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ESTIMATING AVERSION TO RANK INEQUALITY UNDERLYING SELECTED ITALIAN INDICES OF INCOME INEQUALITY

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SUMMARY

In this paper, we estimate aversion to rank inequality (ATRI) underlying selected Italian income inequality indices, I, notably the Pietra index, the Bonferroni index and the "new" Zenga index. We measure ATRI by the parameter v of the generalised Gini index G(v). ATRI is distinct from aversion to income inequality, as measured by parameter ε of Atkinson's index A(ε). We propose eliciting v from the equation I = GE(v). As, in general, an analytical solution to this equality can be cumbersome, we retrieve v from the empirical equation $\hat{I} = \hat{G}(v)$ *where the symbols* $\hat{I}$ *and* $\hat{G}(v)$ *denote the estimates of I and G(v), respectively. We also calculate the benchmark income x* such that adding a small income to it does not affect inequality. In this paper, we solve the equation using the estimates of the Italian inequality indices for Poland from 2000 to 2017. We have found, on average, v≈1.5 for the Pietra index, v ≈ 3 for the Bonferroni index, and v ≈ 11 for the Zenga index.*

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1. INTRODUCTION

Reducing income inequality is an essential aim of many contemporary societies. It is also fundamental to achieving the United Nations Sustainable Development Goals (UNDESA, 2016). The distributional analysis offers plenty of inequality indices. The question is, what index of inequality should an analyst use when appraising inequality-reducing policy?

Applied welfare economics has established several normative criteria for indices of income inequality. There may be some underlying notion of a social welfare function (Kolm, 1969; Atkinson, 1970; Blackorby and Donaldson, 1978; Sen, 1997, pp. 117-119). We discuss some basic normative principles for inequality comparisons in Section 2.1.

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Aversion to income inequality (ATII), as measured by the parameter ε of the constant inequality aversion utility function, is the well-known normative criterion in assessing welfare in income distributions (Atkinson (1970). The higher the value of ε, the more sensitive a social decision maker is to differences *in income* at the bottom end of the income distribution. The parameter ε cannot be measured directly. In the literature, various methods of eliciting ε from empirical data have been proposed (see, e.g. Kot, 2020, for a review). Recently Kot and Paradowski (2022) presented the world atlas of aversion to income inequality.

Aversion to rank inequality (ATRI) is a new normative criterion that Araar and Duclos (2005) proposed. ATRI is captured by the parameter v of the generalised Gini index $GE(v)$ (Kakwani, 1980; Donaldson and Weymark, 1980, 1983; Yitzhaki, 1983). The higher the value of v, the more sensitive a social decision maker is to differences *in income ranks* at the bottom of the income distribution when assessing transfers from the rich to the poor. ATRI cannot be measured directly. Duclos (2000) proposes eliciting v from data obtained by a *leaky bucket* experiment. This experiment suggests the values for v between 1 and 4.

The 'ordinary' Gini index, G, (Gini, 1914), reflects the ATRI level v of 2 (see, e.g. Kakwani, 1980). This observation raises the question of what ATRI levels other income inequality indices could reflect.

This paper aims to answer the above question for selected Italian indices of income inequality, namely the Pietra index (Pietra, 1915), the Bonferroni index (Bonferroni, 1930) and the *new* Zenga index (Zenga, 2007). Recently various properties of these indices have been studied (Giorgi, 1998; Polisicchio, 2008; Maffenini and Polisicchio, 2014; Arcagni and Porro, 2014; Pasquazzi and Zenga, 2018; Zenga, 2016; Zenga and Jedrzejczak, 2020; Zenga and Valli, 2020, 2021). However, ATRI for the inequality indices in question has not been studied yet.

The abovementioned question can be specified as follows: "What would v be if the observed value of an inequality index I were exactly equal to the value of $GE(v)$?" The solution to equation $I = GE(v)$, concerning v, will answer this question.

In general, however, analytical solving of the equation $I = GE(v)$ is cumbersome. Because of that, we estimate v from the empirical equation $\hat{I} = \widehat{GE}(v)$, where the symbols $\hat{I}$ and $\widehat{GE}(v)$ denote estimates of inequality indices I and $GE(v)$, respectively. We solve this equation numerically for the abovementioned Italian inequality indices using data on Poland's household disposable income per capita from 2000 to 2017.

