of integral (9):

$$D[U(T)] = 2\int_0^{T}(T-\tau)K_v(\tau)d\tau, \quad (15)$$

and involving (15), average deviation will be determined using the Formula:

$$\sigma[U(T)] = \sqrt{2}\cdot\sqrt{\int_0^{T}(T-\tau)K_v(\tau)d\tau}. \quad (16)$$

In turn, to integrate the stochastic Function 9 apply operation of mathematical expectation finding for both parts of the Equation:

$$M[U(T)] = M\left[\int_0^{T}V(t)dt\right].$$

Interchange operations of mathematical expectation finding, and integration:

$$M[U(T)] = \int_0^{T}M[V(t)]dt.$$

Then:

$$\bar{u}(T) = \int_0^{T}\bar{v}(t)dt. \quad (17)$$

Mathematical expectation of integral from the stochastic function is equal to integral from mathematical expectation of the function. Hence, consideration of the amount of water accumulation in collectors as a stochastic function (among other things, use of its correlation function) helps identify such numerical characteristics of the accumulated water amount for the given time interval as mathematical expectation and standard deviation.

Conversely, in the practices of iron-ore enterprise, matters dominate constantly connected with the necessity to calculate energy consumption caused by water removal from the specified mine depth for the given period of time, i.e. calculation of integral of the required level of electric capacity of the related engineering systems providing drainage process as a stochastic function:

$$A(T) = \int_0^{T}N(t)dt. \quad (18)$$

If Formulas 7 and 9 are applied then Formula 18 will be as follows:

$$A(T) = \beta\cdot h\cdot U(T).$$

Taking into consideration the abovementioned results, obtained for integral (9), it becomes possible to express numerical characteristics for energy consumption required to pump out water from the specified depth during the given interval as a stochastic function. According to Formula 17, mathematical expectation of energy consumption to pump out water from the specified depth during the given interval will take such form as:

$$\bar{a}(T) = \beta\cdot h\cdot\bar{u}(T). \quad (19)$$

In turn, dispersion of energy consumption amount to provide the drainage during the specified time period will look like:

$$D[A(T)] = 2\beta^2\cdot h^2\cdot\int_0^{T}(T-\tau)K_v(\tau)d\tau, \quad (20)$$

and standard deviation will take the following form:

$$\sigma[A(T)] = \beta\cdot h\cdot\sqrt{2}\sqrt{\int_0^{T}(T-\tau)K_v(\tau)d\tau}. \quad (21)$$

The results make it possible to take into consideration stochastic nature in changes of velocity of water accumulation within mine collectors while calculating energy consumption capacity; water accumulation amount; and energy consumption for drainage from the specified depth during the given interval. Obviously, the characteristics of energy consumption capacity; water accumulation amount; and energy consumption for drainage from the specified depth during the given interval should involve their average values as well as standard deviations describing their dispersion.

If water accumulation velocity within the underground mine level is a normal stochastic process then capacity of energy consumption for water pumping out from one or another level will be a normal stochastic process too which features are determined completely by its average and dispersion [30]. For the purpose, it is quite sufficient to use corresponding normal densities of probability. For example, involving (8), probability density for electric consumption capacity connected with drainage from a mine is as follows:

$$f(n(t)) = \frac{1}{\beta\cdot h\cdot\sqrt{2\pi}\cdot\sigma[V(t)]}e^{-\frac{(n(t)-\beta\cdot h\cdot\bar{v}(t))^2}{2\cdot\beta^2\cdot h^2\cdot D[V(t)]}}. \quad (22)$$

It should be mentioned that stochastic nature of energy consumption capacity for water pumping out from a mine makes it possible to solve "dispersion problems" [31]. If one believes that the analyzed process is a stationary procedure then according to "dispersion problems" the formulas will look like: if average time of a random function is longer than the specified level a during the specified time interval T then dispersion number during the same time period and average dispersion duration are respectively:

$$\bar{t}_a = T\cdot\int_a^{\infty}f(n)dn; \quad (23)$$

$$\bar{m}_a = T\cdot\int_0^{\infty}w\cdot f(a,w)dw; \quad (24)$$

$$\bar{\tau}_a = \frac{\int_a^{\infty} f(n)\,dn}{\int_0^{\infty} w \cdot f(a,w)\,dw}, \tag{25}$$

where:

$w = \frac{dn}{dt}$ – velocity of changes in energy consumption capacity for drainage from the underground mine level, kWh;

$f(n)$ – probability density of distribution of capacity ordinates of energy consumption for drainage;

$f(n, w)$ – two-dimensional probability density of distribution capacity ordinates of energy consumption for drainage and their velocities.

