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Maximum Transportation Growth in Energy and Solute Particles in Prandtl Martial across a Vertical 3D-Heated Surface: Simulations Achieved using by Finite Element Approach

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Abstract

The goal of this study is to determine the maximum energy and solute particles' transportation growth in a 3D-heated region of Prandtl martial through a dynamic magnetic field. The effects of this field on the properties of solvent molecules and heat conduction are studied. A correctly stated functional method and a finite element approach are comparable to a certain type of differential equations. In order demonstrate the effects of various factors such as mass diffusion, heat generation, and thermal diffusivity on the investigation of the diffusion coefficient and thermal mass in a three-dimensional Newtonian flow, the study of viscous and heat conduction rates is presented. The results show that the comparisons of hybrid nanofluid Fe_2O_3 and Al_2O_3 with base fluid and w.r.t Local skin friction coefficient, Nusselt number and Sherwood number.

Keywords: *Galerkin finite element method, 3-D heated surface, Hybrid nanoparticles, Argumentation into heat energy, Variable magnetic field.*

1. Introduction

The creation of solid nanoparticles is now possible because to technological developments. Many cutting-edge engineering applications utilize these nanoparticles. The field of fluid mechanics has a various fields whish use in different scientific approaches. Basically, applications of nanoparticles use for heat absorber many scientists are already studies on the applications of nanoparticles in heat transfer such as [1-7]. Here we study the different types of nanoparticles in heated surface. In this regard, Devi and Devi [8] showed the three-dimensional

(3D) flow behavior of copper and aluminum-oxide hybrid nanoparticles via sheet under the effect of ohmic heating and Lorentz force.

The Carreau Yasuda thermal and mechanical characteristics were taken into consideration while comparing hybrid nanoparticles and nanomaterials in base liquid. Hafeez et al. [9] observed the influence of Williamson liquid on the thermal energy and percentage of nano fluids that were about to melt surfaces. Dogonchi et al. [10] examined the efficacy of fluid heat production in their study of the impact of thermal radiation created by nanoparticles on fluid in the presence parallel surfaces. According to research by Chamkha et al. [11], magnetic fields, spinning barriers, and hybrid nanoparticles all have an impact on the quantity of heat that may be transmitted. By Masayebidarched et al. [12], a theoretical study utilizing hybrid nanoparticles to examine temperature increases in fluid was accomplished. References [13–15] are instances of related articles that investigate how hybrid nanoparticles affect heat generation. Many researchers have studied [16–22] the heat increase of nanofluids by combining various types of nanoparticles with the base liquid. In these investigations, the impacts of genuine variables also on heat rise of nanofluids are investigated, including the joule temperature increase, forces acting, and magnetic influence. It is advised that researchers focus on these most current specialists because they also noticed many mathematical effects, the medium's porosity, and the expanding and contracting of the plates in the no-slip effect. A Casson hybrid nanofluid's thermal properties and the impacts of Dufour and Soret under the impact of the solute's process were investigated in [23]. We saw how nanofluid applications in an automobile radiator improved heat transmission. Analysis of considerable thermodynamic energy production in the partly ionization of hyperbolic tangent material based on ternary hybrid is discovered in [24].

Das et al. [25] investigation of the slip phenomena on MHD boundary layer flow of nanofluid over a vertically stretched sheet addressed non-uniform heat flux effects. Haq et al. [26] investigated at the slip condition over the maxwell fluid flow in the presence of heat transfer. Awais et al. [27] examined the impact of velocity, temperature, and concentration slip on Magnetohydrodynamic nanofluid. Raza et al. study [28] investigated the effects of heat radiation on Casson fluid stagnation point flow under slip circumstances. The entropy production on Peristaltic transport flow for two phases in the existence of slip condition was examined by Ali et al. [29]. Vinita and Poply [30] share their investigation on the slip effects on heat transmission of nanofluid over a stretching cylinder.

The purpose of this investigation is to improve heat transfer by including hybrid nanoparticles into the Prandtl fluid model. Studies in the public domain confirm that no one has undertaken this study to date. This investigation will serve as a starting point for the researchers as they further investigate various elements and analyze a wide range of outcomes. Many energy systems and industries can benefit from this research. A select few major determinants are [31-38]. Due to its extensive uses in industry and many energy systems, energy transfer is a crucial component of the modern period and a hot study field. The base fluid mixture can be blended with the nanoparticles to increase thermal transportation. A number of empirical relationships between substances have been proposed as a result of their properties. When simulating this model in various scenarios with various impacts, researchers pay close attention. Ternary hybrid nanomaterial is used in engineering, cancer treatment, hair care, electrical contacts, ecological tires, dental products, hydrogen fuel, solar energy, optical chemical sensors, bio-sensors, and automotive parts are briefly described in references [39-44].

For analyzing PDEs in two or three spatial variables, the Finite element method (FEM) is a popular simple mathematical methodology described from [45-50]. This technique is widely used in all mathematical demonstrating systems, particularly in the heat and mass transfer systems. It first performed in Ahmad et. al [51] study, which used FEM to simulate NF movement and heat arenas inside a motivated geometry filled by an appliance that produces heat. Hiba et al. [52] used the comprehensive FEM to enhance HNFs. For allied MHD NFs movement, Ali et al. [53] employed a FEM technique to quantify the melting in NFs that are not Newtonian. fluence on HT kinds. Using FEM, Abderrahmane et al. [54] found the best explanation NFs that are not Newtonian. FEM was used by Rana and Gupta [55] to create a result enables HNFs to travel actively and with quadratic convection across a spinning pinecone. By using FEM, Pasha and Domiri-Ganji [56] investigated the effect of HNFs on the widening shallow of Chamfer flippers. By assuming generalized FEM, Redouane et al. [57] investigated the thermal movement flood of hybrid NFs in animated inclusions with switch cylindrical cavities. Alrowaili et al. [58] used FEM to show that NFs convect in a magnetic radioactive single-minded manner. A dynamic system of NFs was described using a homogenization approach and FEM by Zaaroura et al. [59]. Ahmed and Alhazmi [60] used FEM modeling to modify the rotation and different heat conditions of rolls filled with glass spheres when radioactivity was present. Muhammad et al. [61] employed FDM analysis for squeezed flow of HNFs in presence of Cattaneo-Christov heat flux and convective boundary condition. Li et al. [62] developed multi-scale LBM-FDM analysis on HT.

Numerous industrial applications exist for the Prandtl nanofluid past stretched sheet, such as materials sciences, coating and suspend, metal component cooling, heat transfer technique, and so on. Examples of aerodynamic extrusion of materials include the production of paper, the transfer of high - temperature materials between supply and wind-up rollers, and the cooling of an endless vertical surface in a liquid solution.

As a result, this scientific article is divided into five Sections that present many potential answers. Section 2 presents the formulation of the issue. In Section 3, the numerical approach is briefly explained. Section 4 presents the discussion of the result and Section 5 is based on core points of the article.

2. Mathematical Formulation:

In the present study we investigate the maximum transportation growth in energy and solute particles in Prandtl martial across a vertical 3D-heated surface Under the influence of a dynamic magnetic field, solvent molecules are introduced in the direction of the heated area with the features of heat conduction. The combination of Al_2O_3 andFe_2O_3 is referred to as a hybrid nanostructure, and Al_2O_3 is referred to as a nanoparticle. Examples of Al_2O_3 andFe_2O_3 thermal properties are shown in Table 1.