Having estimates of v, which satisfy identity $I = GE(v)$, we can calculate the benchmark income x^* that divides an income distribution into two parts. An increase in someone's income $x < x^*$ reduces inequality, while an increase in someone's income $x > x^*$ enhances inequality (Hoffman, 2001; Lambert and Lanza, 2006; Corvalan, 2014). Although exact formulae for x^* have not been derived yet for Italian indices (except the ordinary Gini index), we can calculate the threshold using the known expression for x^* for $GE(v)$.

The rest of the paper is organised as follows. Section 2 presents the details of the method of retrieving v. Section 3 contains empirical results. Section 4 concludes.

2. THE METHOD

2.1 *The selected italian indices of inequality*

When comparing inequality in alternative income distributions, ambiguity arises when the underlying Lorenz curves intersect (Atkinson, 1970). Mehran (1976) noticed that: "One possible solution is to implicitly formulate preferences in terms of a social welfare function and compare the alternative distributions according to the values of the derived income inequality measure".

A basic normative principle for inequality comparisons is the *Pigou-Dalton* principle of *transfers*. According to it, a positive income transfer from a rich person to a more miserable one reduces inequality when other things remain the same (Sen, 1973, p. 27). The *principle of diminishing transfers* is more demanding, according to which a small positive transfer from a richer to a poorer individual, with a given proportion of the population in between them, decreases the inequality, and the decrease is larger the poorer the recipient is. Loosely speaking, the principle states that inequality among the rich is less important than inequality among the poor.

Below, we present the selected indices of income inequality in the form of Mehran's (1976) linear measures M[1], namely

$$M = 1 - \int_0^1 k(p) L(p)\, dp, \quad p \in [0,1] \tag{1}$$

where $L(p)$ is the Lorenz curve, and $k(p)$ is a function such that $\int_0^1 pk(p)\, dp = 1$. Mehran (1976) proved that M satisfies the *Pigou-Dalton* principle of transfers if and only if $k(p) > 0$. M satisfies the principle of diminishing transfers if and only if $k(p) > 0$ and $k'(p) < 0$, for all $p \in [0,1]$.

The Gini index, G, can be defined as

$$G = 1 - \int_0^1 2L(p)\, dp, \quad p \in [0,1], \tag{2}$$

where $L(p)$ is the Lorenz curve (Lambert, 2001, p. 33). G is the most commonly used descriptive measure of inequality. Note that G ascribes the equal weight of 2 to all ranks.

There is a widespread opinion among economists that the Gini index gives more weight to transfers in the centre of the distribution than at the tails (Atkinson, 1970, Kakwani, 1980, p. 72). Recently Gastwirth (2017) showed that this opinion is incorrect.

Allowing the weight $k(p)$ in (2) to vary along with p leads to the family $GE(v)$ of generalised Gini indices, namely

$$GE(v) = 1 - \int_0^1 v(v-1)(1-p)^{v-2} L(p)\, dp, \tag{3}$$

for $v > 1$, $p \in [0,1]$ (Kakwani, 1980; Donaldson and Weymark, 1980, 1983; Weymark, 1981; Yitzhaki, 1983, Bosset, 1990). Araar and Duclos (2005) interpret v as *the aver-*

[1] Actually, Mehran (1976) uses weights $w(p)$ such that $k(p) = w'(p)$, where $w'(p)$ is the first derivative of $w(p)$.

sion to rank inequality, a distinct concept from aversion to income inequality. For $v < 2$, more ethical weight is applied to higher rank *p*. For $v = 2$, (3) becomes the ordinary Gini index *G* which applies equal weight to all *p*. For $v > 2$, *GE(v)* applies greater weight to lower ranks *p*. In general, the greater the value of *v*, the more sensitive the social decision maker to differences *in ranks* when granting ethical weights to individuals (Araar and Duclos, 2005).

The Bonferroni index, *B,* can be defined as

$$B = 1 - \int_0^1 \frac{1}{p} L(p)\, dp, \quad \text{for } p \in [0,1] \tag{4}$$

(Bonferroni, 1930; Nygård and Sandström, 1981, p. 276; Giorgi, 1998; Chakravarti, 2007). Examining (4) shows that *B* gives more weight to lower ranks among the poor.