Further, it is expedient to consider average ejection number per a time unit:

$$\bar{\nu}_a = \frac{\bar{m}_a}{T}, \tag{26}$$

or, according to (24):

$$\bar{\nu}_a = \int_0^{\infty} w \cdot f(a,w)\,dw. \tag{27}$$

Relying upon (22), in the context of normal stationary stochastic process, probability density of energy consumption capacity for water pumping out from a mine is as follows:

$$f(n) = \frac{1}{\beta \cdot h \cdot \sqrt{2\pi} \cdot \sigma_n} \cdot e^{-\frac{(n-\bar{n})^2}{2\sigma_n^2}}, \tag{28}$$

where:

$\sigma_n = \beta \cdot h \cdot \sigma[V]$ and $\bar{n} = \beta \cdot h \cdot \bar{v}$.

Capacity ordinate of energy consumption for drainage from a mine, and velocity of its changes are independent values; hence, two-dimensional density of probability of distribution of ordinates of energy consumption capacity for water pumping out and their velocities "break down" into a product of normal probability density of capacity ordinates of energy consumption to perform drainage from the underground mine level, and normal probability density of changes in capacity ordinate of the energy consumption making it possible to write the following:

$$f(n,w) = \frac{1}{\sqrt{2\pi} \cdot \sigma_n} e^{-\frac{(n-\bar{n})^2}{2\sigma_n^2}} \cdot \frac{1}{\sqrt{2\pi} \cdot \sigma_w} e^{-\frac{(w-\bar{w})^2}{2\sigma_w^2}}, \tag{29}$$

where:

σ_w – standard deviation of velocity of changes in ordinate of energy consumption for water pumping out from the underground mine level;

$\bar{w}$ – average velocity of changes in ordinate of energy consumption for water pumping out from the underground mine level.

Velocity dispersion of changes in ordinate of energy consumption for water pumping out from one or another mine depth is defined using the Formula:

$$\sigma_w^2 = \left| \frac{d^2 K_n(\tau)}{d\tau^2} \Big|_{\tau=0} \right|. \tag{30}$$

At the same time, resulting from the stationary nature of a random process, average velocity of changes in ordinate of energy consumption for water pumping out from a mine is equal to zero:

$$\bar{w} = 0. \tag{31}$$

For an average ejection number per a time unit, (29) substitution into (27), involving (31), helps shape the Formula:

$$\bar{\nu}_a = \int_0^{\infty} w \cdot \frac{1}{\sqrt{2\pi} \cdot \sigma_n} e^{-\frac{(a-\bar{n})^2}{2\sigma_n^2}} \cdot \frac{1}{\sqrt{2\pi} \cdot \sigma_w} e^{-\frac{w^2}{2\sigma_w^2}} dw. \tag{32}$$

Factor the constant members out of integral in (32):

$$\bar{\nu}_a = \frac{\sigma_w}{2\pi \cdot \sigma_n} e^{-\frac{(a-\bar{n})^2}{2\sigma_n^2}} \cdot \int_0^{\infty} \frac{w}{\sigma_w} e^{-\frac{w^2}{2\sigma_w^2}} dw. \tag{33}$$

Using a variable substitution, calculate integral in (33):

$$\int_0^{\infty} \frac{w}{\sigma_w} e^{-\frac{w^2}{2\sigma_w^2}} dw = \left| \begin{matrix} x = -\frac{w^2}{2 \cdot \sigma_w^2} & dx = -\frac{w}{\sigma_w^2} dw \\ w = 0 \to x = 0 & \\ w = \infty \to x = -\infty & \end{matrix} \right| = . \tag{34}$$

$$= -\sigma_w \cdot \int_0^{-\infty} e^x dx = \sigma_w \cdot e^x \Big|_{-\infty}^{0} = \sigma_w.$$

Substituting (34) into (33), define finally:

$$\bar{\nu}_a = \frac{\sigma_w}{2\pi \cdot \sigma_n} e^{-\frac{(a-\bar{n})^2}{2\sigma_n^2}}. \tag{35}$$

