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A probabilistic-driven framework for enhanced corrosion estimation of ship structural components

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ABSTRACT

The work proposes a probabilistic-driven framework for enhanced corrosion estimation of ship structural components using Bayesian inference and limited measurement data. The new approach for modelling measurement uncertainty is proposed based on the results of previous corrosion tests that incorporate the non-uniform character of the corroded surface of structural components. The proposed framework's basic features are outlined, and the detailed algorithm is presented. Further, the proposed framework is validated by comparison with the classical statistical approach and mass measurements, considering previous experimental work results. Notably, the impact of the number of measuring points is investigated, and the accuracy index is proposed to identify the optimum number of measurements. The developed framework has a significant advantage over the classical approach since measuring uncertainty is incorporated. Additionally, the confidence intervals of both mean value corrosion depth and standard deviation could be gathered due to the probabilistic character of the framework. Thus, the presented approach can potentially be used in the structural health monitoring of ship structural components and reliability analysis.

1. Introduction

Ships and offshore structures are subjected to a severe corrosion environment [1], and a protective coating layer has been applied to protect them. However, it breaks with time [2], and corrosion starts deteriorating the structural components. Various corrosion types could be distinguished, such as general corrosion [3], pitting corrosion [4], crevice corrosion, galvanic corrosion, erosion-corrosion, and microbiologically influenced corrosion [5], recently gaining significant attention from researchers. From these various types, general corrosion and pitting are most common in ship structures [6]. It must be noted that except for general corrosion, the other types could be considered relatively localized phenomena. The ageing structures need to be monitored during the service life to identify excessive corrosion degradations. This is crucial since the consequences of extreme corrosion damage may become severe, including structural collapse and ship hull breaking (e.g. Prestige catastrophe [7]), as well as failures of operating offshore structures [8].

Many works were devoted to investigating the detection and modelling of localized corrosion defects. The excessive review of risk-based decision-making models for microbiologically influenced corrosion (MIC) in offshore pipelines was presented in [5]. The proposal of stochastic modelling of pitting corrosion growth was given in [9], characterizing corrosion volume and depth growth and validated against field data. Other recent works on that problem can be found, e.g. in [4,10,11]. Although localized corrosion defects are essential for structural safety, the current work focuses on proper diagnostics of uniform (general) corrosion, which is discussed in detail in a further part. Even so-called uniform corrosion could be subjected to significant spread in the corrosion depth, even for a single structural element [12].

To monitor the corrosion loss in ship structures, the Classification Societies issued guidelines for periodic inspections of ship hull structures, e.g. [13], related to plating thickness measurements. Currently, the most common method for performing thickness measurements is the one that uses an ultrasonic thickness gauge. It employs relatively simple physical phenomena and measures the time of travel of acoustic signals through the thickness of the plating as fast as a signal is received, as the thinner component is. However, alternative methods are under investigation (e.g. guided waves [14]). Moreover, ultrasonic thickness gauges are commonly used during inspections due to their simplicity, where the measuring ultrasonic thickness gauges are portable, and there is no need for more profound expertise to perform measurements. However, there

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are known drawbacks to this method. As was pointed out in [15], several factors may affect the measurements, resulting in their lower quality, such as cleanliness, lighting, type of inspected area and human factors. In fact, during ship inspections, the hostile conditions for performing measurements are rather typical. The high level of uncertainty generated by this method was also pointed out in different studies, e.g. [16]. Further, if an already corroded structural component is subjected to further structural analysis, the uncertainties can be magnified [17].

Various studies proposed mathematical models to estimate corrosion loss during ship service life, e.g. [18,19]. Further, the time-dependent model to estimate the corrosion loss of deck plates in ballast and cargo tank tankers based on inspection data was proposed in [20] and for ship crude oil tanks in [21]. The advanced time-dependent corrosion degradation model was recently proposed in [22], where the probability distribution of the corrosion depth across the ship hull is estimated during the exploitation period. Based on the available real measurement data, the developed methodology was applied to predict the corrosion depth of the ship's ballast tanks [23]. The simplified method to predict the corrosion level for different ship panels, accounting for the different corrosive environments on each side of the plate and based on the available real measurement data, was proposed in [24], where some probabilistic models were developed to predict corrosion degradation uncertainties. The study presented in [25] uses the corrosion degradation model developed in [18], which was extended using probabilistic representations of the real measurement data. Another probabilistic model to estimate the corrosion degradation rate of fuel tank structures of bulk carriers was proposed in [26]. In [27], the methodology for a risk-based corrosion allowance for oil tankers easily applicable in the design process was also proposed.

The critical issue is to validate the proposed corrosion models given actual field data. In some works, Bayesian updating was used to update the aforementioned general corrosion loss modes based on the available measurement data [28,29]. The Bayesian analysis (including Bayesian inference and networks) was also used to detect localized corrosion defects. In [30], the adaptive approach to predicting pipeline corrosion defects was proposed. The Bayesian inference was proposed to model the growth and generation of localized corrosion defects based on imperfect inspection data [31]. The Bayesian network could also calibrate the corrosion rate instrument, as shown in [32]. These works showed that Bayesian analysis can be used for problems where limited inspection data is available and subjected to significant uncertainty.

Although the intensive development in this field is evident, most of the developed mathematical models rely on the data collected during inspections, where only the mean corrosion degradation value within the structural components (eventually including uncertainty level) is estimated. On the other hand, other developed models treat corrosion only as a localized phenomenon. The current guidelines issued by the Classification Societies related to performing such measurements [4] require a minimum of five-point measurements per square meter and three-point measurements per 1 m of the stiffener. Such guidelines are engineering acceptable since they are sufficient to capture the general trend in the thickness loss of the ship structure. A higher number of measurements will require much more effort and time, whereas each day the ship is out of operation generates costs. However, such an approach may be insufficient when structural components on a local level are considered. The case of the MOL Comfort container ship [33] shows that the failure of a single structural component may trigger severe structural failure of the entire ship hull girder. When considering corrosion degradation, the non-uniform thickness loss may significantly reduce the structural component load-carrying capacity [34]. Thus, the mean corrosion loss value of a single structural component and its standard deviation would be beneficial to estimate the structural capacity of corroded components correctly. This is quite crucial in terms of the safe operation of ship structures. Notably, corrosion is an important degradation phenomenon in other fields of engineering as well, such as civil engineering [35] or the oil and gas branch [36].

Unlike the other studies that were focused on estimating and forecasting the corrosion degradation development as a general average thickness loss in structural components or investigated in detail the process of growth of localized corrosion phenomena, the present work explores the problem of estimating the corroded plate thickness loss within a single structural component based on the limited measurement data and accounting for the non-uniform character of the corrosion degradation. The problem is highly important for the safety and reliability of ships and offshore structures. In view of recent findings only accounting for the non-uniform character of general corrosion, the structural performance could be evaluated adequately [34,37]. Nevertheless, current renewal criteria (decision to either replace or not ship structural element) rely on the gauged corrosion depth, which could be highly uncertain. Thus, the proper treatment of inspection data given the required measurement number and consideration of measuring uncertainty is needed.

To account for that, a novel probabilistic framework is proposed, where the initially assumed corrosion depth characteristics are updated based on the measurement data employing the Bayesian inference. Up to now, no works have investigated the adequacy of Bayesian analysis for assessing the corrosion field characteristics (thickness distribution within structural components). In addition, a new approach for measurement error treatment is proposed due to the significant uncertainty [12] in the corrosion depth measurements. The corrosion depth's mean value and standard deviation are estimated in terms of their probability distributions. The presented framework is validated using real measurement data, showing its credibility. Finally, the impact of the number of points of the corrosion depth measurement within the structural component is investigated, and the concept of accuracy index is proposed to quantify that. Finally, the conclusions coming from the presented work are drawn.

2. Materials and methods

2.1. Case study of corroded specimens

The case study, which will be analyzed in the current work, has been described in [12], where the experimental corrosion was carried out, and further compressive strength tests were performed [34]. However, since it will be used to validate the framework introduced in the other part of the study, the main features and results are introduced herein at the beginning. The work presented in [12] aimed to develop a methodology for accelerated corrosion testing of steel specimens. The normal strength steel specimens (A grade) were subjected to corrosion degradation; a detailed chemical composition can be found in [12]. The nine stiffened plates (1.25 m long and 0.4 m wide with a stiffener of 0.1 m height) were subjected to accelerated corrosion degradation, but only natural factors were controlled. Three different initial thicknesses were investigated (5 mm, 6 mm and 8 mm), leading to different degradation levels in terms of severity. The corrosion degradation level was measured via mass measurements, and the Degree of Degradation (*DoD*) was considered as a governing degradation parameter. The *DoD* was calculated as a percentage loss of the initial mass of the specimen. Detailed thickness measurements were also performed using a certified ultrasonic gauge to capture the corroded thickness distributions. An example of a corroded specimen with measured thicknesses is presented in Fig. 1.

It needs to be noted that the thickness was measured at each point of the grid from both sides of the plate and stiffener. The summary of the mean value thickness from mass and ultrasonic NDT (non-destructive testing) measurements is presented in Table 1. Detailed information about thickness distribution in each specimen can be found in [12]. Since mass measurements consider the difference between the plate's initial (intact) and final mass with high accuracy, we can consider the mean value thickness loss calculated using mass measurements as the most accurate. Thus, it will validate the proposed methodology

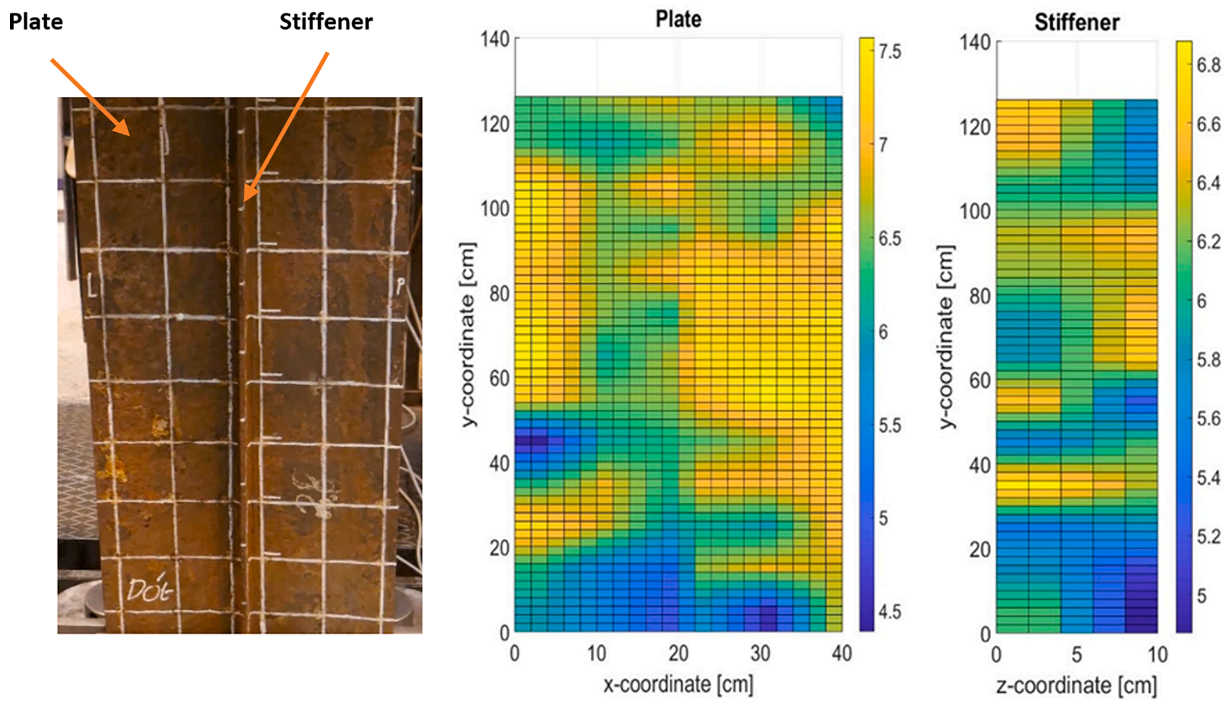

Fig. 1. Corroded specimen – stiffened plate (left) and corroded thickness of plate (central) and stiffener (right) [12].