The *new* Zenga index *Z* can be expressed in terms of the Lorenz curve as follows:

$$Z = 1 - \int_0^1 \frac{1}{p} \cdot \frac{1-p}{1-L(p)} L(p)\, dp, \quad \text{for } p \in [0,1] \tag{5}$$

(Zenga, 2007, Langel and Tillé, 2012). Examining (4) shows that the weight of *Z* is equal to the weight of *B* magnified by a positive and greater than 1 number $(1-p)/(1-L(p))$ for non-egalitarian income distributions.

The Pietra index, *P,* of income inequality has the form

$$P = \max_p \left[p - L(p) \right], \quad \text{for } p \in [0,1] \tag{6a}$$

(Pietra, 1915)[2]. The Pietra index equals half of the mean absolute deviation divided by the mean μ, to wit

$$P = \frac{1}{2\mu} \int_0^\infty |x - \mu|\, dF(x) \tag{6b}$$

Examining equations (2), (4), and (5) shows that indices *G, B* and Z satisfy the Pigou-Dalton principle of transfers since they have positive weight $k(p)$. For the generalised Gini, *GE*(v), the weight $k(p) = v(v-1)(1-p)^{v-2} > 0$ when $v > 1$. Thus if a decision maker wants to approve every transfer from higher to lower income, these indices are appropriate. It is easy to see that *B,* and *Z* also satisfy Mehran's principle of diminishing transfers since $k'(p) < 0$. *GE*(v) satisfies this principle for $v > 2$. If a decision maker wants to detect the reaction of inequality to the transfer between individuals when the ranks of their incomes matter, *B, Z,* and *GE*($v > 2$) are suitable, while the ordinary Gini index is not.

Mehran (1976) demonstrated that the Pietra index *P* violates the Pigou-Dalton principle of transfers. However, *P* satisfies a weaker version of this principle, namely, "(…) a small positive transfer from rich to poor does not increase inequality." (Mehran, 1976).

[2] The Pietra index is also known as the Ricci index, the Schutz index, the Hoover index or the Robin Hood index.

2.2 *The method of retrieving aversion to rank inequality*

As we mentioned in Section 1, we search for an answer to the following question: "What would the level v of ATRI be if the value of an inequality index, I, were exactly equal to the value of $GE(v)$?" The solution to the equation

$$I = GE(v), \tag{7}$$

concerning v will answer this question.

However, a general solution to this equation is difficult to obtain explicitly. Because of this, we propose *to estimate* v from the empirical equation of the following form:

$$\hat{I} = \widehat{GE}(v), \tag{8}$$

where the symbols $\hat{I}$ and $\widehat{GE}(v)$ denote the estimates of inequality indices I and $GE(v)$, respectively. We solve this nonlinear equation using IMSL subroutine NEQNF, which is a modified Powell hybrid algorithm and a finite-difference approximation to the Jacobian. For further description, see More *et al.* (1980).

We can apply the estimates of ATRI for the assessment of social welfare W in an income distribution according to the following formula

$$W = \mu[1 - GE(v)] \tag{9a}$$

(Lambert, 2001, p. 125). According to identity (7), W can be calculated as

$$W = \mu(1 - I) \tag{9b}$$

Social welfare (9) has an interpretation as the *equally distributed equivalent income* (EDEI). EDEI is the income that, if received by all individuals, provides the value of *welfare* precisely the same as the actual distribution (Kolm 1969; Atkinson 1970; Sen 1973: 42; Balckorby, Donaldson, 1978).

We can use the retrieved value of v for calculating the *benchmark income.* Hoffman (2001) observed that income inequality would decrease if low income increased by a small amount. When high-income increases by a small amount, income inequality increases. Therefore, a specific income level, x^*, dividing these effects, must exist. The author called x^* *the relative poverty line* or *dividing line* between the rich and the poor.

The level x^* depends on an inequality measure one is using. Lambert and Lanza (2006) proved the existence of x^* for a broad class of inequality indices. The authors call x^* *the benchmark level of income*[3]. We shall refer to $z^* = x^*/\mu$ as the relative benchmark income.

Unfortunately, exact expressions for x^* have not been derived yet for B, Z, and P indices. We shall overcome this inconvenience by calculating x^* for $GE(v)$ and ascribing x^* to I according to identity (7).

The benchmark income for $GE(v)$ equals $x^* = F^{-1}(\alpha^*)$, where α^* is the position (rank) of x^* defined as

$$\alpha^* = 1 - \frac{1}{v}[1 - GE(v)]^{1/(v-1)} \tag{10}$$

[3] Corvalan (2014) referred to x^* as *the pivotal income*.