Identify average ejection period while substituting (35) into (25):

$$\bar{\tau}_a = \pi \frac{\sigma_n}{\sigma_w} e^{\frac{(a-\bar{n})^2}{2\sigma_n^2}} \int_a^{\infty} f(n)\,dn. \tag{36}$$

Taking into account (28), determine:

$$\bar{\tau}_a = \pi \frac{\sigma_n}{\sigma_w} e^{\frac{(a-\bar{n})^2}{2\sigma_n^2}} \int_a^{\infty} \frac{1}{\sqrt{2\pi} \cdot \sigma_n} e^{-\frac{(n-\bar{n})^2}{2\sigma_n^2}} dn. \tag{37}$$

To calculate the integral in (37), apply Laplace function [30]:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt. \tag{38}$$

Change a variable in (37) integral:

$$\int_a^{\infty} \frac{1}{\sqrt{2\pi} \cdot \sigma_n} e^{-\frac{(n-\bar{n})^2}{2\sigma_n^2}} dn = \left| \begin{matrix} x = \frac{n-\bar{n}}{\sigma_n} & dx = \frac{1}{\sigma_n} dn \\ n = a \to x = \frac{a-\bar{n}}{\sigma_n} & \\ n = \infty \to x = \infty & \end{matrix} \right| = \tag{39}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\frac{a-\bar{n}}{\sigma_n}}^{\infty} e^{-\frac{x^2}{2}} dx.$$

Use the integral linearity on the integration interval:

$$\frac{1}{\sqrt{2\pi}}\int_{\frac{a-\bar{n}}{\sigma_n}}^{\infty} e^{-\frac{x^2}{2}}dx = \frac{1}{\sqrt{2\pi}}\int_{\frac{a-\bar{n}}{\sigma_n}}^{0} e^{-\frac{x^2}{2}}dx + \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{x^2}{2}}dx = \quad (40)$$

$$= \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{x^2}{2}}dx - \frac{1}{\sqrt{2\pi}}\int_{0}^{\frac{a-\bar{n}}{\sigma_n}} e^{-\frac{x^2}{2}}dx.$$

According to (40), integral (39) will be written as follows:

$$\int_a^{\infty}\frac{1}{\sqrt{2\pi}\cdot\sigma_n}e^{-\frac{(n-\bar{n})^2}{2\sigma_n^2}}dn = \quad (41)$$

$$= \Phi(\infty) - \Phi\left(\frac{a-\bar{n}}{\sigma_n}\right) = \frac{1}{2} - \Phi\left(\frac{a-\bar{n}}{\sigma_n}\right).$$

Taking into consideration (41), Formula 37 will take the form:

$$\bar{\tau}_a = \pi\frac{\sigma_n}{\sigma_w}e^{-\frac{(a-\bar{n})^2}{2\sigma_n^2}}\left(\frac{1}{2} - \Phi\left(\frac{a-\bar{n}}{\sigma_n}\right)\right). \quad (42)$$

Note that if deviation from an average value is considered, i.e.:

$$a = \bar{n}.$$

Then Formula 42 is simplified to be:

$$\bar{\tau}_a = \frac{\pi}{2}\cdot\frac{\sigma_n}{\sigma_w}. \quad (43)$$

Integration of a stochastic function is reduced to summation of the stochastic function ordinates and further transition to a boundary. On the other hand, a general course of probability theory states that the total of any number of components, forming a system of normal values, produces a normal value. So, it can be argued that an integral of a normal stochastic process is a normal process as well. Moreover, there is a possibility to characterize the procedure completely using mathematical expectation and correlation function since it is specified by means of normal density probability determined totally by means of mathematical expectation as well as standard deviation. Hence, if water accumulation velocity in a mine underground collector is a normal stochastic process then amount of the accumulated water and energy consumption to remove water from the level will also be normal stochastic processes which features can be determined entirely with the help of mathematical expectations and dispersions. For example, in terms of water accumulation amount in the underground collector of one or another mine level for a certain period, probability density is as follows:

$$f\left(u(T)\right) = \frac{1}{\sqrt{2\pi}\sqrt{\int_0^T (T-\tau)K_v(\tau)d\tau}}e^{-\frac{(u-\bar{u}(T))^2}{4\int_0^T (T-\tau)K_v(\tau)d\tau}}. \quad (44)$$