Table 1
Mean value of residual thickness measurements [12].

Specimen	Residual thickness [mm]		
	Mean - mass	Mean – 1st side	Mean – 2nd side
1.5	4.638	4.735	4.682
2.5	3.948	4.051	4.121
4.5	4.320	4.410	4.318
1.6	5.192	5.428	5.355
3.6	4.778	5.081	5.034
4.6	5.600	5.865	5.753
1.8	6.896	7.301	7.312
2.8	7.453	7.557	7.633
3.8	6.213	6.719	6.691

regarding the mean value thickness loss.

Even from the broad perspective, seen in Table 1, it is noted that the ultrasonic NDT measurements overestimated the residual thickness estimated from the mass measurements even though a significant number of measuring points were considered. Thus, the corrosion depth is underestimated, which leads to a non-conservative assessment. The residual averaged thickness obtained via mass measure is the most accurate since the total material loss is captured. This encouraged the discussion of the uncertainties in ultrasonic NDT measurements in trying to find a solution to catch them, which is also the aim of the presented work.

2.2. *Uncertainties in ultrasonic NDT measurements*

As already outlined, the typical ultrasonic measurements are subjected to significant uncertainties. Two types of uncertainties are identified, i.e. epistemic and aleatory. The first ones are related to insufficient information, thus, due to the limited number of measurements, and they can be reduced by gathering more information, i.e. more measures. A typical example of this type of uncertainty is presented in Fig. 2, showing the relation between the mean value of thickness estimated from specific measurements carried out either on the first or second side of the plating. The black dashed line represents the accurate value of the residual thickness obtained via mass measurements, and the x-axis shows the number of measures. The y-axis shows the estimated mean value of the corroded plate depth as a function of the number of measurements.

With the increase in the number of measurements, it is noted that the mean value of corrosion depth tends to be close to the accurate value of the corroded plate for most of the specimens. Notably, there could be significant differences between the two sides considered for measurements. Further, if only three measuring points are considered, the residual thickness could be overestimated even up to 1 mm. Generally, using up to 10 measuring points, the results are scattered and rather non-reliable. Only for selected cases, the thickness estimates are relatively accurate. However, the mean value is somewhat different from the actual residual corroded thickness for the same number of measurements and different plate side measurements.

Nevertheless, even when significantly increasing the number of measurements, there is still some difference between the mean value of the corroded plate thickness defined by the ultrasonic NDT measurements and those captured via mass measurements. This leads to the second type of uncertainty, the aleatory one. This type relates to the measuring method and phenomena that should be captured, regardless of the number of measurements carried out. Notably, as seen in Fig. 2, even for the maximum number of measures, the mean value of the residual corroded plate thickness is higher than the one defined by mass measurements. This uncertainty is related to corrosion degradation, which causes an irregular thickness distribution (see Fig. 3) on both macro and micro scales. The ultrasonic measurements are based on the time of travel of the acoustic signal perpendicular to the plate surface. The thickness may be accurately estimated for non-corroded specimens since both plating surfaces are flat and parallel. However, for corroded specimens, the plate surfaces are not parallel, leading to a longer time of travel, and local non-uniformities (in both the front and back side of the plate regarding the position of the probe) could cause additional signal reflections [17]. These phenomena disturb the thickness identification significantly. This is the main reason that readings could be substantially different for the same measuring points but performed on the two sides of the plating (see Fig. 3).

In the analyzed case study [12], the measurement conditions were

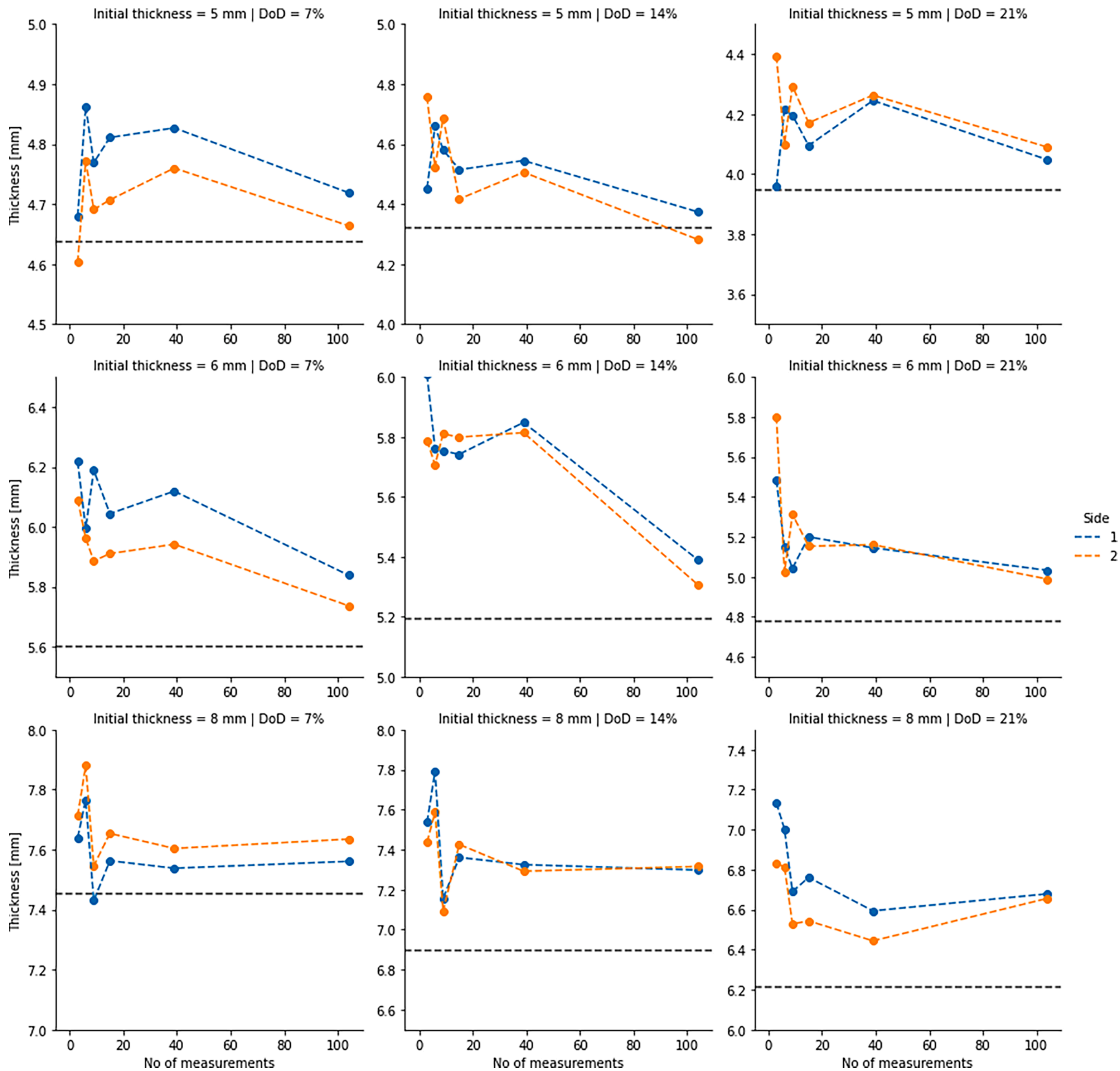

Fig. 2. Mean value thickness of corroded plates as a function of the number of measurements using NDT and mass measurements.

excellent, i.e. the surface was clean, the lighting was good, and the position to carry out the measurements. However, in normal ship structure inspection conditions, additional sources of uncertainties will exist [15]. Finally, one should consider the uncertainty of measuring equipment itself. However, the uncertainties generated by a typical ultrasonic thickness gauge may be regarded as relatively low compared to other sources of uncertainty [16].

The current work considers both uncertainties (i.e. aleatory and epistemic). To account for the aleatory uncertainty related to measuring technique and corrosion character, the novel proposal is made for measurement error, as described in Section 3.2. The way this uncertainty is incorporated into the framework is discussed in Section 2.3. The second type of uncertainty related to several measurements is also considered. This is inherently connected with Bayesian inference since the more measures we have, the more confident our predictions of corrosion depth distribution parameters are, resulting in lower spread. This is demonstrated in Section 4, where the results obtained using the proposed framework are presented.

2.3. Framework outline

As noted, the corroded plate thickness measurements are subjected to significant uncertainties. On the other hand, during the periodic inspection measurements on ships, there is no time to perform such a large number of measures to identify the level of corrosion depth more precisely. Comparing the current recommendations [13] and the analysis presented in Fig. 2, only a coarse estimation of the corroded plate thickness distribution can be identified. Further, due to aleatory uncertainty, the corrosion depth could be significantly underestimated, giving the non-conservative assessment. Since periodical measurements

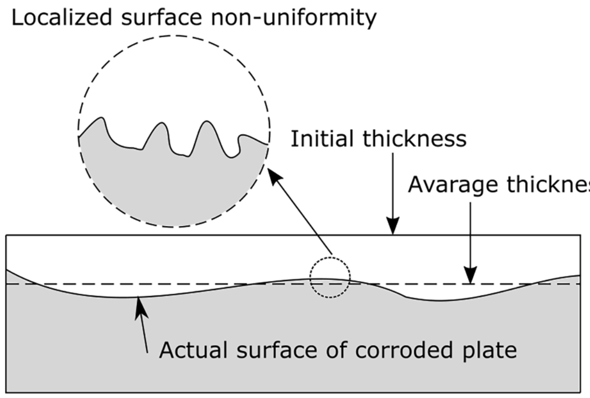

Fig. 3. Corrosion degradation impact on the corroded plate surface.

are typically performed in 5 years, the non-conservative estimation may lead to structural failure before the next measuring campaign. Thus, the presented framework aims to estimate the corrosion loss based on the limited measurement data accounting for the measurement error.