(Lambert and Lanza, 2006). Corvalan (2014) shows that α^* is strictly increasing in *v*. Thus, the stronger a decisionmaker's ATRI, the higher the rank α^* of the benchmark income level x^*.

The knowledge of the benchmark income x^* underlying inequality indices in question is of great importance for an inequality-reducing policy. Imagine transferring a small income, δ, from a person placed at l to a person placed at j, $j < l$. Let the transfer not change the initial position of the donor and recipient. During the transfer, a certain fraction, say q_0, of δ, is lost according to the leaky bucket effect (Okun, 1975). Lambert and Lanza (2006, theorem 7) demonstrated that case $0 < q_0 < 1$ occurs if the donor and recipient are positioned below α^*. Otherwise Seidl's (2001) paradox will appear; notably, the leakage will either exceed the amount taken away ($q_0 < 1$), so the recipient may lose too, or be negative, so the recipient may receive more than the donor gives up ($q_0 > 1$), without no adverse effect on inequality.

3. EMPIRICAL RESULTS

We estimate inequality indices *P, G, GE*(*v*), *B,* and *Z* using statistical micro-data data from the Polish Household Budget Surveys for 2000-201. The household monthly disposable incomes per capita are in constant 2011 prices PPP adjusted. We omit null and negative incomes. We use household sizes as weights.

Table 1 presents the estimates of Italian income inequality indices and ATRI's corresponding level v. Figures 1-3 display the changes in ATRI over the years.

TABLE 1. - *Aversion to rank inequality underlying Italian indices for Poland*

Year	Pietra		Gini	Bonferroni		Zenga	
	Index	v	Index	Index	v	Index	v
2000	0.23539	1.53037	0.33488	0.45136	3.01910	0.67309	10.84102
2001	0.23531	1.54117	0.33283	0.45017	3.01391	0.67107	10.66139
2002	0.24203	1.53766	0.34176	0.45842	2.99856	0.68002	10.93047
2003	0.24497	1.54067	0.34533	0.46245	2.99142	0.68389	10.91439
2004	0.25027	1.53873	0.35297	0.47110	2.98353	0.69262	10.72133
2005	0.24512	1.53575	0.34632	0.46290	2.99357	0.68475	11.05941
2006	0.24117	1.53634	0.34085	0.45657	2.99855	0.67840	11.22831
2007	0.23921	1.52440	0.33981	0.45355	3.01248	0.67623	11.60742
2008	0.23448	1.53078	0.33245	0.44649	3.01670	0.66854	11.44110
2009	0.23421	1.53407	0.33222	0.44722	3.01233	0.66885	11.16552
2010	0.23709	1.52654	0.33672	0.44979	3.00581	0.67228	11.66467
2011	0.23584	1.52910	0.33549	0.45106	3.01896	0.67301	11.07414
2012	0.23629	1.52664	0.33638	0.45152	3.01459	0.67361	11.18084
2013	0.23776	1.52557	0.33892	0.45553	3.01742	0.67755	10.93264
2014	0.23113	1.53618	0.32856	0.44572	3.02090	0.66664	10.68248
2015	0.22721	1.53384	0.32370	0.44037	3.03389	0.66096	10.78372
2016	0.21161	1.53939	0.30077	0.41182	3.05426	0.63085	11.57549
2017	0.20373	1.53060	0.29030	0.39635	3.07463	0.61538	12.67484
Mean	0.23460	1.53321	0.33279	0.44791	3.01559	0.66932	11.17439
Std.Dev.	0.01125	0.00540	0.01525	0.01769	0.02192	0.01852	0.49093

Source: own calculations using data from Polish Household Budget Surveys 2000-2017.