In turn, in terms of energy consumption to pump out water from the underground mine level for a certain period, probability density looks like:

$$f\left(a(T)\right) = \frac{1}{\beta h\sqrt{2\pi}\sqrt{\int_0^T (T-\tau)K_v(\tau)d\tau}}e^{-\frac{(u-\beta\cdot h\cdot\bar{u}(T))^2}{4\beta^2h^2\int_0^T (T-\tau)K_v(\tau)d\tau}}. \quad (45)$$

3. Results and discussion

As an example of drainage simulation as a stochastic process, Figure 3 demonstrates a graph of water accumulation velocity in the operating mine of Kryvyi Rih iron ore basin during 50 hours (the first part of 100-hour implementation) in a collector at the 1000 m depth.

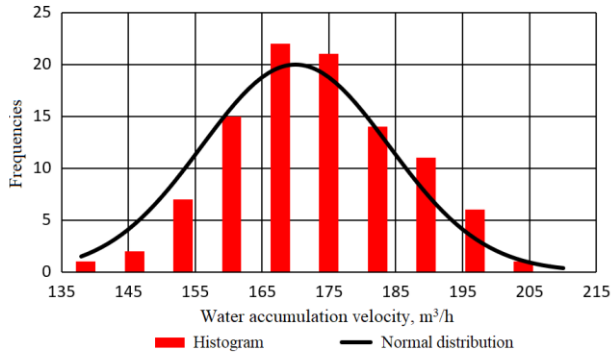

Figure 3. Statistical indicators of water accumulation velocity in the underground collector of Ternivska mine (Kryvyi Rih)

Analysis of the graph, shown in Figure 3, supports the idea that water accumulation velocity is a stochastic process. Average velocity of the procedure for a certain period, being 100 hours, is:

$$\bar{v} = \frac{1}{100}\sum_{i=1}^{100} v_i = 170. \quad (46)$$

In turn, dispersion is equal to:

$$D[V] = \frac{1}{99}\sum_{i=1}^{100}(v_i - \bar{v})^2 = 196. \quad (47)$$

To verify normality of the stochastic process of water accumulation velocity in the analyzed underground mine collector, which depth is 1000 m, apply Pearson's criterion [31][31], [32] or the purpose, corresponding calculation have been performed, shown in Table 2.

To apply Pearson's criterion, statistical data on water accumulation velocity in the underground mine collector should be divided into groups represented in column two. Column three demonstrates middles of the intervals. According to the statistical data and using a Histogram function, being a part of Data analysis complex of Excel spreadsheets, a histogram is developed, which values column four shows. Column five calculates probability of mine water velocity getting for a normal distribution law if average velocity of water accumulation is $\bar{v} = 170$ m^3/h and standard deviation is $\sigma_v = 14$ m^3/h. Column six demonstrates theoretical frequencies of mine water accumulation velocity getting into the specified interval. The last column contains calculation of differences between theoretical and empiric frequencies based upon Pearson's criterion. The total of the last column elements produces the observed value of Pearson's criterion, i.e. $\chi_c^2 = 12.98$.

Table 2. Verification of the stochastic process normality

#	Interval of velocities	Average velocity	Empiric frequencies of getting into interval	Probability of getting into interval	Theoretical frequencies of getting into interval	Difference between theoretical and empirical frequencies
1	138-145	141.5	1	0.03	3	0.98
2	145-152	148.5	2	0.06	6	2.86
3	152-159	155.5	7	0.12	12	1.87
4	159-166	162.5	15	0.17	17	0.27
5	166-173	169.5	22	0.20	20	0.26
6	173-180	176.5	21	0.18	18	0.59
7	180-187	183.5	14	0.13	13	0.17
8	187-194	190.5	11	0.07	7	2.42
9	194-201	197.5	6	0.03	3	3.05
10	201-209	205	1	0.02	2	0.50
Σ			100	1.00	100	12.98

Identify $\chi^2_{кр}(0.05;7)=14.067$ for the number of degrees of freedom $k = 10 - 3 = 7$ and significance level $\alpha = 0.05$ using a table of critical points.

Since $\chi^2_c = 12.98 < \chi^2_{кр}(0.05;7) = 14.067$ then one can say that normal distribution law with $\bar{v} = 170$ and $\sigma_v = 14$ parameters is at 0.95 confidence level.