2.1 Research hypothesis for the current model

The following criteria must be applicable to the stream structure, including the requirements:

3-D laminar steady flow,	model of phase flow,
Hybrid Prandtl-Eyring nanofluids,	permeable medium,
MHD,	heat source
variable thermal conductivity,	viscous dissipation.

2.2 Flow geometry

The overall system is shown in Fig. 1. The magnetic field was found to be injected along the $y-axis$, with the $x-axis$ presumed to be vertical and the y-axis considered to be horizontal.

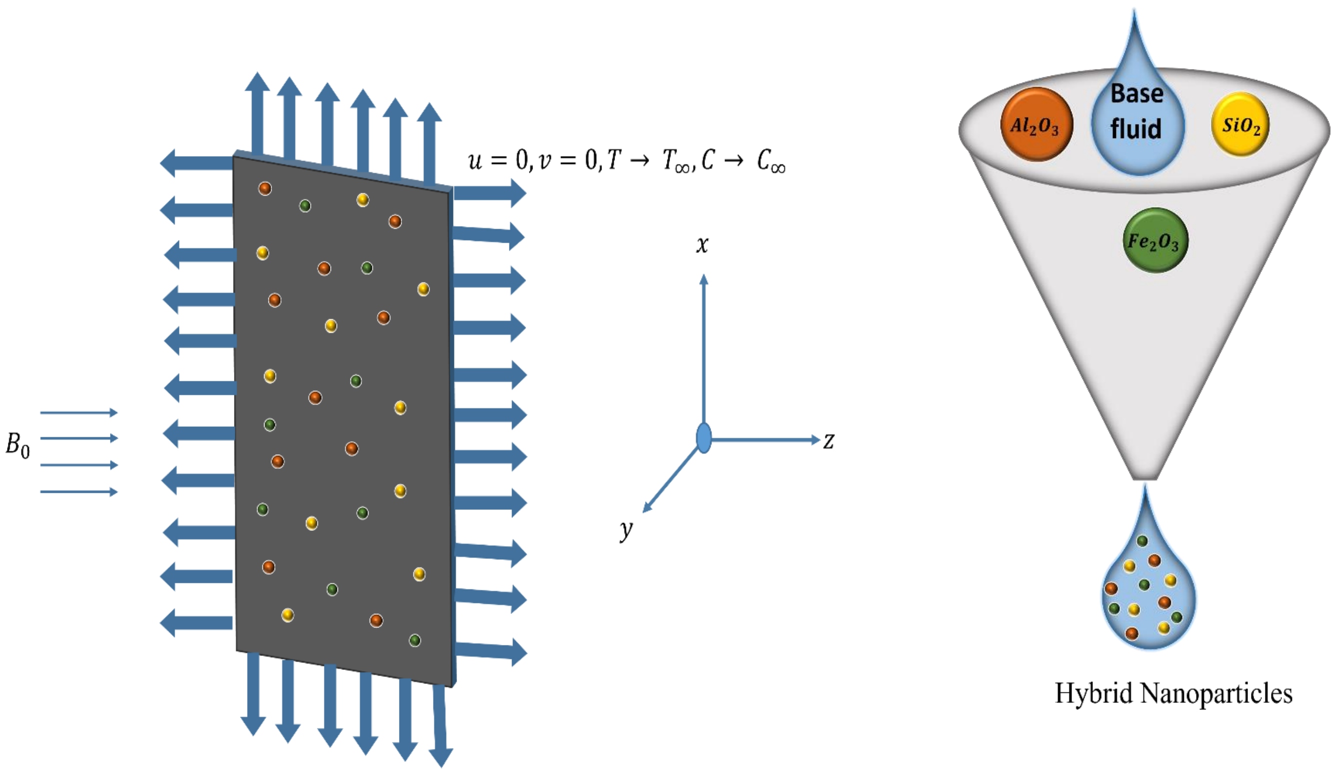

Fig. 1. 3D vertical surface.

2.3 Prandtl-Eyring Fluid Stress Tensor

The following mathematical formula represents the Prandtl-Eyring fluid stress tensor: (see, for example, Mekheimer and Ramadan [63]).

$$\tau = \frac{A_w \ \mathrm{Sin}^{-1}\left[\frac{1}{C_1}\left[(u_y)^2+(u_y)^2\right]^{\frac{1}{2}}\right]}{\left[(u_y)^2+(u_y)^2\right]^{\frac{1}{2}}}(u_y). \tag{1}$$

The curved velocity in this case reveals the mechanics $\overleftarrow{V} = [\mathrm{u}(x,y,z), \mathrm{v}(x,y,z), \mathrm{w}(x,y,z)]$. A_w and C are fluid parameters.

PDEs that describe the problem are as follows:

$$u_x + v_y + w_z = 0, \tag{1}$$

$$uu_x + vu_y = g^*(T - T_\infty) + (\beta_{hnf})_C g^*(C - C_\infty) - \frac{\sigma_{nf}B_0^2}{\rho_{nf}}u - \mu_{hnf}\frac{u}{K_1} + \nu_{nf}u_{zz} - wu_z, \tag{2}$$

$$uv_x + vv_y = \nu_{hnf} v_{zz} + (\beta_{hnf})_T g^*(T - T_\infty) + (\beta_{hnf})_C g^*(C - C_\infty) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} v - \mu_{hnf} \frac{v}{K_1} + \nu_{nf} v_{zz} - w v_z, \tag{3}$$

$$uT_x + vT_y = \frac{1}{(\rho c_p)_{Thnf}} \frac{\partial}{\partial z}\left(K_{Thnf}(T) T_z\right) + \frac{Q_0}{(\rho c_p)_{hnf}}(T - T_\infty) + \frac{DK_T}{C_s C_p} \frac{\partial^2 C}{\partial z^2} + + \frac{\mu}{(\rho c_p)_f}(u^2 + v^2) - wT_z, \tag{4}$$

$$uC_x + vC_y = \frac{\partial}{\partial z}\left(D_{hnf} T_{z'}\right) + \frac{D_T}{T_\infty} T_{zz'} - wC_{z'} \tag{5}$$

System of Eqs. 1-5 BCs are [9];

$$\left.\begin{array}{c} u = U_w, v = V_w, w = 0 \\ T = T_w, C = C_w \quad as\ y = 0 \\ u = 0, v = 0, T \to T_\infty, C \to C_\infty \qquad as\ y \to \infty. \end{array}\right\} \tag{6}$$