Typically, based on the small sample, statistically, we cannot draw general conclusions using a specific statistical approach (e.g., calculating mean value, standard deviation, etc.). Additionally, it is not clear the confidence interval of such estimations (e.g., how precisely the mean value of corrosion depth is estimated). From the various existing statistical approaches, the Bayesian updating [30,38] allows drawing conclusions based only on the limited sample size. Firstly, some prior assumptions are needed, i.e., the general overview of how the parameters analyzed may be statistically distributed. The prior knowledge can be considered from either boundary conditions or previous experience. Then, using the Bayesian inference (based on the Bayes theorem), the posterior knowledge (probability distribution of a particular variable) having the observed data may be estimated.

Whereas the typical (i.e. frequentists) inference is a statistical method that draws results directly from data, the Bayesian inference defines probability as the degree of belief for particular events. The belief is further verified and modified, given the observation data. The basic form of the Bayesian inference relies on the Bayes theorem as follows:

$$P(H|D) = \frac{P(D|H)\cdot P(H)}{P(D)} \tag{1}$$

where H and D stand for the hypothesis and data, respectively.

$P(H)$ is the prior probability for the hypothesis, and $P(H|D)$ is the posterior probability for the hypothesis, but considering new data. Further, $P(D|H)$ is the likelihood of the hypothesis, which is not the same as probability. The likelihood shows how well the given hypothesis can represent the data. Finally, the $P(D)$ is the prior probability of the data acting as a normalizing factor.

Thus, in view of the problem formulated in the current work, the Bayes inference is formulated as follows to determine the distribution of corrosion depth:

$$p(\beta|D) = \frac{p(D|\beta)\cdot p(\beta)}{p(D)} \propto p(D|\beta)\cdot p(\beta) \tag{2}$$

where β are the corrosion depth distribution parameters, i.e., d_{Mean} and d_{StDev}; D are the measurement data containing gauged corrosion depth values. The $p(D|\beta)$ is the likelihood of the hypothesis that measurement data follows the truncated normal distribution described by d_{Mean} and d_{StDev}. The $p(\beta)$ is the prior distribution of the β parameters.

The likelihood $p(D|\beta)$ for N measurement points could be calculated as follows:

$$p(D|\beta) = p(D|d_{Mean}, d_{StDev}) = \prod_{i=1}^{N} p(d_i|d_{Mean}, d_{StDev}) \tag{3}$$

where d_i is considered as the single measurement point with or without consideration of measurement error. Thus, it could be either a deterministic value or a random representation.

Which is more important, additional iterations having new prior knowledge based on the previous runs of the Bayesian inference can be performed. In this view, the observed data may be limited since, having some prior information, there is some background knowledge about the distributions of the analyzed variables. Firstly, only some essential data must be gathered based on classical statistics. Such methodology is perfectly applicable in the corrosion degradation process and inspection measurements since, having previous information on how the corroded plate thickness is distributed within structural components and regarding measurement error, the corroded plate thickness distribution in newly inspected areas can be estimated.

The approach employed uses the Bayesian inference, and the framework diagram is presented in Fig. 4. Below, the additional description is given in a point-by-point manner:

1 Firstly, the prior assumptions are adopted, preferably based on experience. From the beginning, the corrosion depth within the corroded plating follows a truncated normal distribution. The justification for choosing the specified distribution is given in Section 3.1. Thus, the truncated to zero-value normal distribution is employed, which has two governing parameters, i.e. mean value ($\widetilde{d_{Mean}}$) and standard deviation ($\widetilde{d_{StDev}}$). Since, in the beginning, the level of corrosion degradation is unknown, these parameters are assumed to be uniformly distributed. The mean value of the corrosion depth can be between 0 mm and the initial non-corroded plating thickness (t_0). The standard deviation is between 0 mm and half of the initial plating thickness ($t_0/2$).
2 The first run of the Bayesian inference (see Eq. 2) is conducted. It allows us to estimate the distribution parameters of the corrosion depth within the plate ($\widetilde{d_{Mean}}$, $\widetilde{d_{StDev}}$), based on the given measurement data (see Section 2.1). Notably, the distributions of the mean value and standard deviation (non-deterministic values) and their confidence levels are estimated, i.e. histograms of both $\widetilde{d_{Mean}}$ and $\widetilde{d_{StDev}}$. This first iteration allows an estimate of the corrosion depth distribution parameters and their uncertainties without consideration of the measurement error.
3 The measurement error is estimated depending on the corrosion depth obtained for each measurement (the measurement error is introduced in Section 3.2), i.e. $d_{i,err} = f(d_i)$. The measurement error follows the truncated normal distribution. This allows transferring the measurement data from a deterministic to a probabilistic representation. Thus, each measuring point has a specific probability distribution.
4 The second run of the Bayesian inference is performed to account for the measurement error. For the second iteration of the Bayesian inference, the informed prior regarding the mean value of corrosion depth ($\widetilde{d_{Mean}}$) based on the results from the first iteration can be used. Unlike in the first run, it is considered that the mean corrosion depth follows the truncated to zero normal distribution having the following parameters:

$$\widetilde{d_{Mean,2nd}} \sim N\left(Mean\left(\widetilde{d_{Mean,1st}}\right),\ Mean\left(\widetilde{d_{StDev.1st}}\right)\right) \tag{4}$$

where $Mean(d_{Mean,1st})$ is the mean of d_{Mean} distribution from the first run, and $Mean(d_{StDev.1st})$ is the mean of the d_{StDev} distribution from the first run.

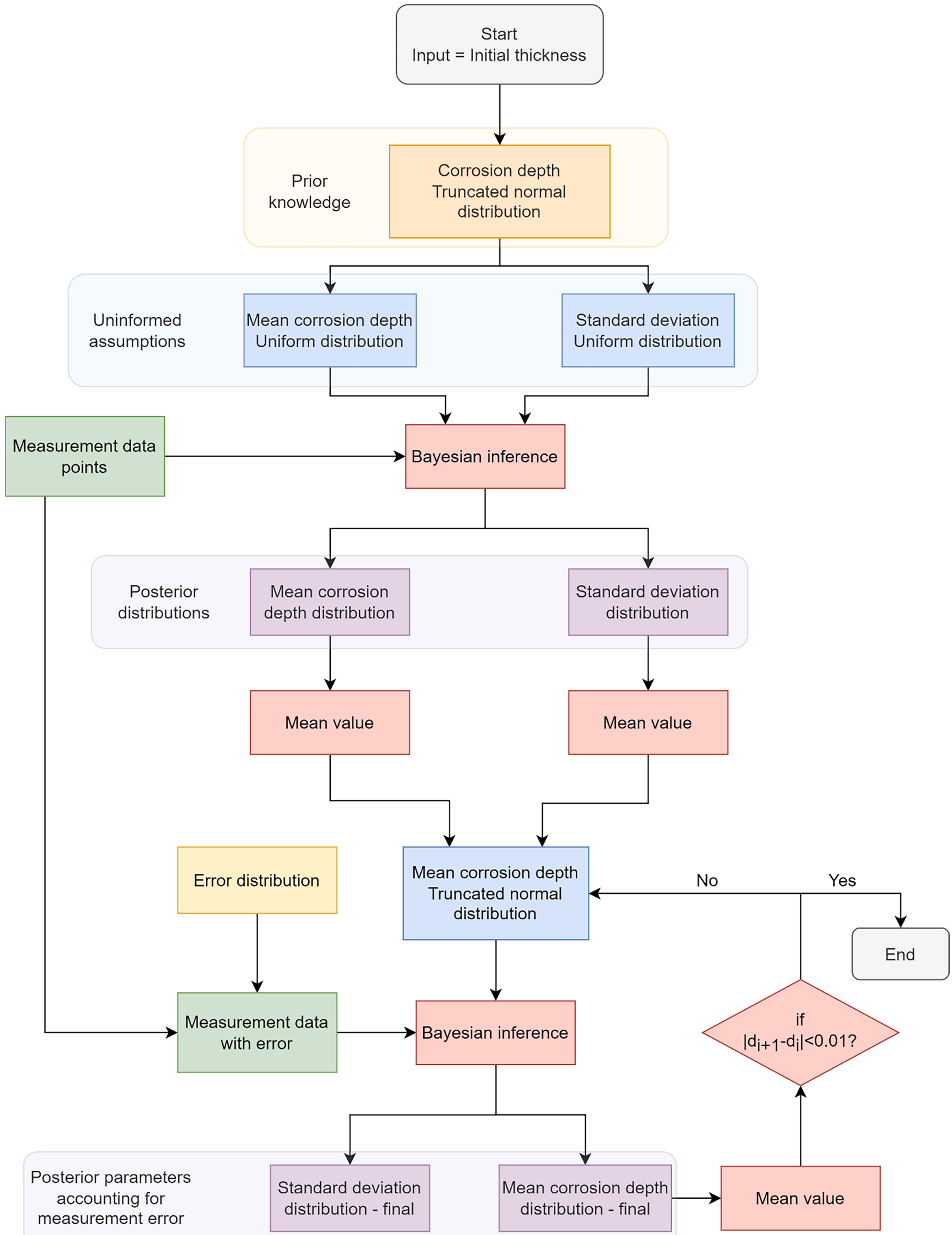

Fig. 4. Proposed framework.

In terms of the prior corrosion depth standard deviation, the same prior as in the first iteration is used. Based on the results of the second iteration, the final posterior distributions of mean value and standard deviation of corrosion depth are gathered, accounting for the measurement error.

1 Additional iterations are performed since better prior knowledge may result in a better estimate. Thus, the prior knowledge regarding the mean value corrosion depth in the subsequent iterations is updated ($\widetilde{d_{Mean,i}} \sim N(Mean(\widetilde{d_{Mean,i-1}}),\ Mean(\widetilde{d_{StDev,i-1}}))$, resulting in the minimum difference between the mean value corrosion depth obtained in subsequent runs.
2 If the difference between the two subsequent runs of the Bayesian inference satisfies the condition that $|Mean(\widetilde{d_{Mean,i}}) - Mean(\widetilde{d_{Mean,i-1}})| < 0.01\ mm$, the entire algorithm is finished.
3 The distributions obtained in the final step ($\widetilde{d_{Mean}}$, $\widetilde{d_{StDev}}$) are considered as results of the framework.

The adopted prior distribution of the corrosion depth is described in Section 3.1, and a description of the measurement error is presented in Section 3.2. Finally, the proposed framework is validated using experimental data, as presented in Section 4.1.