Figure 4 shows graphs derived while analyzing the statistical data on water accumulation velocity in the underground collector of the operating iron-ore mine.

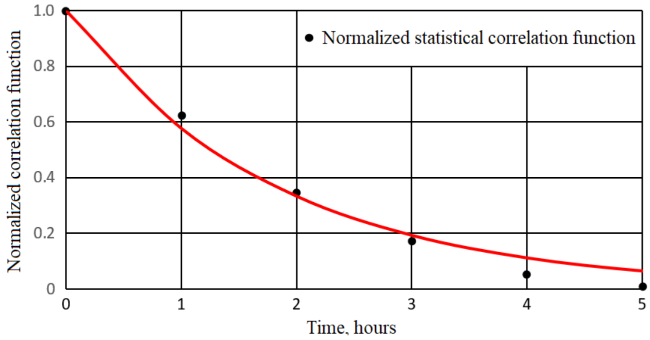

Figure 4. Graph of the normalized statistical correlation function

Analysis of data in Figure 4 shows rather sufficient histogram approximation to the normal distribution curve which confirms the results obtained using Pearson's criterion.

Since water accumulation velocity in the underground mine collector is quite a normal stochastic process then, according to (28), (46), and (47), formula of standard density of energy consumption probability to pump out water from the specified mine depth is as follows:

$$f(n) = 0.0105 \cdot e^{-3.4 \cdot 10^{-4}(n-463)^2}. \tag{48}$$

Formula 48 helps analyze completely energy consumption to pump out water from the specified mine depth as a stochastic process.

Correlation function of the water accumulation velocity is determined with the help of the Formula:

$$K_v(l) = \frac{1}{100-l+1} \sum_{j=0}^{100-l} \left(v_j - \bar{v}\right)\left(v_{j+l} - \bar{v}\right), \tag{49}$$

where:

$l = 0, 1, \ldots, 5$.

Figure 4 represents a graph of the normalized correlation function calculated by means of the Formula:

$$\tilde{K}_v(l) = \frac{K_v(l)}{D[V]}, \tag{50}$$

and the function approximation using the Formula:

$$\tilde{K}_v^a(\tau) = e^{-\alpha \cdot |\tau|}. \tag{51}$$

A method of the least squares, applied to ordinates of the normalized statistical correlation Function 50, has helped identify a parameter value in Formula 51 being:

$$\alpha = 0.55. \tag{52}$$

As a result, Formula 51 takes the form:

$$\tilde{K}_v^a(\tau) = e^{-0.55 \cdot |\tau|}. \tag{53}$$

Analysis shows rather satisfactory approach of the normalized statistical correlation Function 50 as well as its approximation (53). To verify the result, determination index has been calculated on the Formula:

$$R^2 = 1 - \frac{S_a^2}{D\left[\tilde{K}_v\right]}, \tag{54}$$

where:

$$S_a^2 = \frac{1}{5}\sum_{l=0}^{5}\left(\tilde{K}_v(l) - e^{-0.55 \cdot l}\right)^2.$$

The calculation used Formula 54, which involved numerical results, provided the value:

$$R^2 = 1 - \frac{0.0015}{0.1459} = 0.990. \tag{55}$$

The determination index value (55) makes it possible to state that according to Chaddock scale, rather high correlation takes place between variables [31], [32].

Taking into consideration Formula 7, the normalized correlation function of energy consumption to pump out water from the specified mine depth coincides with the normalized correlation function of water accumulation velocity within a corresponding collector, i.e. it is possible to apply matching approximation (53). Consequently, taking into account the fact that dispersion of energy consumption capacity to pump out water from the specified mine depth is:

$$\sigma_n^2 = 1105.6, \tag{56}$$

correlation function of energy consumption capacity to pump out water from the specified mine depth is:

$$K_n^a(\tau) = 1105.6 \cdot e^{-0.55 \cdot \tau}. \quad (57)$$

According to Formula 30, it becomes possible to write:

$$\sigma_w^2 = 334.5. \quad (58)$$

Hence, in accordance with (35), the average of ejection number per unit of time beyond n_0 limit is:

$$\bar{\nu}_{n_0} = 0.0875 \cdot e^{-4.52 \cdot 10^{-4}(n_0 - 457.47)^2}. \quad (59)$$