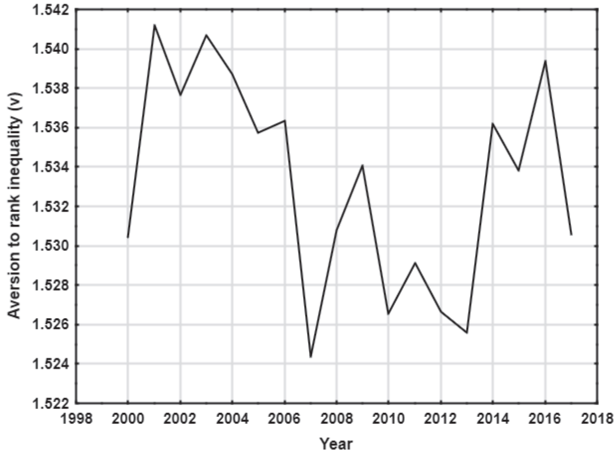

FIGURE 1. - *Aversion to rank inequality underlying the Pietra index*

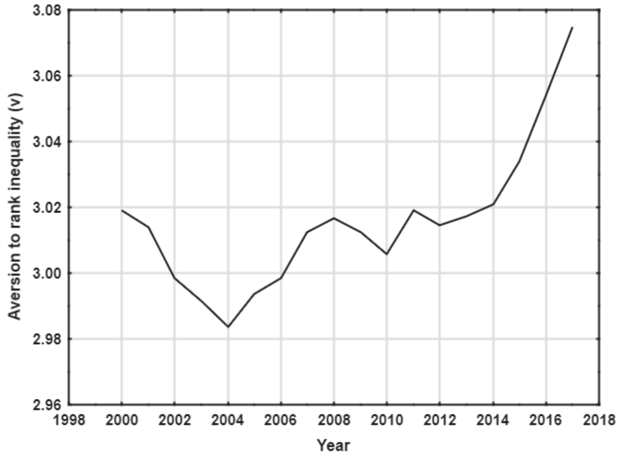

FIGURE 2. - *Aversion to rank inequality underlying the Bonferroni index*

Examining Table 1 and Figures 1-3 shows two main features. First, ATRI varies over the years for all analysed inequality indices. To explain ATRI time-specific, one can imagine various competitive policies being characterised by a distinct level of ATRI, thus offering a different extent of the redistribution of incomes. Every year a society may promote a policy most suitable for the current challenges of an economic and social environment.

Second, each inequality measure reveals a different level of ATRI. Policymakers who use the Zenga index, the Bonferroni index, or $G(v > 2)$ for assessing inequality in a given income distribution show much stronger ATRI than a politician using the ordinal Gini index. The Pietra index reveals the lowest level of ATRI.

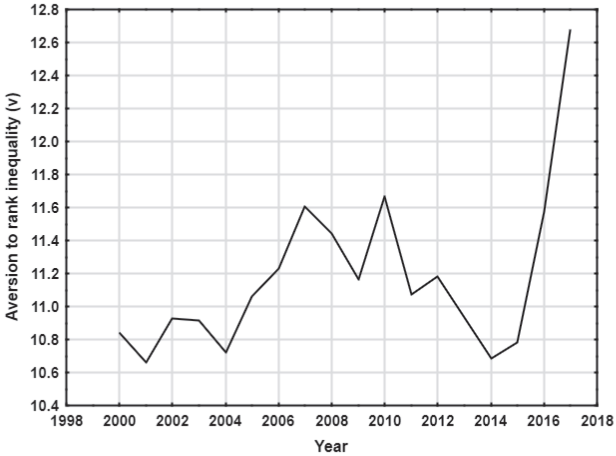

FIGURE 3. - *Aversion to rank inequality underlying the Zenga index*

Table 2 presents estimates of the relative benchmark incomes z^* and their ranks in income distributions.

Examining Table 2 shows that the greater level of ATRI, the greater the rank predicted by an index of inequality and the greater the relative benchmark z^*. The Pietra