In ethylene glycol, hybrid nanostructures and nanomaterials are related [45]

$$\left.\begin{array}{c} \rho_{hnf} = \left[(1-\phi_2)\{(1-\phi_1)\rho_f + \phi_1 \rho_{s_1}\}\right] + \phi_2 \rho_{s_2}, \rho_{nf} = (1-\phi)\rho_f + \phi \rho_s \\ (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_{s'} \\ (\rho C_p)_{hnf} = \left[(1-\phi_2)\left\{(1-\phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_{s_1}\right\}\right] + \phi_1(\rho C_p)_{s2} \end{array}\right\} \tag{7}$$

$$\left.\begin{array}{c} \frac{k_{nf}}{k_f} = \left\{\frac{k_s + (n+1)k_f - (n-1)\phi(k_f - k_s)}{k_s + (n-1)k_f + \phi(k_f - k_s)}\right\}, \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \\ \mu_{hnf} = \frac{\mu_f}{(1-\phi_2)^{2.5}(1-\phi_1)^{2.5}}, \frac{\sigma_{hnf}}{\sigma_f} = \left(1 + \frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right) \end{array}\right\} \tag{8}$$

$$\left.\begin{array}{c} \frac{k_{hnf}}{k_{bf}} = \left\{\frac{k_{s_2} + (n-1)k_{bf} - (n-1)\phi_2(k_{bf} - k_{s_2})}{k_{s_2} + (n-1)k_{bf} - \phi_2(k_{bf} - k_{s_2})}\right\} \\ \frac{\sigma_{hnf}}{\sigma_f} = \left(\frac{\sigma_{s_2} + 2\sigma_f - 2\phi_2(\sigma_{bf} - \sigma_{s_2})}{\sigma_{s_2} + 2\sigma_f + \phi_2(\sigma_{bf} - \sigma_{s_2})}\right) \end{array}\right\}. \tag{9}$$

The definitions of heat capacity and mass transfer in relation to temperature are

$$K_{hnf}(T) = K_{hnf}\left(1 + \epsilon_1 \frac{T - T_\infty}{T_w - T_\infty}\right), D_{hnf}(T) = K_{hnf}\left(1 + \epsilon_2 \frac{T - T_\infty}{T_w - T_\infty}\right). \tag{10}$$

Similarity transformation is,

$$\left.\begin{array}{c} u = a(x+y)^{\frac{1}{3}}, v = a(x+y)^{\frac{1}{3}}, \eta = \sqrt{\frac{a}{\nu_f}}(x+y)^{-\frac{1}{3}} z, \\ w = -\sqrt{a\nu_f}\,(x+y)^{-\frac{1}{3}}\left(\frac{2}{3}(f+g) - \frac{1}{3}\eta(F+G)\right), \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty} \end{array}\right\} \tag{11}$$

In Eqs. $(1) - (6)$, similarity transformation utilized, we have

$$\left.\begin{array}{c}\frac{\nu_{hnf}}{\nu_f}\left(\alpha_1 F''' + \alpha_2 F''^2 F'''\right) + \frac{2}{3}(F+G)F'' - 2F'(F'+G') + +4\lambda G' + (Gr)_t\theta \\ +(Gr)_c\phi - \left(\frac{\sigma_{hnf}}{\sigma_f}\right)\left(\frac{\rho_f}{\rho_{hnf}}\right)Mf' - \left(\frac{\mu_{hnf}}{\mu_f}\right)K^*f' = 0 \\ f'(0)=1,\;\; f(0)=0,\;\; f'(\infty)\to 0.\end{array}\right\} \quad (12)$$

$$\left.\begin{array}{c}\frac{\nu_{hnf}}{\nu_f}\left(\alpha_1 G''' + \alpha_2 G''^2 G'''\right) + \frac{2}{3}(F+G)G'' - 2G'(F'+G') + +4\lambda G' + (Gr)_t\theta \\ +(Gr)_c\phi - \left(\frac{\sigma_{hnf}}{\sigma_f}\right)\left(\frac{\rho_f}{\rho_{hnf}}\right)Mg' - \left(\frac{\mu_{hnf}}{\mu_f}\right)K^*g' = 0 \\ g'(0)=\beta,\;\; g(0)=0,\;\; g'(\infty)\to 0.\end{array}\right\} \quad (13)$$

$$\left.\begin{array}{c}\frac{K_{hnf}}{K_f}[(1+\epsilon_1\theta)\theta'' + \epsilon_1(\theta')^2] + \left(\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}\right)\frac{2}{3}Pr(F+G)\theta' - \left(\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}\right)\frac{2}{3}Pr(F'+G')\theta \\ -Pr\beta^*\theta + \left(\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}\right)DuPr\phi'' + \left(\frac{\sigma_{hnf}}{\sigma_f}\right)MPrEc(F'+G')^2 = 0 \\ \theta(0)=1,\;\; \theta(\infty)\to 0\end{array}\right\}, \quad (14)$$

$$\left.\begin{array}{c}\frac{D_{hnf}}{D_f}[(1+\epsilon_1\varphi)\varphi'' + \epsilon_2\varphi'\theta'] + \frac{2}{3}Sc(F+G)\phi' - \frac{2}{3}Sc(F'+G')\phi + SrSc\theta'' = 0 \\ \phi(0)=1, \phi(\infty)\to 0.\end{array}\right\} \quad (15)$$

Dimensionless numbers are:

$$\left.\begin{array}{l}(Gr)_t = \frac{(\beta_{hnf})_T g^* cT_0}{a^2},\ (Gr)_c = \frac{(\beta_{hnf})_C g^* dC_0}{a^2},\ M = \frac{\sigma_f}{\rho_f}\frac{B_0{}^2A^2}{a},\ K^* = \frac{\mu_f}{ak_1}, \\ Ec = \frac{1}{(c_p)_f}\frac{a^2}{cT_0},\ \beta^* = \frac{Q_0}{a(\rho c_p)_f},\ Du = \frac{DK_T dC_0}{C_s C_p V_f cT_0}, Sc = \frac{V_f}{d_f},\ Sr = \frac{D_T T_0}{(T_\infty C_0)V_f}.\end{array}\right\} \quad (16)$$

Table 1 explains the group of variables that were employed in this inquiry to achieve its practical goals.

Table 1. Thermal components of SiO_2, Al_2O_3 and Fe_2O_2.

Fe_2O_3	Al_2O_3	SiO_2
$\rho = 999$	$\rho = 6490$	$\rho = 2213.5$
$C_p = 766$	$C_p = 679$	$C_p = 1989$
$k = 0.155$	$k = 33.01$	$k = 1.5034$
$\beta = 2.8424 \times 10^{-5}$	$\beta = 6.09 \times 10^{-5}$	$\beta = 5.8 \times 10^{-4}$
$\sigma = 0.149 \times 10^{-11}$	$\sigma = 6.30 \times 10^{7}$	$\sigma = 5.4 \times 10^{6}$

In terms of surface-based forces,

$$C_{fx} = \frac{u_z|_{z=0}}{\rho_f(U_w)^2} = \frac{(1-\phi_1)^{-2.5}}{(1-\phi_2)^{2.5}(Re)^{1.5}}[\alpha_1 f''(0) + \alpha_2(f'''(0))^3],$$

(17)

$$C_{gy}=\frac{\left.\frac{\partial v}{\partial z}\right|_{z=0}}{\rho_f(U_w)^2}=\frac{(1-\phi_1)^{-2.5}}{(1-\phi_2)^{-2.5}(Re)^{1.5}}[\alpha_1 g''(0)+\alpha_2(g'''(0))^3]. \tag{18}$$

Nusselt number is

$$Nu=-\frac{(x+y)K_{hnf}T_z\big|_{y=0}}{k_f(T-T_\infty)}=-\frac{K_{hnf}}{k_f\,(Re)^{1.5}}\theta'(0),$$

Mass diffusion is

$$Sh=\frac{(x+y)D_{hnf}C_z\big|_{y=0}}{D_f(C-C_\infty)}=-\frac{D_{hnf}}{D_f(Re)^{1.5}}\phi'(0). \tag{19}$$

where $Re=\frac{xU_w}{\nu_f}$, the Reynolds number.