In turn, as provided by (42), average ejection interval beyond n_0 limit is:

$$\bar{\tau}_{n_0} = 1.728 \cdot e^{-4.52 \cdot 10^{-4}(n_0 - 457.47)^2} \times \times \left(0.5 - \Phi\left(0.03 \cdot (n_0 - 457.47)\right)\right). \quad (60)$$

If $n_0 = \bar{n}$, then, in compliance with (43), average ejection interval will be:

$$\bar{\tau}_{\bar{n}} = 1.728.$$

Apply Formula 17 to identify the accumulated water amount within the specific collector of the analyzed mine depending upon time:

$$\bar{u}(T) = \int_0^T \bar{\nu} dt = \bar{\nu} \cdot T = 167.94 \cdot T, \quad (61)$$

where:

$0 \le T \le 100$.

To determine dispersion of the accumulated water amount, apply Formula 14. Taking into consideration (28), the formula becomes:

$$D\left[U(T)\right] = 2D\left[V\right] \int_0^T (T - \tau) \cdot e^{-0.55 \cdot \tau} d\tau. \quad (62)$$

Calculation of integral in Formula 60 in parts shows the following:

$$\int_0^T (T - \tau) \cdot e^{-0.55 \cdot \tau} d\tau = -\frac{1}{0.55}(T - \tau) e^{-0.55 \cdot \tau} \Big|_0^T - \frac{1}{0.55} \int_0^T e^{-0.55 \cdot \tau} d\tau = = \frac{T}{0.55} + \frac{1}{0.55^2}\left(e^{-0.55 \cdot T} - 1\right),$$

or, after ordering it will take the form:

$$\int_0^T (T - \tau) \cdot e^{-0.55 \cdot \tau} d\tau = \frac{e^{-0.55 \cdot T} + 0.55 \cdot T - 1}{0.55^2}. \quad (63)$$

Involving (63), Formula 62 will look like:

$$D\left[U(T)\right] = 985.12 \cdot \left(e^{-0.55 \cdot T} + 0.55 \cdot T - 1\right), \quad (64)$$

where:

$0 \le T \le 100$.

In turn, standard deviation is identified by means of the Formula:

$$\sigma\left[U(T)\right] = 31.39 \cdot \sqrt{e^{-0.55 \cdot T} + 0.55 \cdot T - 1}, \quad (65)$$

where:

$0 \le T \le 100$.

To calculate average energy consumption for drainage, use Formula 19 which, involving numerical values of the parameters, will take the form:

$$\bar{a}(T) = 456.8 \cdot T, \quad (66)$$

where:

$0 \le T \le 100$.

Dispersion of energy consumption to perform drainage is defined by means of Formula 20 which, involving numerical values of the parameters, will take the form:

$$D\left[A(T)\right] = 7288.34 \cdot \left(e^{-0.55 \cdot T} + 0.55 \cdot T - 1\right). \quad (67)$$

where:

$0 \le T \le 100$.

In turn, standard deviation of energy consumption for drainage will be defined as follows:

$$\sigma\left[A(T)\right] = 85.37 \cdot \sqrt{e^{-0.55 \cdot T} + 0.55 \cdot T - 1}, \quad (68)$$

where:

$0 \le T \le 100$.

If water accumulation velocity in the underground mine collector is a normal stochastic process then water accumulation amount for the specified period should be described by means of following formula of normal probability density:

$$f\left(u(T)\right) = \frac{0.013}{\sqrt{e^{-0.55 \cdot T} + 0.55 \cdot T - 1}} \cdot e^{-\frac{5.1 \cdot 10^{-4}(u - 167.94 \cdot T)^2}{e^{-0.55 \cdot T} + 0.55 \cdot T - 1}}. \quad (69)$$

In turn, formula of normal density of energy consumption capacity to perform drainage from the specified mine depth for the given period is described using the procedure:

$$f\left(a(T)\right) = \frac{4.67 \cdot 10^{-3}}{\sqrt{e^{-0.55 \cdot T} + 0.55 \cdot T - 1}} \cdot e^{-\frac{6.9 \cdot 10^{-5}(a - 456.8 \cdot T)^2}{e^{-0.55 \cdot T} + 0.55 \cdot T - 1}}. \quad (70)$$

Formulas 48, 69 and 70 help analyze sufficient fullness level to identify energy consumption for water removal from underground mine collectors; amount of water accumulation in them; and adequate expenditures connected with the water pumping-out from the specified mine depths for the given period as stochastic processes.