TABLE 2. - *The rank and the relative benchmark income z* for Poland, 2000-2017*

Year	Pietra		Gini		Bonferroni		Zenga	
	Rank	z*	Rank	z*	Rank	z*	Rank	z*
2000	0.6061	0.9732	0.6674	1.0602	0.7540	1.2180	0.9177	1.8498
2001	0.6048	0.9747	0.6664	1.0687	0.7535	1.2341	0.9164	1.8512
2002	0.6116	0.9770	0.6709	1.0697	0.7546	1.2343	0.9184	1.8770
2003	0.6140	0.9780	0.6727	1.0739	0.7552	1.2385	0.9184	1.8972
2004	0.6193	0.9829	0.6765	1.0778	0.7569	1.2438	0.9174	1.9126
2005	0.6148	0.9789	0.6732	1.0734	0.7554	1.2330	0.9194	1.8945
2006	0.6109	0.9759	0.6704	1.0672	0.7542	1.2304	0.9203	1.8741
2007	0.6105	0.9677	0.6699	1.0580	0.7542	1.2072	0.9225	1.8746
2008	0.6051	0.9666	0.6662	1.0570	0.7528	1.2160	0.9214	1.8613
2009	0.6045	0.9741	0.6661	1.0636	0.7527	1.2184	0.9197	1.8425
2010	0.6082	0.9685	0.6684	1.0563	0.7530	1.2070	0.9228	1.8778
2011	0.6067	0.9685	0.6678	1.0577	0.7539	1.2139	0.9192	1.8508
2012	0.6074	0.9742	0.6682	1.0605	0.7538	1.2125	0.9199	1.8398
2013	0.6090	0.9797	0.6695	1.0669	0.7548	1.2147	0.9184	1.8395
2014	0.6013	0.9822	0.6643	1.0749	0.7528	1.2299	0.9164	1.8038
2015	0.5977	0.9776	0.6619	1.0668	0.7522	1.2221	0.9170	1.8048
2016	0.5820	0.9585	0.6504	1.0465	0.7471	1.2038	0.9214	1.7835
2017	0.5747	0.9425	0.6452	1.0271	0.7450	1.1759	0.9273	1.7979
Mean	0.6049	0.9723	0.6664	1.0626	0.7531	1.2196	0.9197	1.8518
Std.Dev.	0.0101	0.0096	0.0076	0.0119	0.0028	0.0161	0.0027	0.0364

Source: own calculations using data from Polish Household Budget Surveys.

index has x^* equal to about 60% percentile, whereas the Zenga index has x^* equal to about 90% percentile of income distributions. Notice that the Gini index has x^* close to the mean income.

From Seidl's (2001) paradox point o view, the Zenga index is 'the safest'. If a policymaker assesses inequality with this index, the paradox might appear only in 10 per cent of transfers since 90 per cent of potential donors and recipients are positioned below the benchmark x^*. On the other extreme, there will be a 40 per cent of chance for this paradox if a policymaker uses the Pietra index for assessing income inequality.

Table 3 presents the values of the absolute benchmark x^* and EDEI (social welfare). Figure 4 displays trends of x^* in the years.

The table shows that the inequality indices in question assess the benchmarks x^* and social welfare differently. However, Figure 4 shows that the measures of inequality exhibit a consistent pattern of changes over the years.

Table 3 shows that the benchmark income x^* is not the right candidate for a poverty line, as Hoffman's (2001) term 'the relative poverty line' suggests. Note that all inequality indices in Table 3 predict the values of x^* higher than EDEI. Thus, using x^* as a poverty line would present a peculiar theoretical situation in which the social decision maker promised the eradication of economic inequality for the price of prevalent poverty. Such a promise will not gain conscious attention in any reasonable society.

TABLE 3. - *The absolute benchmark income x^* and the equally distributed equivalent income (EDEI) of Italian indices of inequality for Poland*

Year	Pietra		Gini		Bonferroni		Zenga	
	x^*	EDEI	x^*	EDEI	x^*	EDEI	x^*	EDEI
2000	429	337	467	293	537	242	815	144
2001	429	337	470	294	543	242	815	145
2002	435	337	476	293	550	241	836	142
2003	443	342	487	297	561	244	860	143
2004	467	356	512	307	591	251	909	146
2005	453	350	497	303	571	249	877	146
2006	489	380	535	330	616	272	939	161
2007	527	415	576	360	658	298	1021	176
2008	570	452	624	394	717	327	1098	196
2009	590	464	644	405	738	335	1116	201
2010	610	480	665	418	760	346	1182	206
2011	606	478	661	415	759	343	1157	204
2012	609	478	663	415	758	343	1151	204
2013	622	484	678	420	772	346	1168	205
2014	643	503	704	440	805	363	1181	218
2015	668	528	729	462	835	382	1233	232
2016	724	596	791	528	910	445	1348	279
2017	733	619	799	552	915	470	1399	299
Mean	558	441	610	385	700	319	1061	192
Std.Dev.	102	89	111	81	125	70	184	47

Source: own calculations using data from Polish Household Budget Surveys.