3. Numerical Procedure:

The Galerkin finite element method is used to resolve the given issue. Here [27-31] is a collection of the FEMs that describe the approach. The FEM capacity to manage solution domains with arbitrary geometry is its most major practical benefit. The fact that the governing differential equations are converted into a set of algebraic equations in a theoretically correct manner without the use of heuristics or numerical approximations is another significant benefit. The finite element method is additionally supported by a solid theoretical framework. It may be demonstrated that the finite element approach and a correctly stated functional minimization method are equivalent for a particular class of differential equations. The list below includes several finite element method restrictions.

1. Compared to other numerical methods, analysis of finite elements is thought to be more difficult to comprehend;
2. Comparing the computational cost of the finite element approach to other methods, it can be expensive;
3. Large amounts of data are required for mesh-free analysis.
4. The residual equations have been constructed.
5. When the discretization integrals are produced by the Galerkin technique, stiffness matrices are produced.

6. The nonlinear system's equations are converted to linear ones by following to the limits of element assembly. The linearized system is solved 10^{-3} under the calculation.

The FEM's flowchart is described in Fig. 2.

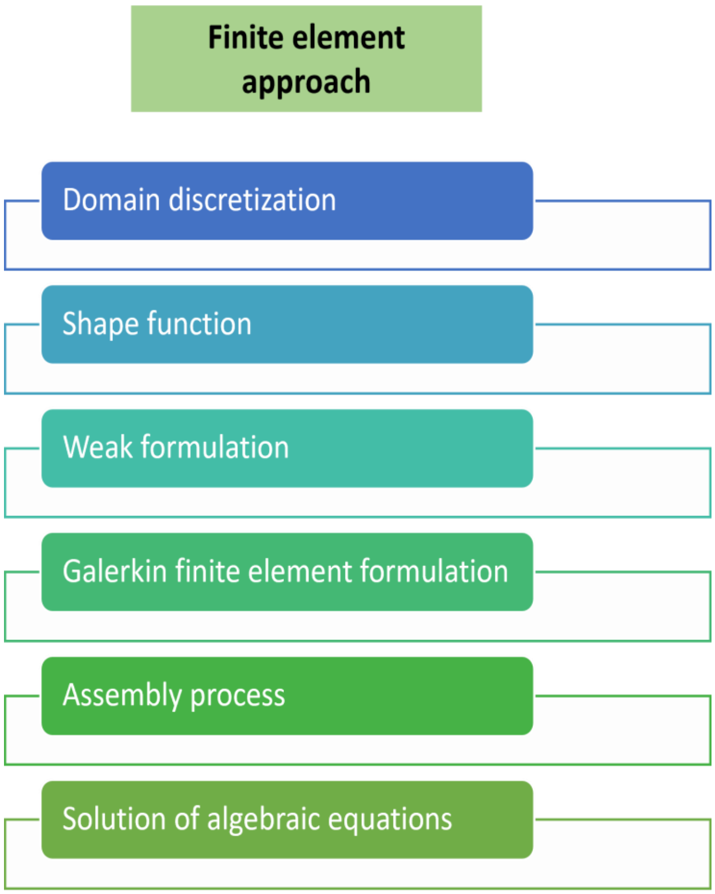

Fig. 2. Flow chart of finite element method.

As soon as the confluence has been confirmed, results that are grid independent are obtained. It makes use of the error analysis criterion. In order to illustrate the effects of heat flux, diffusion, porous media, and Examples of parametric study are presented for the rate of viscous dissipation and Heat conduction on investigation of thermal mass and momentum transfer in three-dimensional Newtonian fluid flow. The investigation of 300 pieces without mesh is shown in Table 3 along with the results.

Table 3 displays a study of temperatures, velocities, and concentrations assuming 300 grid elements.

e	$F'\left(\frac{\eta_{max}}{2}\right)$	$G'\left(\frac{\eta_{max}}{2}\right)$	$\theta\left(\frac{\eta_{max}}{2}\right)$	$\phi\left(\frac{\eta_{max}}{2}\right)$
30	0.7840956617	0.003662478537	0.0036624785	0.00010687187
60	0.8208393123	0.09000953164	0.1110267939	0.00506883934
90	0.8299235799	0.002650512986	0.01342155803	0.00005691660
120	0.6909729670	0.0004285360556	0.01039153713	0.04544824477
150	0.6949838844	0.0004160495348	0.01033124362	0.04500018680
180	0.6979030185	0.0004100790678	0.01029442761	0.04477827150
210	0.7002801242	0.0004086029651	0.01027192133	0.04470747392
240	0.7023798401	0.0004105345687	0.01025902553	0.04474784442
270	0.7043301851	0.0004151139823	0.01025290562	0.04487205678
300	0.7061806472	0.0004216405675	0.01025152760	0.04505585937

Table 4. Studying the temperature gradient in comparison to data reported by Qureshi et al. [64] for nanofluid

Qureshi et.al [64]	Present results
Nusselt number	Nusselt number
0.62	0.6768954
0.72141	0.7233317
0.82458	0.8247624