4. Conclusions

The study of water removal process from the underground collectors of iron-ore mines considered as a stochastic process, based upon actual experimental data, and used statistical material on the water accumulation velocity in the underground collector of operating iron-ore mine, has helped develop adequate mathematical model.

The synthesized model has made it possible to identify such stochastic characteristics of drainage process as averages and dispersions of energy consumption capacity for water removal; water amount accumulation in the underground mine collector; and corresponding energy consumption for water pumping out from the specified mine depth for the given period. Moreover, if ordinates of water accumulation velocity in a mine belong to a normal distribution law (i.e. its distribution density is known) then the normal distribution law also covers the model-based capacity of energy consumption to drain; and amounts of water accumulation and its pumping out from the specified mine depth for the given period, i.e. they are characterized as completely random values.

Consequently, the findings help characterize water removal from the underground mine collectors not only using average values (as it has been done before and is being done now) but also applying dispersions, i.e. their distribution. If one knows that ordinates of water accumulation in a mine belong to a normal distribution law then it is possible to characterize drainage as a stochastic process.

Mathematical modelling of water drainage from the underground mine collectors as a stochastic process has made it possible to identify its deeper characteristics connected with a natural randomness factor. The water removal parameters, early developed with the use of only average values, have been complemented by new factors characterizing their dispersion. In addition, certain drainage feature, being determined with the help of normal distribution law of ordinates of water accumulation velocity in the mine collectors, may involve a complete expectation characteristic; and water accumulation amount as well as energy consumption capacity to pump out water from the specified mine depth for the given period as random processes.

The developed methods to analyze water removal process from the underground collectors as a stochastic process in terms of operating iron-ore mine help apply them for other auxiliary activities in the functioning technology of mining enterprises while iron-ore raw material developing, extracting, and processing.

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Математична модель оцінки енергозатрат комплексом "водоприплив-водовідлив" залізорудних шахт у варіанті стохастичності цього процесу

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Мета. Розробка уніфікованої математичної моделі оцінювання енергоефективності процесу щодо "водоприплив-водовідлив" як реального варіанту стохастичності технології функціонування комплексу відкачування вод з підземних виробок залізорудних шахт.

Методика. Процес досліджень базувався на методах теорії ймовірностей та стохастичного моделювання. Інтегрування стохастичної функції зведено до підсумовування її ординат і подальшого переходу до відповідного кордону.

Результати. Побудовано математичну модель комплексу "водоприплив-водовідлив" як стохастичного процесу на прикладі вхідних параметрів типової діючої залізорудної шахти. Це дозволило реально оцінити, обґрунтувати та отримати можливості як для конкретного виробничого об'єкта, так і для узагальнення результатів із визначенням стохастичних характеристик процесу водовідведення.

Наукова новизна. Вперше побудована математична модель водовідведення з підземних горизонтів залізорудних шахт як стохастичного процесу. Визначені характеристики цього процесу, шо ґрунтуються на випадковості протікання технології водовідведення. На відміну від існуючих, запропоновані нові параметри моделі, які характеризують їх розкид. Доведена можливість отримання повної характеристики енергозатрат: на водовідведення; обсягів накопичення води в підземних водозбірниках; на водовідлив з заданих глибин шахт за визначений час як випадкових процесів. Досліджено і диференційовано ряд особливостей водовідведення, що визначаються нормальним законом розподілу ординат швидкості накопичення води в підземних водозбірниках залізорудних шахт.

Практична значимість. Побудовано узагальнену математичну модель водовідведення як стохастичного процесу проведені з використанням статистичних даних щодо швидкості накопичення води в підземних виробках на прикладі діючої залізорудної шахти. Доведено, що, якщо ординати швидкості накопичення води в підземних водозбірниках шахт мають нормальний закон розподілу, то доцільно повністю характеризувати водовідведення як стохастичний процес. Розроблена методика дослідження водовідведення як стохастичного процесу дозволяє розширити кордони досліджень поширивши її на інші допоміжні роботи, які виконуються в процесі технології видобутку залізорудної сировини підземним способом.

Ключові слова: *ефективність, методологія, модель, шахта, водовідлив, стохастичний процес*