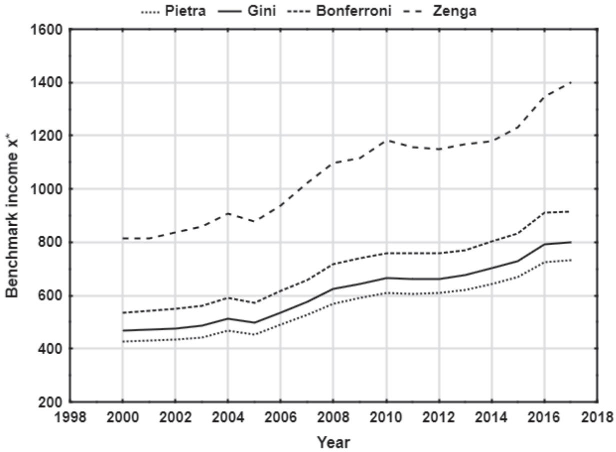

FIGURE 4. - *Trends of benchmark income x* according to the Italian indices of inequality*

4. CONCLUSIONS

This paper estimates aversion to rank inequality (ATRI) that underlines the selected Italian income inequality indices. ATRI differs from a well-known aversion to income inequality, commonly called *inequality aversion*. We show how to estimate the level of ATRI from the generalised Gini index.

Equality $I\text{-}GE(v)$ does not imply, in general, that Italian indices are particular cases of the generalised Gini index. Equality only holds when we are interested in the position (rank) of someone in the distribution of incomes, not in the absolute value of income itself. Because of that, we presented Italian indices in the 'positional' form, according to Lambert and Lanza's (2006) specification of 'non-positional' and 'positional' indices of income inequality. When income matters, one might use the indices' original (non-positional) expressions in question.

The values of v enable us to calculate the benchmark incomes x^* for analysed Italian inequality indices. The knowledge of x^* seems to be crucial for developing anti-inequality programs as well as for assessing leakages of transfers of incomes.

The obtained numerical values of ATRI are consistent with the known theoretical properties of the selected inequality indices. All estimates of ATRI for the Bonferroni and the Zenga indices are greater than 2. It confirms these indices' fulfilment of the positional principle of diminishing transfer. On the other hand, we get $v < 2$ for the Pietra index. This result is consistent with Mehran's (1976) claim that this index of inequality violates the principle in question.

A generalisation of our empirical findings is limited as they concern one country and a short period. Further empirical analyses covering more countries and the years could shed more light on ATRI underlying the Italian income inequality indices.

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REFERENCES

Araar A., Duclos J-Y. (2005). An Atkinson-Gini family of social evaluation functions: Theory and illustration using data from the Luxembourg Income Study. LIS Working Paper Series, No. 416, Luxembourg Income Study (LIS), Luxembourg.

Arcagni A., Porro F. (2014). The graphical representation of inequality. *Revista Colombiana de Estadistica*, **37**(2), 419-436.

Atkinson A. (1970). On the measurement of economic inequality. *Journal of Economic Theory,* **2**(3), 244-263.

Blackorby C., Donaldson D. (1978). Measures of relative equality and their meaning in terms of social welfare. *Journal of Economic Theory*, **18**, 59-80.

Bonferroni C. (1930). *Elementi di Statistica Generale*. Seeber, Firenze.

Bossert W. (1990). An axiomatisation of the single-series Ginis. *Journal of Economic Theory*, **18**, 180-187.

Chakravarty S.R. (2007). A deprivation-based axiomatic characterisation of the absolute Bonferroni index of inequality. *Journal of Economic Theory*, **5**, 339-351.

Corvalan A. (2014). The impact of a marginal subsidy on Gini indices. *Review of Income and Wealth,* **60**(3), 596-603.

Donaldson D., Weymark J. (1980). A single-parameter generalisation of Gini indices of inequality. *Journal of Economic Theory*, **22**, 67-86.

Donaldson D., Weymark J. (1983). Ethically flexible Gini indices for income distributions in the continuum. *Journal of Economic Theory,* **29**, 353-358.

Duclos J.-Y. (2000). Gini indices and the redistribution of income, *International Tax and Public Finance*, **7**, 141-162.

Gastwirth J.L. (2017). Is the Gini index of inequality overly sensitive to changes in the middle of the income distribution? *Statistics and Public Policy*, **4**, 1-11.