3. Results and discussion

An Analytical investigation has been given to look at the physics of the problem mentioned in the previous section. Finite element method is already used in many heat transfer problems to generate a numerical solution [65-68]. analysis of the temperature gradient in comparison to published Solving such issues requires the application of the mathematical model FEM for thermal energy and mass flow of non-Newtonian flows over a material with thermal and boundary concentrations variations. Since the quality that keeps flow from deforming until a particular scientific stress is reached, applied stress. analysis of the temperature gradient in comparison to published for the fluid to endure the produced strain and attain equilibrium, the maximum stress must increase. temperature gradient comparison with reported The The velocity profile as a result (in both the $\boldsymbol{x}$, $\boldsymbol{y}$ *and* $\boldsymbol{z}$ components) is seen to decrease. Fig.3 to Fig. 5 shows the velocity profile with different parameters with different values. Contrary to increased influences of fluid parameter, fluid is shown to become thin. Diverse samples of variable of values are used in various numerical investigations. The parameter $\boldsymbol{k}^*$ is also shown in Figs. 3 and 8 which is shows the effect of porous parameter with different impacts. The Lorentz force and the magnetic field are intimately linked. The change of $\boldsymbol{M}$ can be used to assess how the Lorentz force affects flow. With rising values of $\boldsymbol{M}$ and porous medium, the Lorentz force's negative effects become more pronounced. As seen in Figs. 7 and 8, the Lorentz force causes the flow to slow down as a result. To reduce the thickness of the boundary layer, a change in the magnetic field is used. It is also claimed that the Lorentz force for the flow of $\boldsymbol{Fe_2O_3} - \boldsymbol{Al_2O_3}$ nanofluid is larger than the Lorentz force for the flow of $\boldsymbol{Fe_2O_3}$ nanofluid. The interactions between $\boldsymbol{M}, \boldsymbol{Ec}\ \boldsymbol{Pr}$, $\boldsymbol{Du}$ **and** $\boldsymbol{Gr}$ by thermal energy is investigated for alumina-ferrofluids hybrid nanofluid. From Figs. 9–13 show how various parameters have been observed to have an impact. A reference to the input variable $\boldsymbol{Du}$ is the Dufour number. When the heat energy transcript then it shows up in the heat equation's dimensionless form. It examines how the variations brought on by dispersed soluble chemicals and nanoparticles cause changes in the fluid's thermal heat transfer. The temperature of $\boldsymbol{Fe_2O_3}$-nanofluid and $\boldsymbol{Fe_2O_3} - \boldsymbol{Al_2O_3}$-hybrid nanofluid is affected by $\boldsymbol{Du}$, as shown in Fig. 9. The temperature of both kinds of fluids tends to increase as a function of $\boldsymbol{Du}$. Both types of fluids' temperatures tend to increase as a function of $\boldsymbol{Du}$. $\boldsymbol{Fe_2O_3}$-temperature hybrid nanofluids is more sensitive to $\boldsymbol{Du}$ than $\boldsymbol{Fe_2O_3} - \boldsymbol{Al_2O_3}$-temperature nanofluid's. The impact of liquid particles on a fluid temperature copper nanofluid and $\boldsymbol{Fe_2O_3} - \boldsymbol{Al_2O_3}$ −nanofluid are shown in Fig. 10. As a result, Fig. 11 shows the temperature profile with impact of magnetic field. In this fig it clearly shows that when we increasing the values of Magnetic field the temperature profile will be increasing. We can also observer that the comparison of hybrid nano fluids and simple nanofluids. The parameter $\boldsymbol{\epsilon_1}$ and $\boldsymbol{Ec}$ also shows the results from temperature profile. When we increase the value the parameter $\boldsymbol{\epsilon_1}$ and $\boldsymbol{Ec}$ the temperature is also increasing. This observation is supported by the data in Fig. 12 and Fig.13. Additionally, Fluid velocity significantly raises the temperature of the fluids ($\boldsymbol{Fe_2O_3} - \boldsymbol{Al_2O_3}$ – nanofluid) in these figs. The parameters $\boldsymbol{Sr}$, $(\boldsymbol{Gr})_{\boldsymbol{c}}$, and $\boldsymbol{Sc}$, respectively, Analyze the effects of temperature gradient, buoyancy force caused by concentration differences, and diffusion coefficient on the concentration field. Fig. 14 and Fig. 15 show how their impact on concentrations profile with $\boldsymbol{Sr}$ and $\boldsymbol{Sc}$. It shows that when $\boldsymbol{Sc}$ and $\boldsymbol{Sr}$ increases, the concentration field shrinks. For both the $\boldsymbol{Fe_2O_3} - \boldsymbol{Al_2O_3}$-fluid the wall stress data in numbers in the $\boldsymbol{x}$ and $\boldsymbol{y}$

directions, wall rate of heat transfer, and wall mass flux are examined in Table 4. A summary of the numerical findings is shown in Table 5. The number of vacancies in the porous material seem to have a negative correlation with the $\boldsymbol{k}^*$. Thus, the stress per unit area rises as a result. As a result, wall shear strains in both the $\boldsymbol{x}$ and $\boldsymbol{y}$ axes are rising functions of $\boldsymbol{k}^*$. The difference in temperature and the mass-flux are both descending functions of $\boldsymbol{k}^*$. In addition, it has been discovered that raising $\boldsymbol{Du}$ raises wall shear stress. The wall mass transfer coefficient against $\boldsymbol{Du}$, however, is seen to increase. $\boldsymbol{Sr}$ is the factor that ultimately determines how hot or cold solute particles are, and an increase in $\boldsymbol{Sr}$ causes the wall shear stress to decrease. $\boldsymbol{Sc}$ clearly exhibits the opposite tendency.

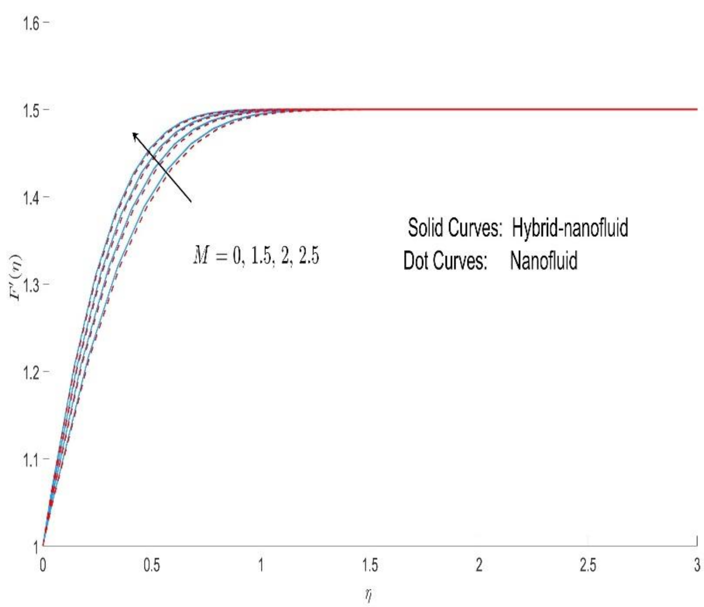

Fig. 3. Influence of M on F'

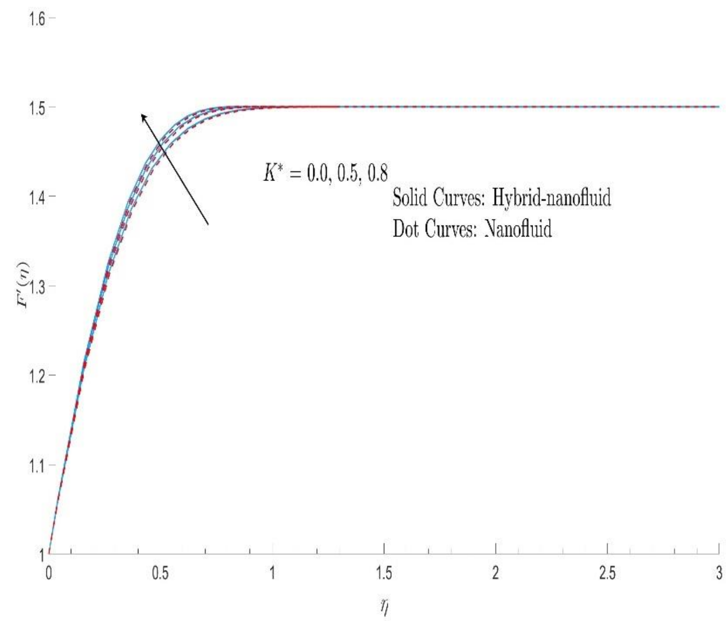

Fig. 4. Influence of K^* on F'.