The presented framework was implemented in specially developed code in Python programming language, and the Bayesian inference was implemented using the PyMC3 library [39]. The PyMC3 library uses the

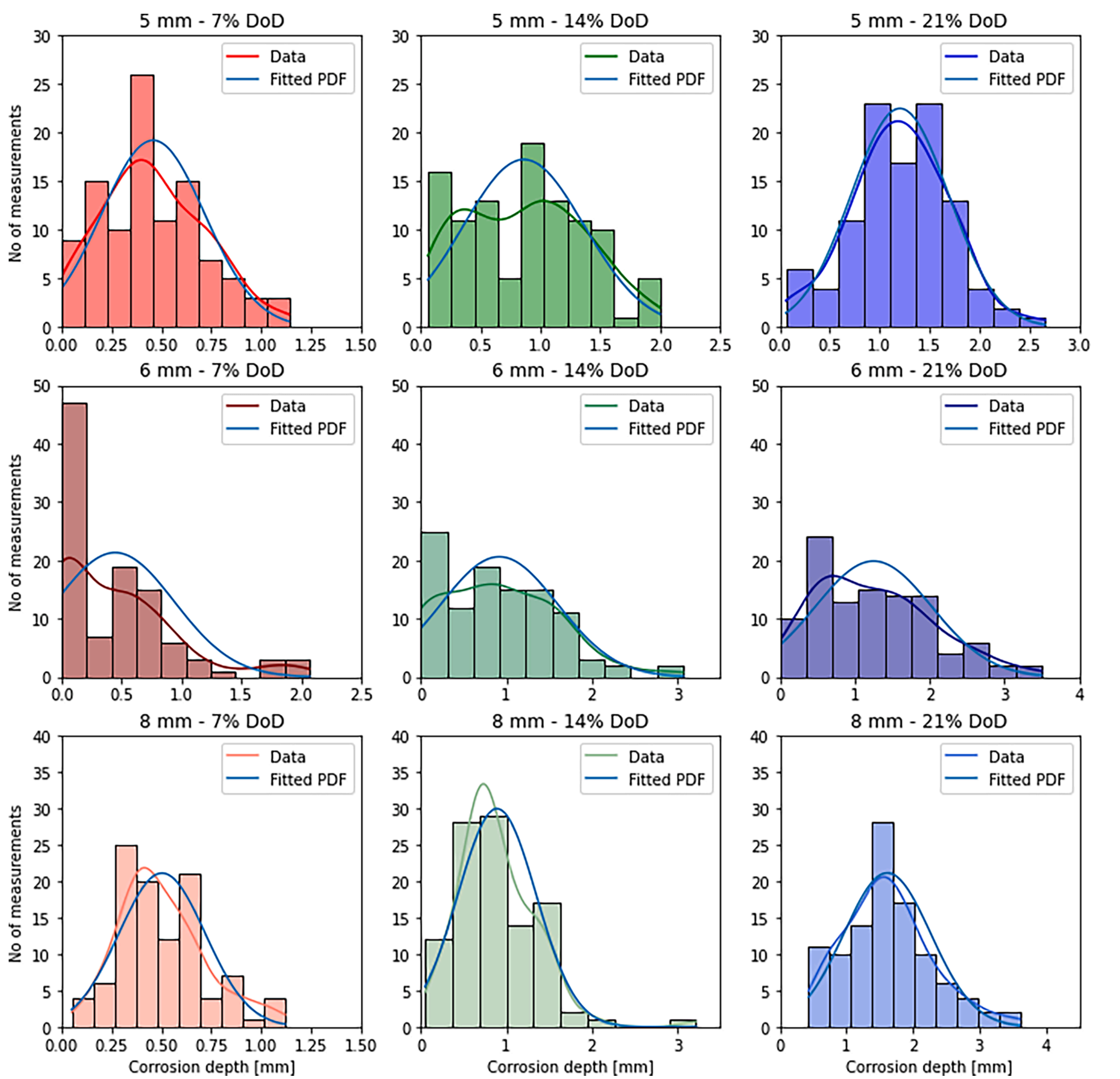

Fig. 5. Distributions of corrosion depths.

Markov chain Monte Carlo method for sampling the probability distributions of the variables considered in the framework.

3. Parameters distributions

3.1. Corrosion depth distribution

The corrosion depth distribution may be assumed priorly based on the detailed measurements from the analyzed case study (see Section 2.1). Notably, previous works also analyzed thickness distribution, but usually for the much more extensive area considered, such as the plating of an entire deck of a tanker ship [40] with limited measurement points, as typically required during inspections. In this case, the corrosion depth was modelled using log-normal distribution. Paik et al. [41] suggested using either log-normal or normal distribution to model the corrosion depth based on the surveys of ballast tank structures. However, the log-normal distribution has two disadvantages. The first one is those parameters that describe the log-normal distribution are not directly a mean value and standard deviation. Thus, they are not intuitive. Secondly, the log-normal distribution is considered from 0 to infinity. Therefore, theoretically, the corrosion value depths exceeding the initial thickness of the plating can be seen. These two disadvantages may be quickly obeyed when using the truncated normal distribution. On the one hand, truncation does not allow the corrosion depth to be below 0 and above the initial thickness. On the other hand, the intuitive parameters, i.e., mean value and standard deviation are used to model the distribution.

The corrosion depth distribution for each specimen of the analyzed case study (see Section 2.1) is presented in Fig. 5. The corrosion depth at each point on the plating was calculated as the difference between the initial thickness of the specimen and the gauged thickness.

Fig. 5 presents histograms of corrosion depths for each specimen and their probability density functions (PDF) obtained via kernel density estimates (KDE). The KDE allows showing the PDFs precisely based on accurate data since it does not follow any known distribution. The exact estimation is visible, e.g. for a 5 mm specimen with 14 % of *DoD*, since two local extrema were found (typical distributions cannot capture this). However, the KDE leads to quite different distributions for each specimen, so it is non-practical to use it for modelling. Nevertheless, it is considered the best possible estimate of the actual PDF. Further, for each specimen, the PDF that follows the truncated to zero normal distribution is fitted to the results (marked as fitted PDF on graphs).

Based on the comparison of distributions, the truncated normal distribution represents the corrosion depth distribution very well. The distribution is almost the same for a couple of specimens as the ones of KDE. Only for two specimens (6 mm – 7 % DoD and 5 mm – 14 % DoD) the deviation between the KDE and fitted distributions is significant. Thus, the truncated normal distribution is considered the prior distribution of the corrosion depth in the framework.

It is noted that even for a considerably low level of corrosion degradation (7 % of DoD), the residual thickness is strongly non-uniform. Further, with the increase of the degradation level, the range of corrosion depth increases, and the variance, too. For the low level of corrosion degradation, more measurement data are closer to 0 mm, and left-bound distribution truncation has a significant value. With the increase of the mean value corrosion depth, the distribution became closer to the normal distribution. There are no significant differences between the distributions obtained for various thicknesses. However, for the 8 mm specimens, the distributions are closer to the normal ones, even for lower levels of degradation.

3.2. Measurement error

It can be seen from Table 1 that even with very dense points of measurement, the corrosion depth may be significantly underestimated. Notably, the exact mass measurements are possible only for isolated, disintegrated specimens but not for the structural components of the integral ship hull structure. Thus, it can be related only to non-destructive ultrasonic measurements. The suggested probabilistic framework accounts for the measurement error and allows for a conservative assessment of real corrosion depths. Based on the study in [12], it was found that when the minimum value from two measurements performed on both sides of the corroded plates at each point was considered, the resulting mean value of corrosion depth was closest to the one calculated based on the mass loss.

However, the measurement is usually performed on one side of the plating. The ultrasonic measurements assume that the two surfaces are parallel and that the time of travel of the acoustic signal is correlated with the thickness of the plating. However, corroded surface irregularities make the surfaces unregular and not parallel on each point. The acoustic signal travels a longer distance than one of the mean thicknesses of a particular corroded plate place.

Thus, when comparing the two-sided thickness measurements, it can be assumed that the lower value is accurate (closest to the actual corroded plate thickness) and the higher value is overestimated. Thus, the measurement error may be defined as a difference between the corroded plate thickness estimated for both sides of the corroded plate. Although this methodology does not fully capture the physical phenomena behind the measurement error (reflection of the acoustic signal, etc.), it allows us to quantify the aleatory uncertainty and implicitly capture this problem. It is then fully justified from an engineering point of view and explains collected measurement data.

Fig. 6 presents the measurement error calculated for three selected specimens described earlier. It is noted that the mean error increases with the degradation level (i.e., the mean corrosion depth). Thus, the newly introduced measurement error should depend on the corrosion depth to comply with the observed phenomenon fully. The following finding is easily explained since, with the increase of the corrosion depth, the irregularities are also more significant, resulting in growth in the measurement error.

The mean value and standard deviation of the measurement error for each plate were estimated and given in Table 1. The power regression functions represent the relationship between the corrosion depth's mean value and the measurement error's standard deviation. Additionally, the function tends to be zero for non-corroded material, excluding the equipment's fundamental measurement error.

The functions used in Fig. 7 are employed in the developed framework. Each measurement point's second loop of the Bayesian inference accounts for the measurement error. The truncated to zero normal distribution was chosen to model the measurement error (see Fig. 6).

4. Results and discussion

4.1. Validation of the framework

The corroded plate thickness distribution is estimated for the plates given in Table 1 to validate the proposed framework. However, a different number of measurements is used to see the dependency between the accuracy of the estimation of the thickness distribution parameters and the number of measures. Thus, the minimum number of 3 measurements was used.

The results of using the proposed framework are presented in Fig. 8 for six measurements considered (for specimen 1.8, see Table 1) measurements on the first side. The obtained probability distributions are shown on the left, whereas the sampling process is given on the right. The total number of 20,000 samples was considered to obtain the variables' probability distributions. After the first run of the Bayesian inference, the distributions of the mean value and standard deviation are estimated without the measurement error and the prior information about the distributions of the mean value and standard deviation of the corrosion depth. In the next run, the measurement error is estimated for each measurement (see the top chart in Fig. 8), and the distributions are

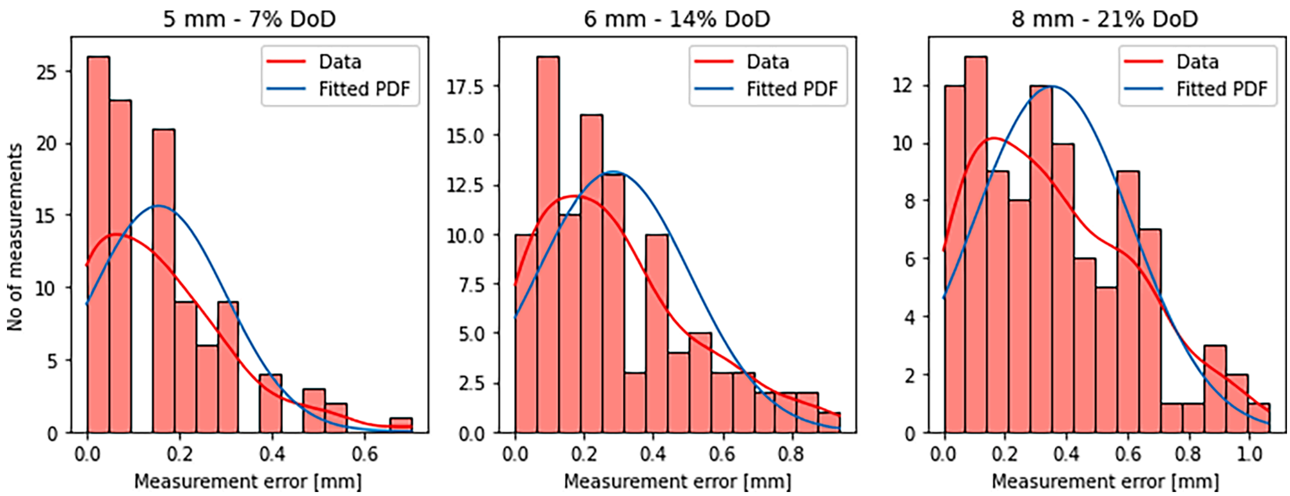

Fig. 6. Measurement error distribution for three selected specimens.