Gini C. (1914). Sulla misura della concentrazione e della variabilità dei caratteri, *Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti*, 73, 1203-1248. [Trad. It. F. de

Santis (2005). On the measurement of concentration and variability of characters. *Metron,* **63**, 3-38].

Giorgi G.M. (1998). Concentration Index, Bonferroni. In S. Kotz, (ed.) *Encyclopedia of Statistical Science*, Update Volume 2, Wiley- Interscience, (pp. 141-146).

Hoffmann R. (2001). Effect of the rise of a person's income on inequality. *Brazilian Review of Econometrics*, **21**, 237-262.

Kakwani N.C. (1980). *Income Inequality and Poverty*. *Methods of Estimation and Policy Applications.* Oxford University Press, New York.

Kolm S.-C. (1969). The optimal production of social justice. In J. Margolis and H. Guitton (eds), *Public Economics* (pp. 145-200), Macmillan, London.

Kolm S.-C. (1976). Unequal inequalities, I and II. *Journal of Economic Theory*, **12**, 416-442 and **13**, 82-111.

Kot S.M. (2020). Estimating the parameter of inequality aversion on the basis of a parametric distribution of incomes. *Equilibrium. Quarterly Journal of Economics and Economic Policy,* **15**(3), 391-417.

Kot S.M., Paradowski P.R. (2022). The atlas of inequality aversion: theory and empirical evidence on 55 countries from the Luxembourg Income Study database. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, **17(**2), 261-316.

Lambert P.J. (2001). *The distribution and redistribution of income: a mathematical analysis.* Manchester University Press, Manchester.

Lambert P.J., Lanza G. (2006). The effect on inequality of changing one or two incomes. *The Journal of Economic Inequality,* **4**, 253-277.

Maffenini W., Polisicchio M. (2014).The potential of the $I_{(p)}$ inequality curve in the analysis of empirical distributions. *Statistica & Applicazioni,* **12**(1), 63-85.

Mehran F. (1976). Linear measures of income inequality. *Econometrica*, **44**, 805-809.

More J., Garbow B., Hillstrom K. (1980). User guide for MINPACK-1, Argonne National Labs Report ANL-80-74, Argonne, Illinois.

Nygård F., Sandström A. (1981). *Measuring Income Inequality.* Almqvist & Wiksell International, Stockholm.

Okun A.M. (1975). *Equality and efficiency.* Washington, DC, Brookings Institution.

Pasquazzi L., Zenga M. (2018). Components of Gini, Bonferroni, and Zenga inequality indexes for EU income data. *Journal of Official Statistics*, **34**(1), 149-180.

Pietra G. (1915). Delle relazioni fra indici di variabilità note I e II, *Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti,* **74** (2), 775-804.

Polisicchio M. (2008). The continuous random variables with uniform point inequality measure $I_{(p)}$. *Statistica & Applicazioni*, **6**(2), 137-157.

Seidl C. (2001). Inequality measurement and the leaky-bucket paradox. *Economics Bulletin* **4**(6), 1-7.

Sen A. (1973). *On economic inequality.* Oxford, Clarendon Press.

Sen A. (1997). *On economic inequality.* Oxford, Clarendon Press.

UNDESA (2016). Transforming Our World: The 2030 Agenda for Sustainable Development. New York, NY: United Nations Department of Economic and Social Affairs.

Yitzhaki, S. (1983). On an extension of the Gini inequality index. *International Economic Review,* **24**(3), 617-628.

Zenga M. (2007). Inequality curve and inequality index based on the ratio between Lower and upper arithmetic means. *Statistica & Applicazioni,* **1**, 3-27.

Zenga M. (2016). Decomposition by sub populations of the point and synthetic Zenga (2007) inequality indexes. *Metron*, **74**(3), 375-405.

Zenga M., Jędrzejczak A. (2020). Decomposition of the inequality index I(Y) into the contributions of macro-Regions and income component. An application to data from Poland and Italy. *Argumenta Oeconomica*, **1**(44), 101-125.

Zenga M., Valli I. (2020). Decomposition by subpopulations of Gini, Bonferroni and Zenga inequality measures. *Statistica & Applicazioni*, **18**(1), 69-113.

Zenga M., Valli I. (2021). Decomposition by sources, by subpopulations and joint decomposition by subpopulations and sources of Gini, Bonferroni and Zenga 2007 inequality indexes. *Statistica & Applicazioni*, **19**(1), 1-34.