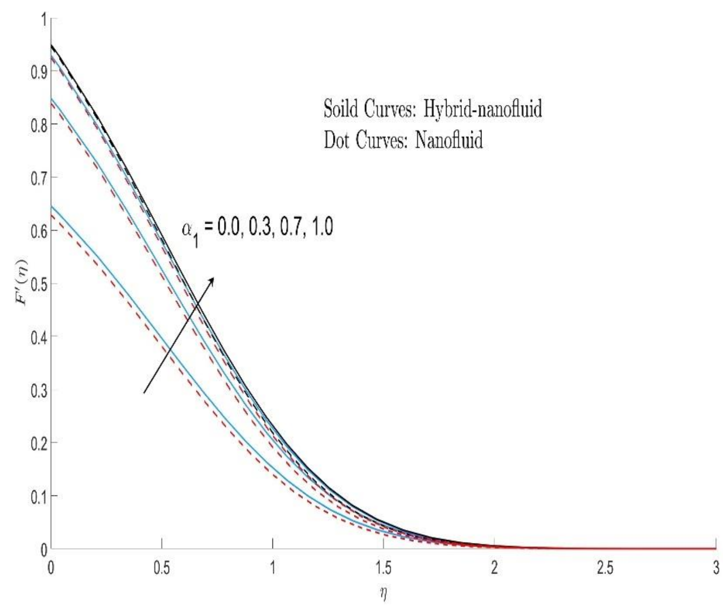

Fig. 5. Influence of α_1 on F'.

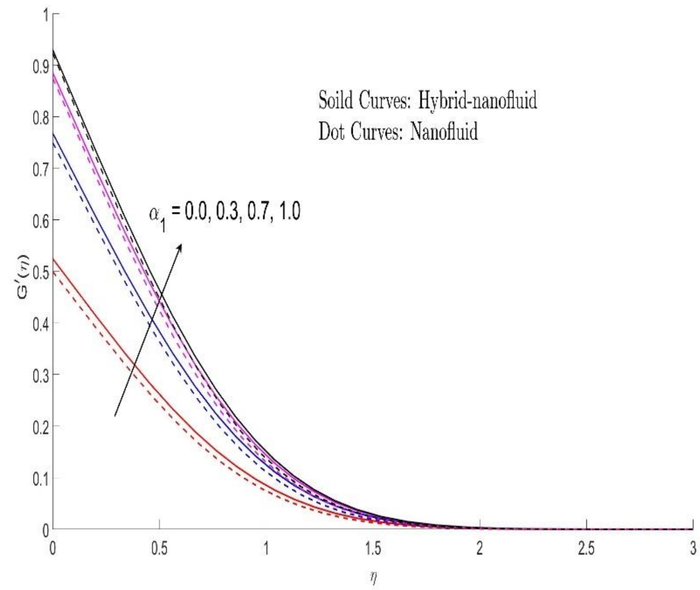

Fig. 6. Influence of α_1 on G'.

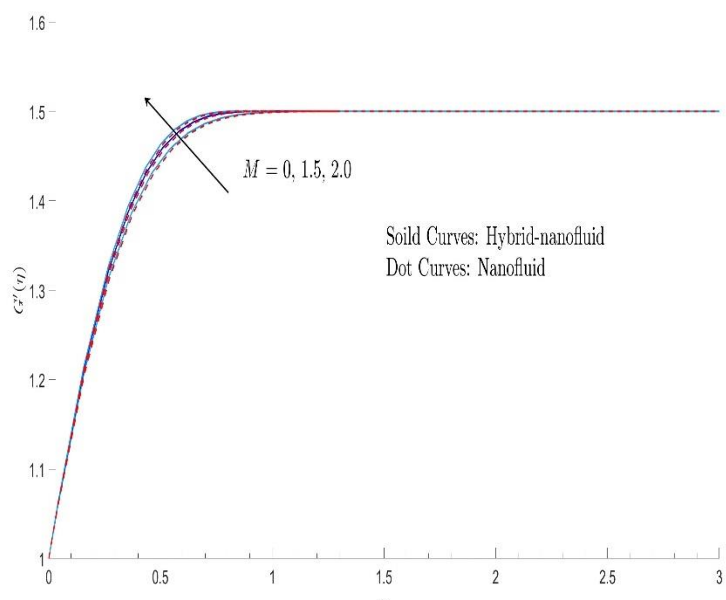

Fig. 7. Influence of M on $G.'$

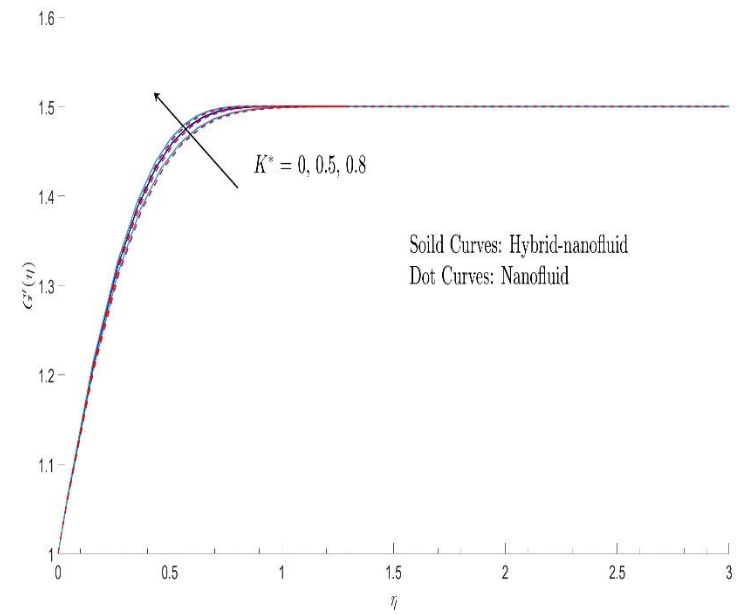

Fig. 8. Influence of K^* on G'.

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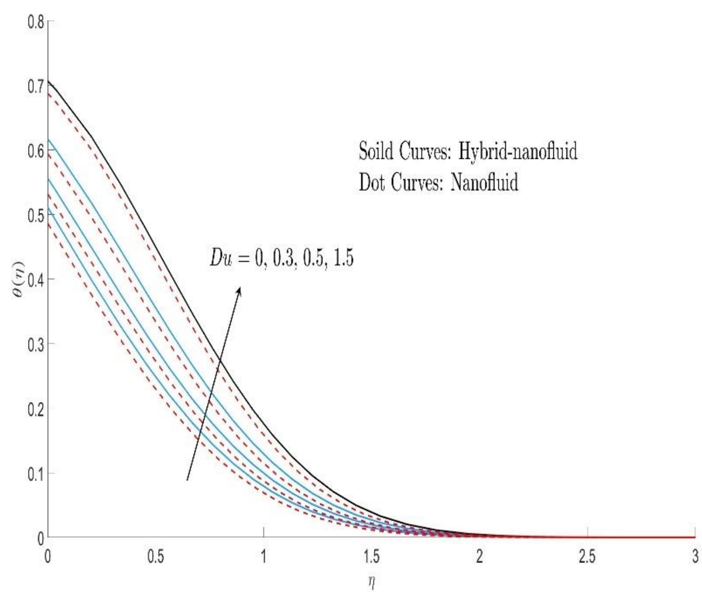

Fig. 9. Influence of Du on θ.