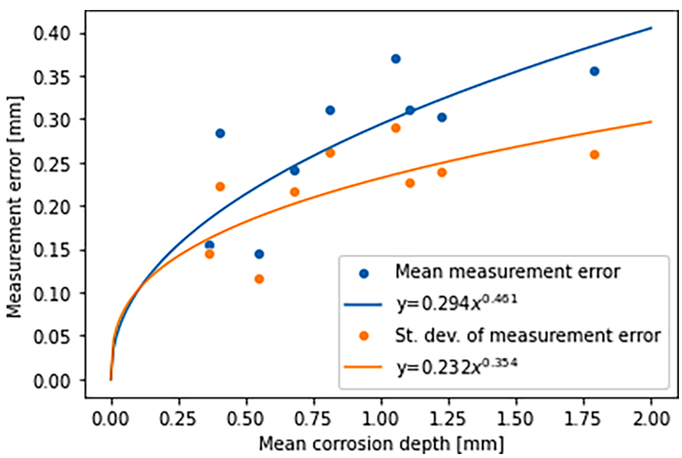

Fig. 7. Measurement error mean value and standard deviation as a function of the corrosion depth.

consistent with the model proposed in Section 3.2. Thus, increasing the measured corrosion depth makes the measurement error distribution more spread, bringing considerable uncertainty. Each measurement is then extended from the deterministic representation into a probabilistic one with specified distributions (see Fig. 8, second from the top chart). The corrosion depth's mean value and standard deviation are described by probability distributions (see Fig. 8). Thus, the estimated mean value and standard deviation of the corrosion depth will have the most expected values as mean values of the distributions and specified confidence intervals (see Fig. 9).

Based on the histograms in Fig. 9, it can be seen that the most conservative corrosion distribution, resulting in the 95 % confidence level of corrosion degradation, is achieved for a mean value of the corrosion depth of 0.63 mm with a standard deviation of 0.83 mm. The means of the mean values and standard deviations for the same analyzed case are 0.35 mm and 0.43 mm, respectively.

To compare the classical approach with the proposed framework, the mean value of the corrosion depth for various measurements is compared for the specimen (1.8), considering measurements performed on the first side of the specimen, as seen in Table 2.

It can be noted from Table 2 that the classical approach of measurements significantly underestimated the corrosion degradation level. In the case of the proposed framework, except for the six measurements, the upper boundary was higher than the actual degradation level or very close to it. Thus, the framework's accuracy for this specimen was better than the classical approach.

Further, the results obtained for various measurements and corrosion degradation levels are compared to validate the framework. For each specimen (see Table 1) and the 1st or 2nd sides of the measured corroded plating, the results are given in terms of the mean value of the corrosion depth distribution and its standard deviation. A total of 18 cases were analyzed (nine specimens measured from both sides). Notably, the upper and lower bounds (considering 95 % of the confidence interval) and mean values of distributions of both parameters are analyzed, depending on the number of measurements carried out. Thus, with a 95 % probability, the distribution's mean value and standard deviation should lie between the lower and upper bound. For the mean value of the corrosion depth distribution, the additional horizontal lines are presented, showing the corrosion depth value estimated from the mass measurements, which are considered the most accurate (although some uncertainty levels could still be considered). The results from the classical approach regarding mean value corrosion depth from measurements are also presented.

The results of the analysis are presented in Figs. 10—13. Figs. 10 and 11 present the mean value corrosion depth estimation as a function of the number of measurements carried out on the 1st and 2nd sides of the specimens, respectively. In Figs. 12 and 13, the standard deviation of the corrosion depth is presented as a function of the number of measurements carried out on the 1st and 2nd sides of the specimens, respectively. In addition to the point values for each number of measurements, the multi-linear trends are plotted for each parameter to show the tendencies.

Based on the presented results in Figs. 10 and 11, the spread between the upper and lower bound decreases with the number of measurements. However, the spread is similar between 39 and 104 measurements and stabilizes constantly. Thus, when a specific number of measurements are achieved, the fundamental measurement uncertainty will result in the spread in estimating the mean corrosion depth. Further, the corrosion depth estimated from mass measurements mainly lies between the upper and lower bounds of the estimated mean corrosion depth. It is often very close to the trend of the mean values estimated from the framework. The exceptions from that observation are seen for 6 mm specimens with 7 % and 14 % of the degradation level, where for the lower number of measurements, the framework slightly underestimates the corrosion depth. However, the corrosion depth was predicted accurately when considering the higher number of measurements (N=39 and N=104) in almost all cases. The mean error between framework and mass measurements is at the level of fundamental measuring error of the equipment used. Thus, the corrosion depth predicted using mass measurements is the most accurate and proves the accuracy of the

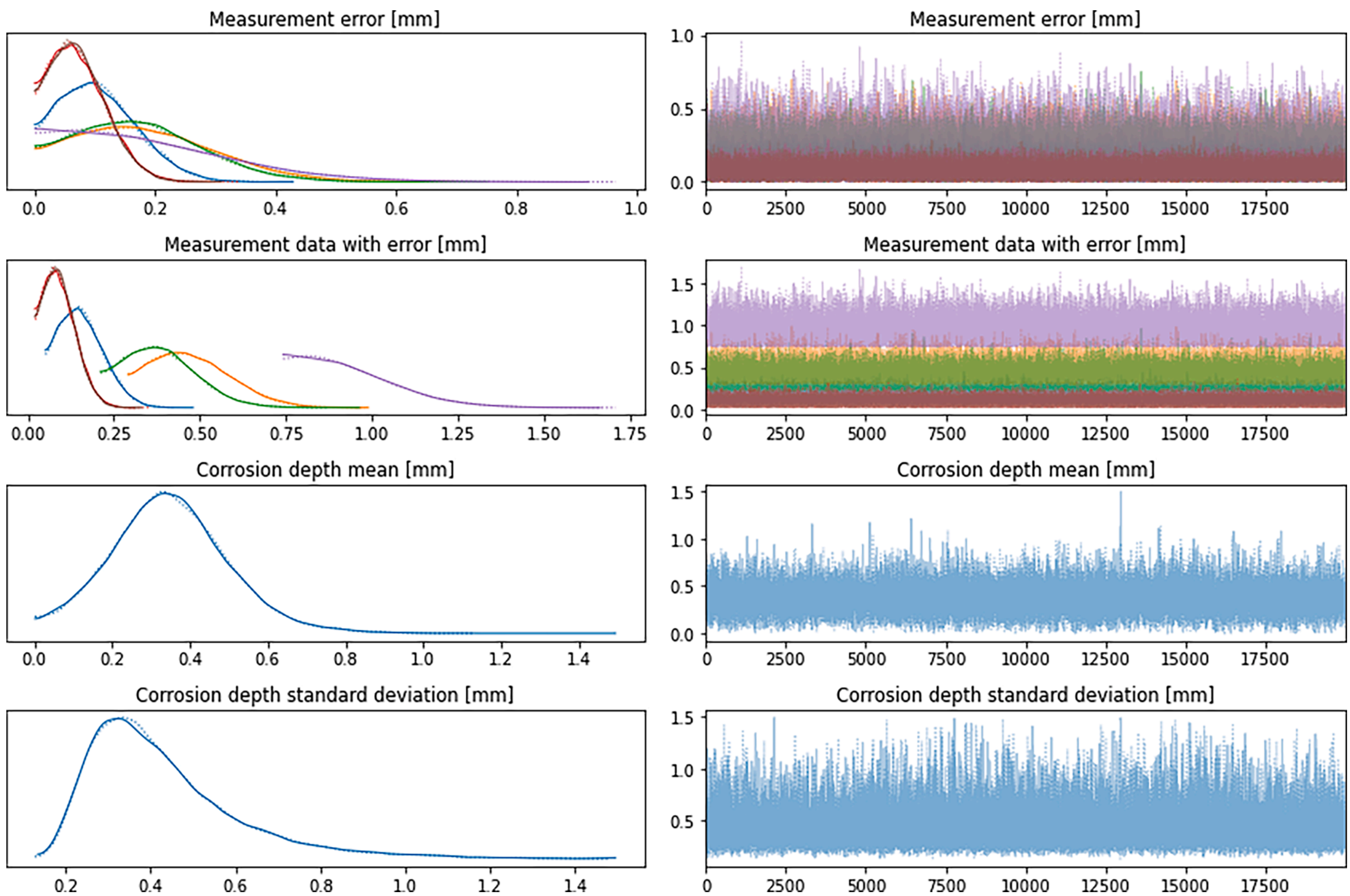

Fig. 8. Probability density as a function of measurement error, data with error, mean value and standard deviation (left) and spread of measurement error, data with error, mean value and standard deviation as a function of the number of simulations (right).

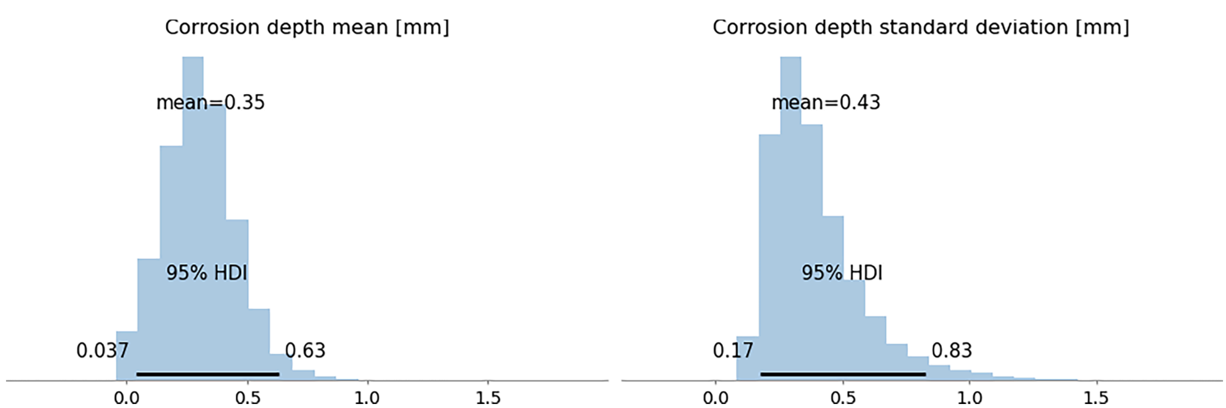

Fig. 9. The confidence intervals for mean value (left) and standard deviation (right) of corrosion depth.