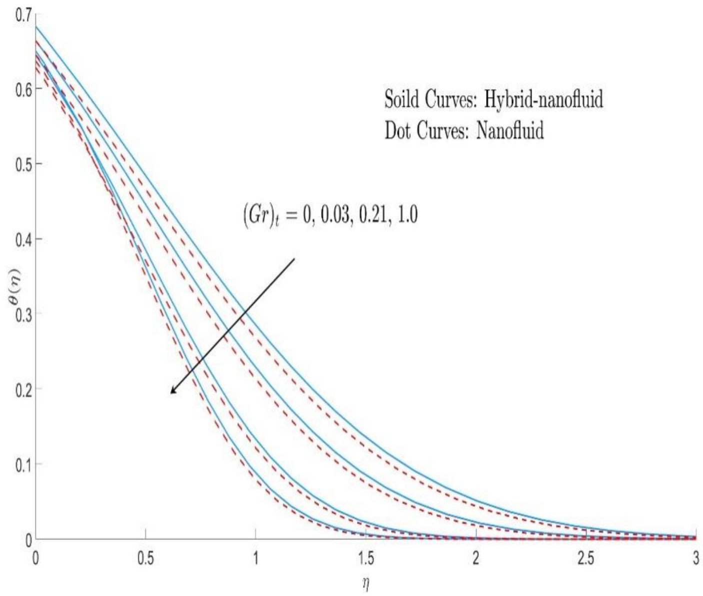

Fig. 10. Influence of $(Gr)_t$ on θ.

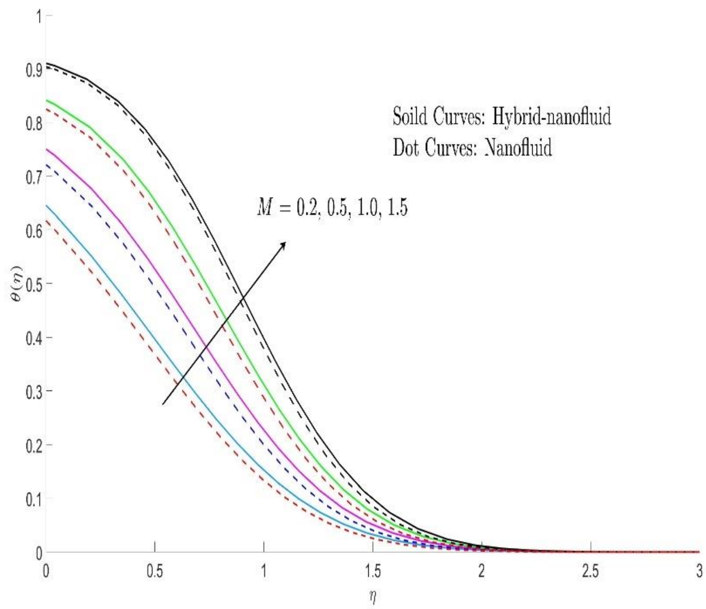

Fig. 11. Influence of M on θ.

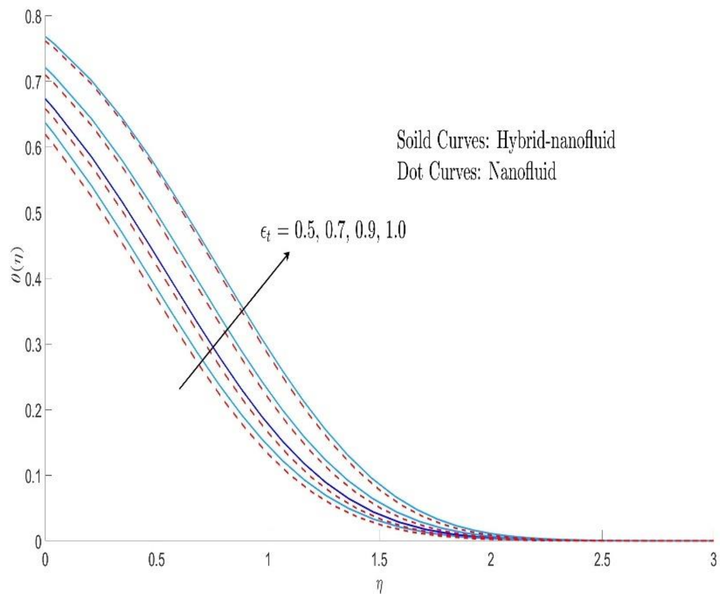

Fig. 12. Influence ϵ_1 on θ.

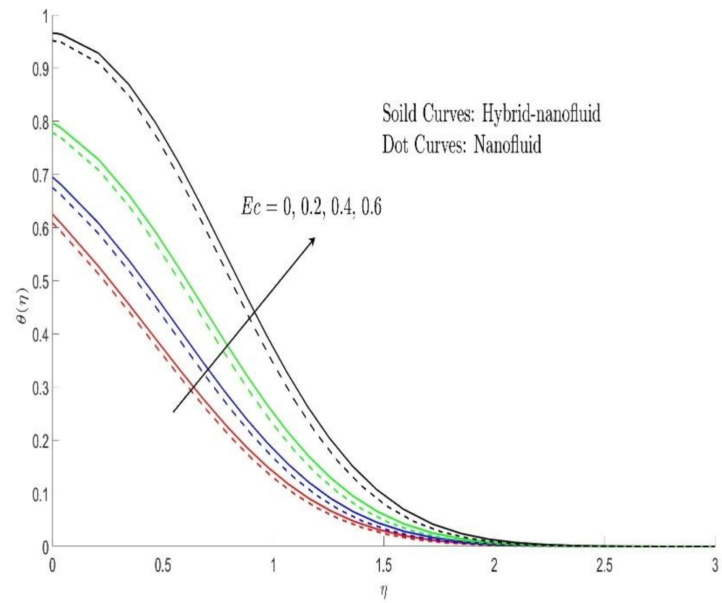

Fig. 13. Influence of Ec on θ.

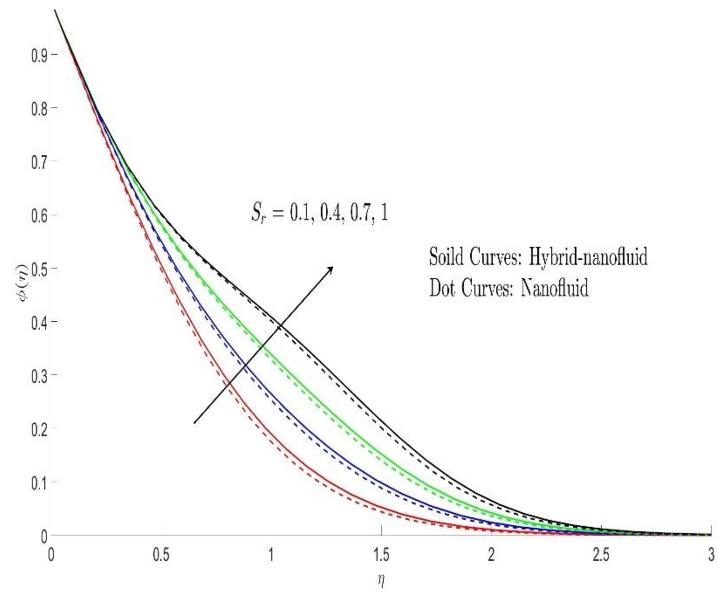

Fig. 14. Influence of Sr on ϕ.