Table 2
Comparison between classical approach and proposed framework.

No measurements	Mean value [mm]				
	Mass	Classical Approach (M)	Framework (Mean)	Framework (Lower Bound)	Framework (Upper Bound)
3	1.104	0.477	0.792	0.002	1.548
6	1.104	0.222	0.37	0.07	0.675
9	1.104	0.85	1.123	0.648	1.617
15	1.104	0.607	0.825	0.468	1.191
39	1.104	0.678	0.918	0.735	1.101
104	1.104	0.705	0.954	0.855	1.054

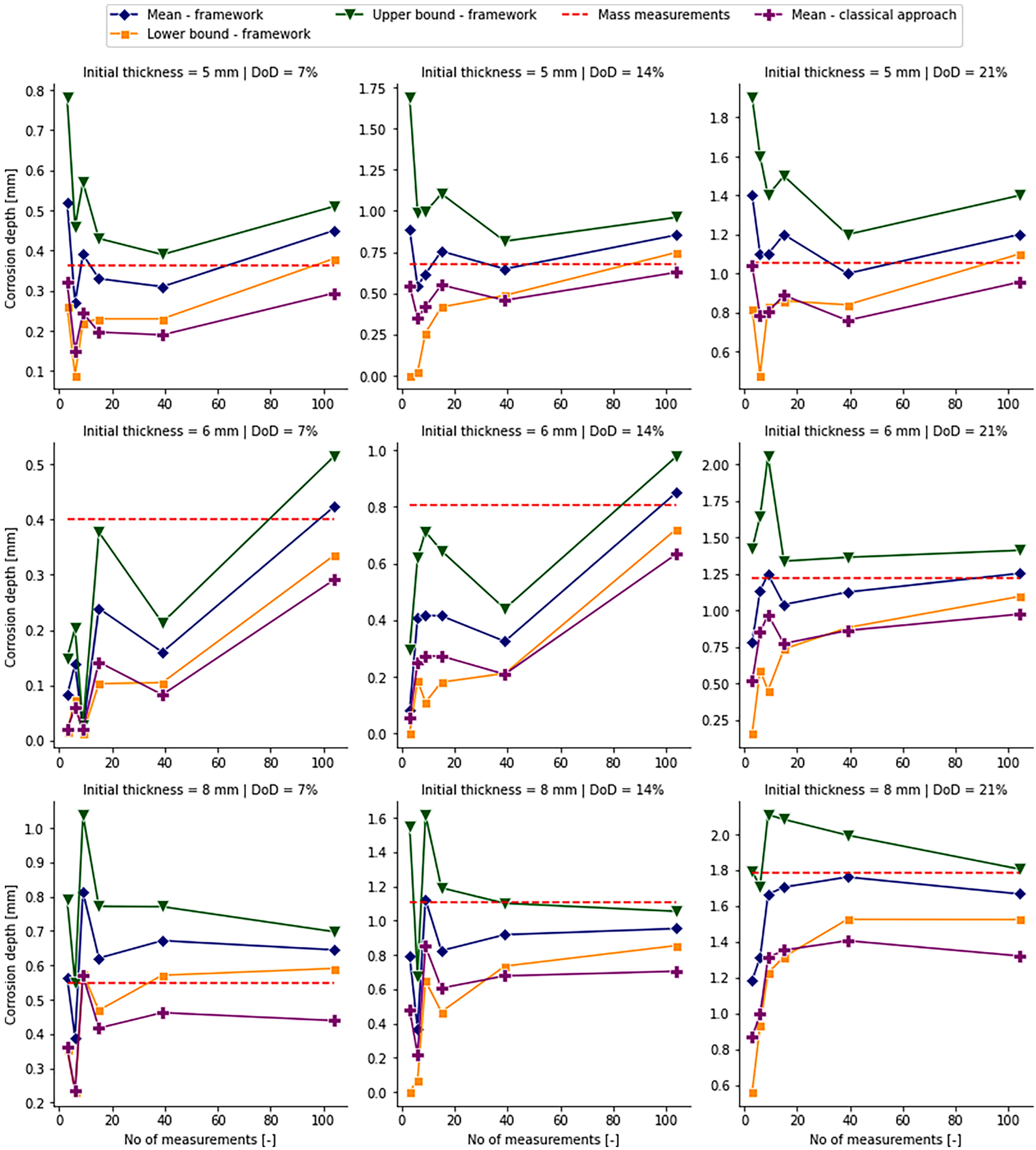

Fig. 10. Framework results – mean corrosion depth – side 1.

proposed framework for predicting actual thickness loss caused by corrosion.

Most importantly, only for 2 cases from 18, the estimated corrosion depth was higher than the estimated upper bound. Thus, in comparison to the classical approach, where only for one case the corrosion depth was assessed higher than the one obtained in mass measurements, the introduction of the measurement error improved the obtained estimated values. For most cases, the classical approach shows lower values than the lower bound from the framework, thus significantly underestimating the actual corrosion depth value. This is consistent with findings presented in [42], where comparing the gauged thickness of corroded plating using the classical technique and the actual thickness values were presented, and the classical method underestimated corrosion depth between 0.6 mm up to 2.2 mm for severely corroded plates. This shows that incorporating the measurement error in the presented work enables us to efficiently and accurately predict the actual thickness loss.

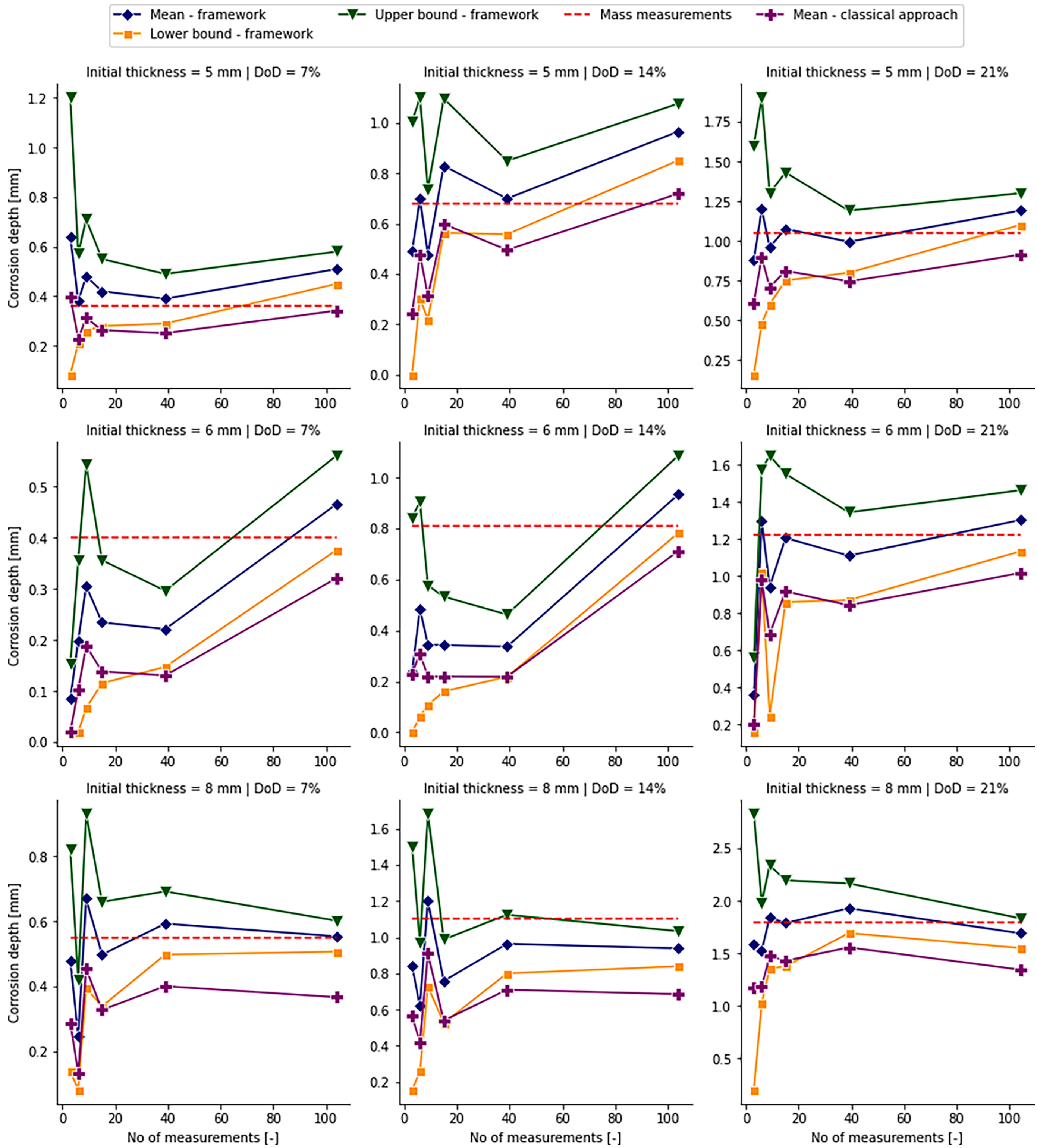

Fig. 11. Framework results – mean corrosion depth – side 2.

Finally, when only three measurement points are considered, a huge spread is observed, and for some specimens, the difference between the lower and upper bound reaches the level of 2.6 mm. It is highlighted that the current industrial norms require a number of measurements as the minimum (the area of the considered plating is around 0.5 m^2). Nevertheless, even with three measurements only, the presented framework will result in the conservative estimation of the corrosion depth (i.e., the upper bound of the mean value is higher than the corrosion depth estimated from mass measurements) for most cases (for 15 of 18 analyzed cases).