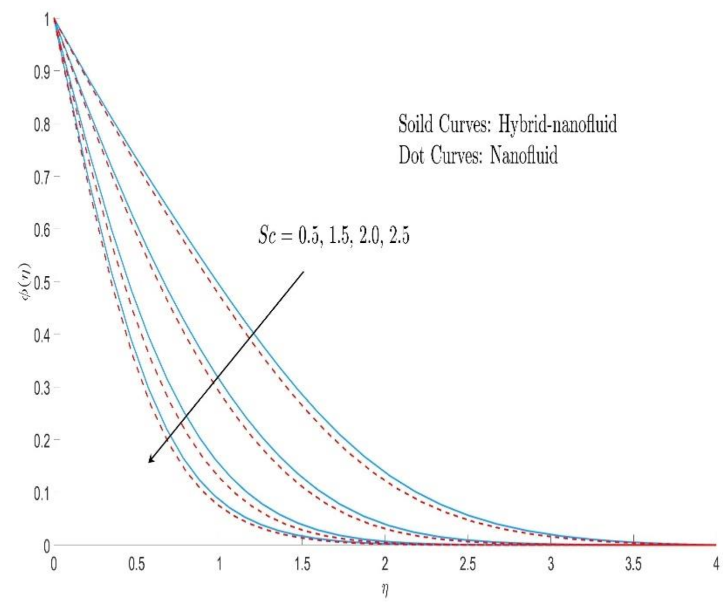

Fig. 15. Influence of Sc on ϕ.

Table 5. physical quantizations in simulation when $Du = 0.3, M = 0.6, Sr = 0.2, \beta = 0.1, Pr = 0.7, (Gr)_c = 1.6, K^* = 0.9, \beta^* = 2.9, Ec = 6, Sc = 10,$ and $(Gr)_t = 0.1$.

		-$C_{fx}(Re)^{1.5}$	-$C_{fy}(Re)^{1.5}$	-$Nu(Re)^{1.5}$	-$Sh(Re)^{1.5}$
k^*	0.1	0.5854736654	0.4251622865	1.469615409	1.336014008
	0.3	0.5908495342	0.5872447698	1.477060316	1.342782105
	0.5	0.6643955706	0.594703677	1.480258693	1.376598812
Du	0.1	0.5264339688	1.137228409	1.457396180	1.324905619
	0.3	0.5042650248	1.119093231	1.424234505	1.310213186
	0.5	0.5000114186	1.010257923	1.417014987	1.302740897
Sr	0.1	0.5000114159	1.210257919	1.778926707	1.617206098
	0.3	0.4856589213	1.138660768	1.726486241	1.604078401
	0.5	0.4835117384	1.108346245	1.718368259	1.525789326
Sc	0.1	0.4835117388	1.238346247	2.235444588	2.032222353
	0.3	0.4835117399	1.238346247	2.563934111	2.330849192
	0.5	0.4835117384	1.238346245	2.805765261	2.550695691

Table 6. Shows that the comparisons of hybrid nano fluids ($Fe_2O_3 - Al_2O_3$) and nanofluid Fe_2O_3) w.r.t Local skin friction coefficient, Nusselt number and Sherwood number.

		Hybrid Nanofluid ($Fe_2O_3 - Al_2O_3$)			**Nanofluid (Fe_2O_3)**		
		$Re^{\frac{1}{2}}C_f$	$Re^{-\frac{1}{2}}Nu$	$Re^{-\frac{1}{2}}Sh$	$Re^{\frac{1}{2}}C_f$	$Re^{-\frac{1}{2}}Nu$	$Re^{-\frac{1}{2}}Sh$
E_i	0.2	0.797275	0.584809	0.471631	0.366215	0.515542	0.392980
	0.3	0.781452	0.582914	0.553971	0.361498	0.513625	0.437901
	0.4	0.775078	0.582143	0.587303	0.358824	0.512534	0.463449
	0.6	0.813044	0.696015	0.702877	0.359494	0.612130	0.591962
B_i	0.4	0.831321	0.565637	0.360010	0.747694	0.623337	0.346218
	0.5	0.829379	0.782424	0.382862	0.737070	0.645848	0.344631
	0.6	0.810632	0.602874	0.357482	0.729593	0.661806	0.343501
	0.8	0.799329	0.623448	0.356074	0.746463	0.832262	0.372349
$(Gr)_t$	0.0	1.097611	0.564346	0.350003	0.995086	0.618957	0.336153
	0.1	0.954371	0.576780	0.354613	0.862183	0.633571	0.340746
	0.2	0.819197	0.587380	0.358536	0.737070	0.645848	0.344631
	0.3	0.690322	0.596671	0.361961	0.617928	0.656470	0.348014

5. Conclusions

We look at the characteristics of thermal energy and mass transfer that have a big impact on how hybrid nanoparticles and nanoparticles behave along the vertical 3D plate. Principal conclusions include the following:

- In comparison to nanofluid, the use of hybrid nanoparticles is thought to be more effective at maximizing energy production into fluidic particles;
- The parameter of the magnetic field reduces particle velocity;
- The development of heat energy is influenced by variable thermal conductivity numbers.
- The non-approach Fourier's has less of an impact on thermal diffusion and heat amplification when compared to larger values of the heat source number.
- Lorentz force diminishes distribution into fluid flow motion; however, an investigation found the reverse trend instead.

Future physical and technological difficulties could be addressed by the FEM [69-79]. The papers [80-90] go over a few recent developments that aim at the value of the study domain in consideration.

Compliance with Ethical Standards

The authors have not declared any conflicts of interest.

Data availability

This published publication contains all of the data created or analyzed during this investigation.

Author Contributions

MBH presented the idea and topic. MBH solved the problem and present the results. MBH, MK validated the results via literature. MBH,MK and WJ review and supervision. Each author participated equally to the drafting and editing of the paper. The paper was examined by each author.

Nomenclature:

Du	Dufour number	Nu	Nusselt number
T	Nanofluid temperature	PDEs	partial differential equations
T_∞	prevailing temperature	Al_2O_3	alumina
g^*	force of gravity	Fe_2O_3	ferrofluids

u, v	velocity component in x, y direction $(m\ s^{-1})$	Sh	rate of mass diffusion
k	thermal efficiency (W m^{-1} K^{-1})		
Ec	Eckert number	q_w	flow of wall heat
h_{nf}	hybrid nanoparticles		
x, y, z	dimensional space coordinates (m)	***Greek symbols***	
Sr	Soret number	ρ	Density
Sc	Schmidt number	β^*	energy source
K^*	permeable medium element	ρc_p	capacity for heat
C_{fx}	Surface tension	ϕ	volume fractions
Re	Reynolds number	σ	conductive to electricity $(\Omega\ m)^{-1}$
M	magnetic factor	D_{hnf}	Diffusivity of the hybrid nanofluid's temperature
β_{hnf}	thermal expansion factor for volume		

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