The results for standard deviation estimation of the corrosion depth distribution are presented in Figs. 12 and 13. As can be noticed, the spread between the lower and upper bound significantly decreases with more measurements and stabilizes on the constant level for many samples. When only three or six measurements are considered, the spread of results is significant since estimating the entire distribution's standard

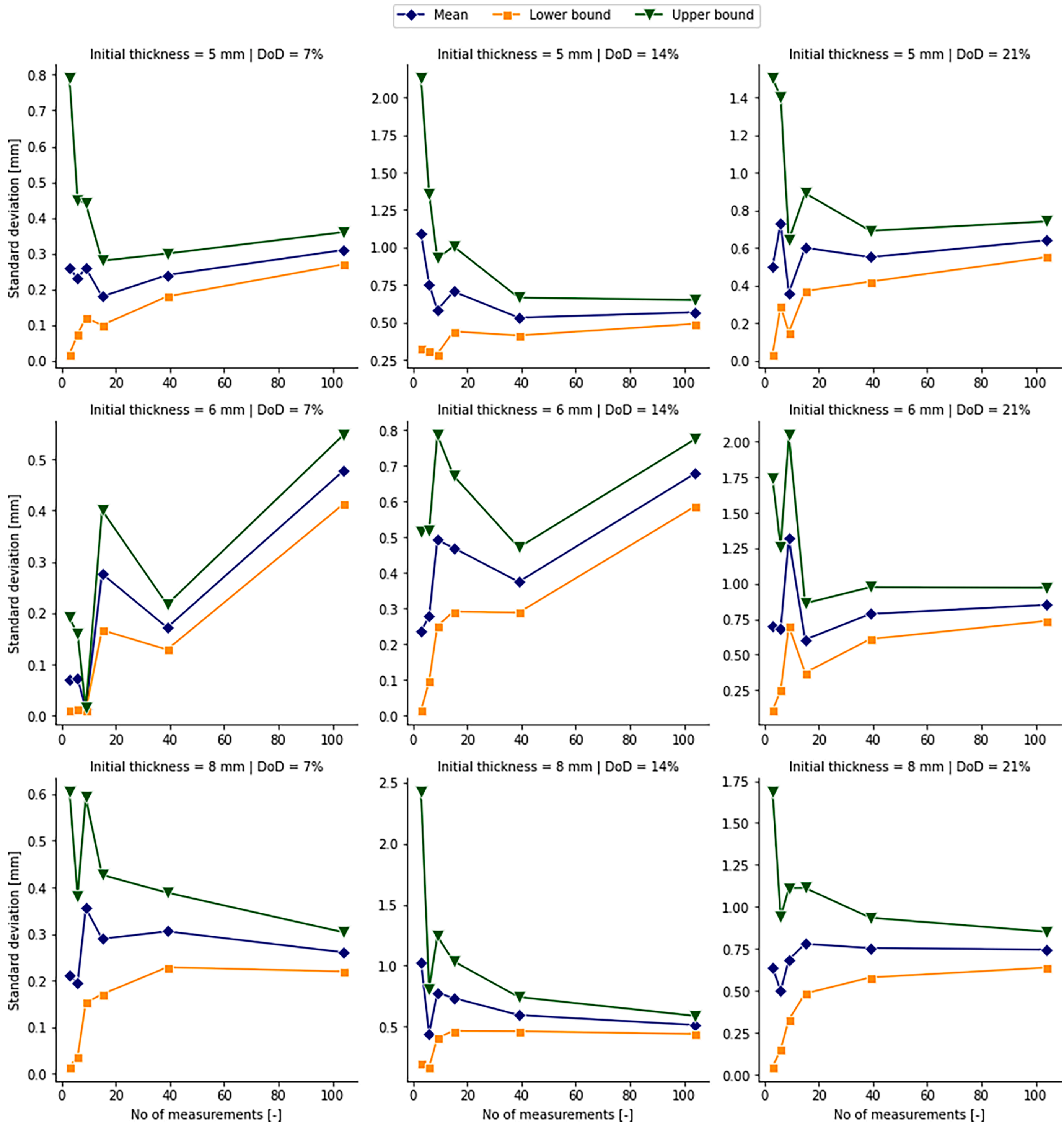

Fig. 12. Framework results – standard deviation of the corrosion depth – side 1.

deviation with a limited number of observations is strict. However, there is a substantial reduction in the spread for nine or more observations. Similarly to the mean value corrosion depth, when the upper bound from the framework is considered, it will result in a conservative estimation of the standard deviation.

To investigate the dependency between the corrosion degradation level and the number of measurements, the distribution descriptors of the mean value estimated in the framework are plotted as a function of the corrosion depth evaluated based on the mass measurements (see Fig. 14). It is noted that if there is a low number of measurements (3,6,9), the uncertainty of the estimation increases significantly with the corrosion degradation level. However, for the high number of measurements (104), there is no significant increase in confidence interval value between lower and higher degradation levels. Thus, more measurements will be needed to correctly estimate the corrosion depth and its probabilistic descriptors, especially for higher degradation levels.

4.2. Optimum number of measurements

As noted in Figs. 10 and 11, the framework defines the mean value of

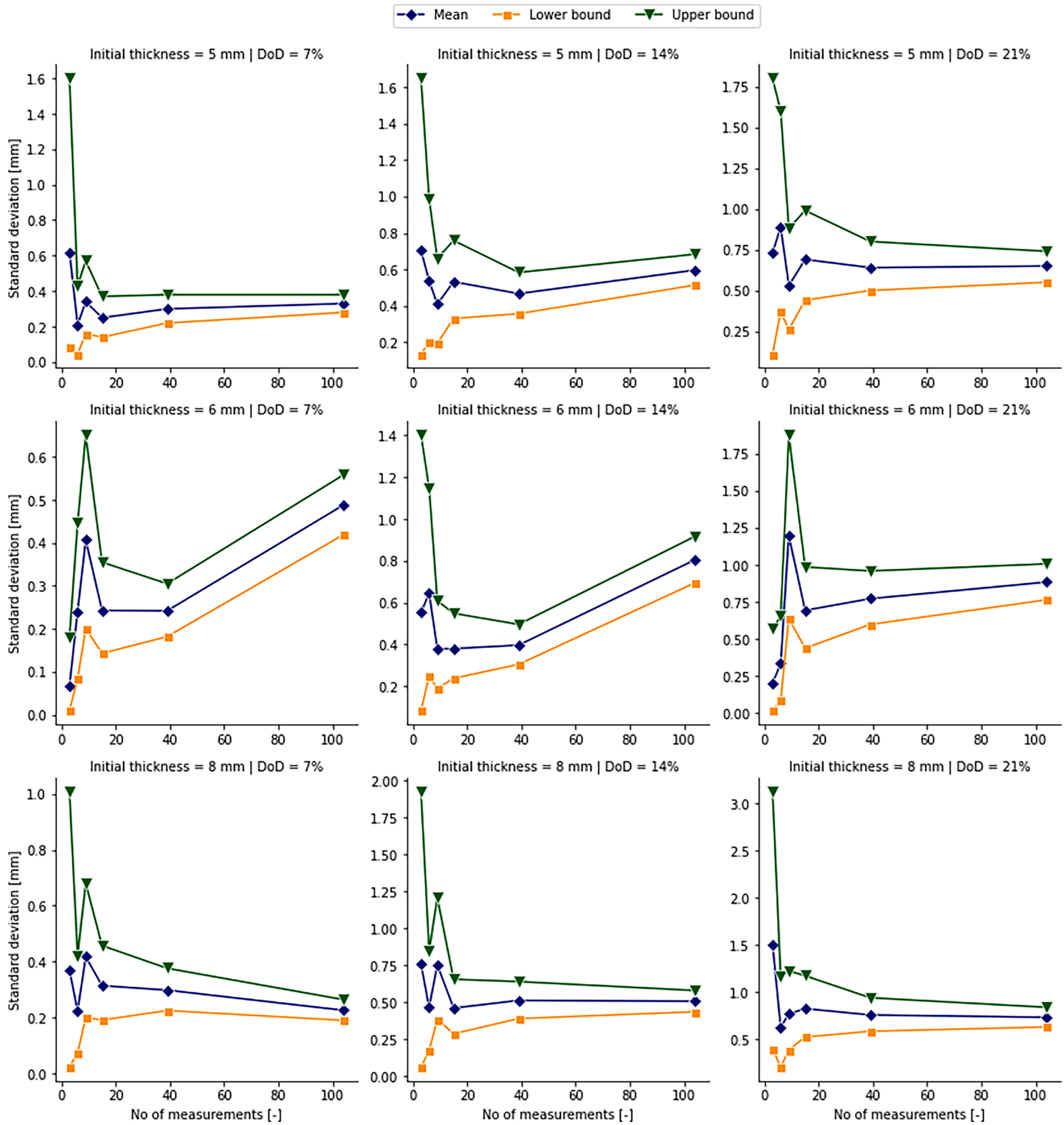

Fig. 13. Framework results – standard deviation of the corrosion depth – side 2.

corrosion depths closest to the mean value estimated from mass measurements is not always for the highest number of measurements. One reason may be the so-called accumulation of the basic error resulting in a cumulative error [43]. Thus, if each measurement is subjected to primary uncertainty, the infinite increase of measurements will not result in cancelling the cumulative error, but in some cases, increase due to the cumulation of basic errors of each measurement.

The accuracy margin $AM_{depth} = depth_{mass} - depth_{framework}$ is defined as a function of the corrosion depth defined by the mass measurement and the depth estimated by the framework, which are independent truncated to zero normally distributed variables, and the probability that the $depth_{mass}$ matches the $depth_{framework}$ is expressed as:

$$P_{AM_{depth}} = P(AM_{depth} = 0) \tag{5}$$

Since the accuracy margin, AM_{depth} is a function of the random variables $depth_{mass}$ and $depth_{framework}$, it is a random variable, too and also truncated to zero normally distributed.

The two first moments of the accuracy margin AM_{depth} are determined by:

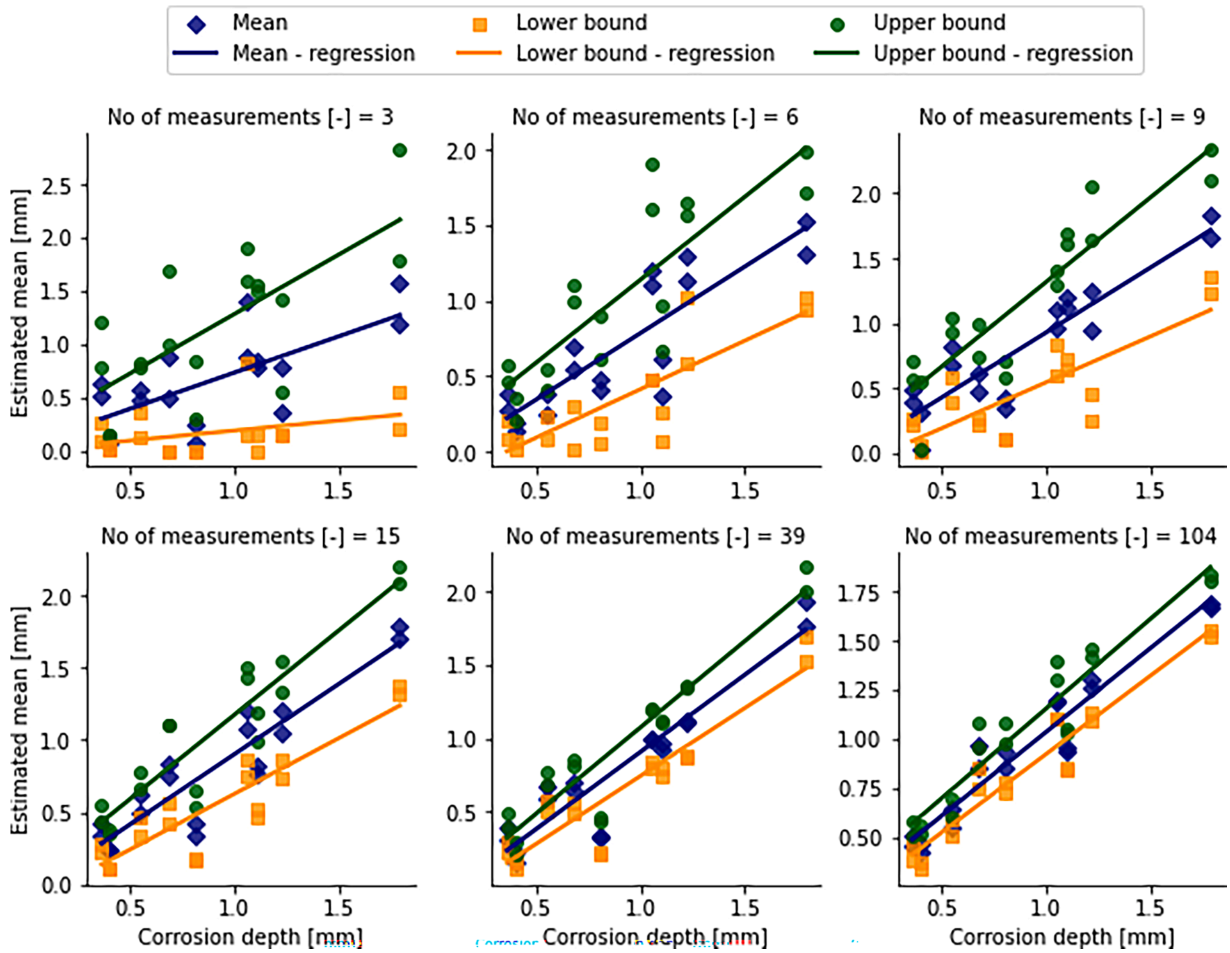

Fig. 14. Mean value corrosion depth as a function of corrosion depth from mass measurements.

$$E_{AM_{depth}} = E(depth_{mass}) - E(depth_{framework}) \quad (6)$$

$$\sigma_{AM_{depth}} = \sqrt{\sigma^2_{depth_{mass}} + \sigma^2_{depth_{framework}}} \quad (7)$$

where $E_{AM_{depth}}$ is the mean value margin, and $\sigma_{AM_{depth}}$ is the standard deviation of the accuracy margin. In the present study $\sigma^2_{depth_{mass}}$ is considered zero.

The accuracy margin is a linear function of the corrosion depth defined by the mass measurement and estimated by the framework, and the probability of their matching is defined as:

$$P_{AM_{depth}} = \frac{\Phi\left(\frac{0 - E_{AM_{depth}}}{\sigma_{AM_{depth}}}\right) - \Phi\left(\frac{0}{\sigma_{AM_{depth}}}\right)}{1 - \Phi\left(\frac{0}{\sigma_{AM_{depth}}}\right)} \quad (8)$$

and the accuracy matching index is defined by:

$$\alpha = -\Phi^{-1}\left(P_{AM_{depth}}\right) \quad (9)$$

By this definition, the closer the accuracy index is to zero, the more accurate the estimation is. An accuracy index higher than zero means overestimating the corrosion depth (conservative), and less than zero means an underestimate.

Fig. 15 plots the accuracy index for each specimen and side of measurement depending on the number of measurements.

It is noted that in most cases, determining the optimum number of measurements is not straightforward. The blue dashed line represents the 0-value of the index, showing an ideal estimation of the corrosion depth. For some specimens, the optimum number of measurements will be the one with the alpha index closest to zero, where the plot lies on one side of the 0-line. However, in some cases, the alpha index oscillates between negative and positive values. In that case, we can choose the higher value since we generally have a lower spread in the mean value for a higher number of measurements. The proposed optimum number of measurements is presented as red dots in graphs (see Fig. 15). In all cases, approximately 60 measurements result in the lowest alpha index.

5. Conclusions

The study aimed to introduce and validate a probabilistic framework based on the limited measurement data for a corrosion depth distribution estimation. It was shown that background knowledge about the corrosion degradation process and its impact on structural integrity makes it possible to estimate the corrosion depth distribution within the single structural component by incorporating the measurement error having a probabilistic representation. Compared to the simple statistical parameters obtained from measurements, the developed methodology allows quantifying the confidence intervals of those parameters. In that case, mean value and standard deviation are considered random variables instead of deterministic ones.

It was found that with a low number of measurements, the corrosion depth distribution parameters (mean value and standard deviation) are highly uncertain. Nevertheless, even in that case, the upper bound of these parameters could effectively be used as a conservative estimation of the corrosion depth distribution. Thus, the incorporation of measurement error, as well as the treatment of corrosion field parameters as

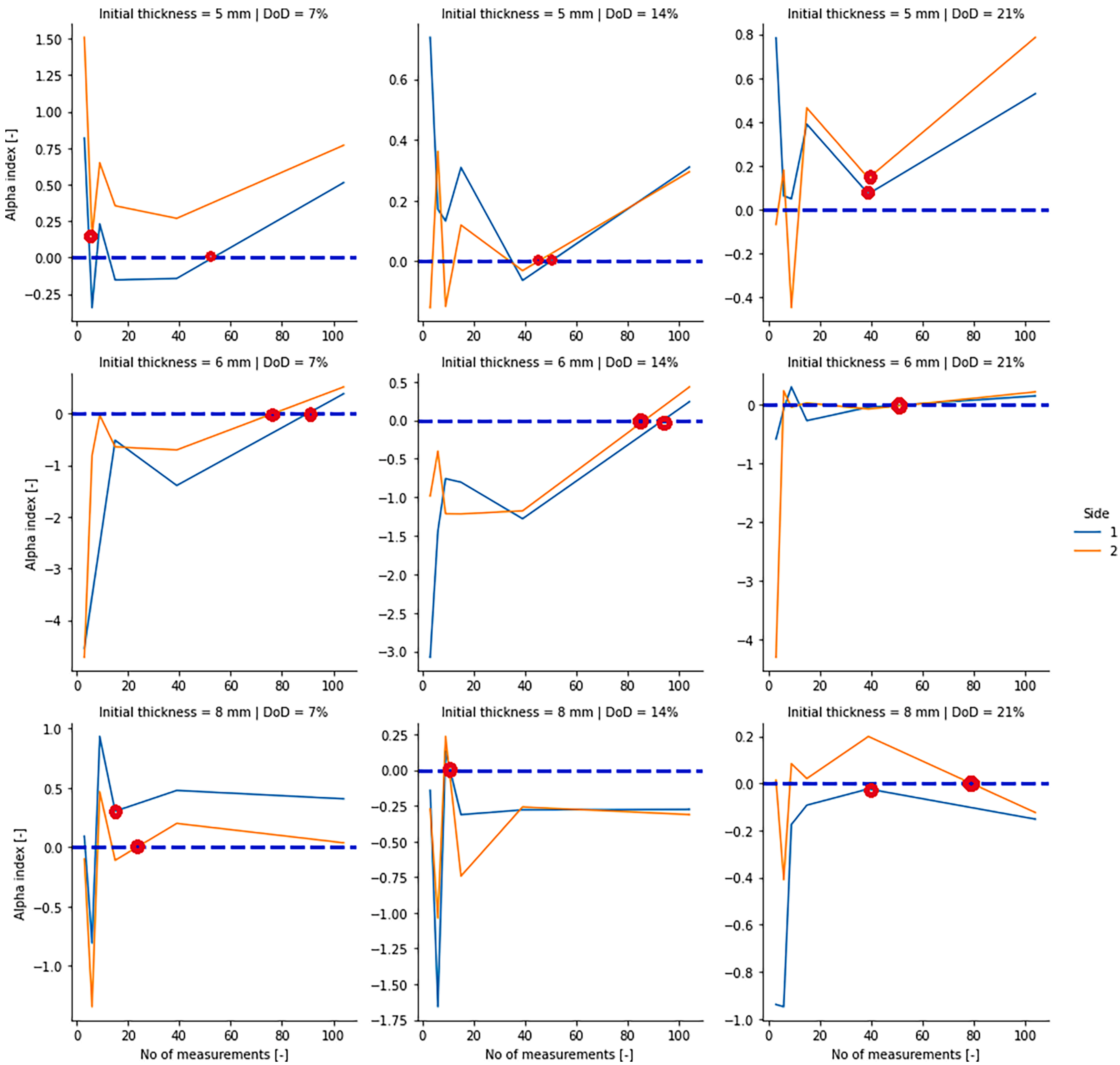

Fig. 15. Accuracy index as a function of a number of measurements.

random variables, provides a more accurate and safer estimation of actual corrosion loss. On the contrary, the significant corrosion degradation may be easily omitted where only deterministic values from measurements are considered.

The presented study may also trigger the discussion about reconsidering the current Classification Societies' approach regarding currently used guidelines for performing such measurements, where the required minimum measurement points number is relatively low. The proposed accuracy index could help determine the necessary number of measurements. It must also be noted that introduced measurement error could be extended in terms of other sources of uncertainties, such as measurement conditions and precision of the measuring tool itself. In addition, the renewal criteria (decision about the eventual replacement of structural element) are currently set based on the results of the classical approach. Incorporating the presented framework may lead to another decision-making regarding renewals during inspections.

Compared to the classical approach, the developed framework is more accurate for estimating corrosion degradation. The possible weakness of the solution is the considerable computation time since the Bayesian inference is based on random sampling. Thus, more studies are needed when this solution is applied in practice. Finally, the measurement error adopted in the framework was calculated based on previous corrosion testing. Depending on the corrosion type and surrounding environmental conditions, more studies are needed to define the measurement error properly. Finally, benchmarking with other possible solutions that may come up in future should be performed.

The developed probabilistic framework may be used as a decision-making tool. Firstly, more dense measurements can be made based on the observed uncertainty level. Secondly, the decision can be made about the eventual replacement of structural components. Since the developed framework is also an uncertainty-quantification tool, it may also be used for the reliability assessment of the residual strength of the corroded structural components.

CRediT authorship contribution statement

Krzysztof Woloszyk: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Writing – original draft, Writing – review & editing. **Yordan Garbatov:** Formal analysis, Methodology, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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