Acta of Bioengineering and Biomechanics
Vol. 25, No. 3, 2023

Original paper
DOI: 10.37190/ABB-02331-2023-02

Enhancing rheological muscle models with stochastic processes

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Purpose: Biological musculoskeletal systems operate under variable conditions. Muscle stiffness, activation signals and loads change during each movement. The presence of noise and different harmonic components in force production significantly influences the behaviour of the muscular system. Therefore, it is essential to consider these factors in numerical simulations. *Methods*: This study aimed to develop a rheological mathematical model that accurately represents the behaviour of the actual muscular system, taking into account the phenomena described by the stochastic model in the form of stationary processes. Stochastic disturbances were applied to simulate variable conditions in which musculo-skeletal system operates. Numerical simulations were conducted for two dynamic tasks, where the internal force generated by the system (task 1), and its displacement (task 2) were calculated. These simulations were performed using two different datasets sourced from the literature. In the next step, simulation results were compared with our own experiment. *Results*: The considered mathematical model was successfully tuned and compared with both the literature data and our own experimental results. During the analysis of muscle model behaviour, depending on the data source for model tuning, we observed distinct frequency characterized by a sine-type pattern and a higher frequencies marked by stochastic perturbations. *Conclusions*: The proposed model can be customized to simulate systems of varying sizes, levels of maximum voluntary contraction, and the effects of perturbations, closely resembling real-world data. The presented approach can be applied to simulate the behaviour of the musculoskeletal system as well as of individual muscles.

Key words: *modelling, stochastic disturbances, muscle and muscle system rheological model*

1. Introduction

Biological systems operate in variable conditions [12], [20], [30], [37], adapting to both internal factors (e.g., perturbations and/or noise originating from the neural or muscular systems of the human body) and external factors (e.g., perturbations from the environment). Internal factors arise due to the variability in the activation of muscle motor units caused by stochastic discharge from motor neurons [8], [12], [43], the coordination of agonist and antagonist muscular groups, body core, body shell and muscle temperature [11], [41], musculoskeletal disorders or pain [5], [15] or injury [19], [24], as well as noise and sensory thresholds of the body sensors [20] or the level of muscle contraction [1]. External factors occur due to the influence of environmental temperature, variations in applied external mechanical loads [4], [20], [34], the history of muscle loading [11], types of sports performed [18] and mechanical vibrations that are acting on the human body. Likely, this list is not exhaustive, however, we can assume that not all factors are known. Nonetheless, all types of perturbations act in various and frequently unpredictable combinations, exerting an influence on both movement and the stability of force production.

Considering the variability of the force produced by the musculoskeletal system, one can agree that this variability depends on factors such as muscle size, the level of maximal voluntary contraction (MVC), the individual's sex, age, and their physical and mental condition [8], [10]–[12], [14], [16], [17], [27], [30], [33].

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Received: October 13th, 2023
Accepted for publication: January 3rd, 2024

Furthermore, these perturbations can lead to movement inaccuracies [6], [12], which are observed during quasi-repetitive cycles of the musculoskeletal system when attempting to follow the same movement trajectory [6], [9], [20], [40]. This results in an inability to repeat the movement perfectly, maintaining the same trajectory, speed, acceleration or level of force/torque production [33]. For instance, in the paper [3], it was reported that even EMG characteristics for the same task changed over time, even for the same individual, after a few hours of rest.

From a mechanical standpoint, the musculoskeletal-neural system is considered redundant [20], [25]. This implies that the set of inverse dynamics equations describing the movement of this system is insufficient to yield a unique solution [40]. To find a solution, it is necessary to consider a group of admissible trajectories that adhere to physiological constraints [13]. However, it is worth noting that similar movements of body segments are typically executed using different muscular activation patterns in successive cycles [12], [16], which makes the problem even more challenging.

Various hypotheses exist to explain the strategies chosen by living species to execute specific movements. From a mathematical perspective, the inverse dynamics problem is often resolved using optimization approaches, such as minimizing locomotion energetic costs [12], [26], [27], optimizing muscle stress and dynamic parameters [34], [36], [40], optimizing different cost functions [7], [12], [29], and/or considering muscle synergy [21], [28]. These methods help limit the range of potential solutions created by the mathematical equations.

Measurements of forces and torques in musculoskeletal systems reveal that during the isometric contraction of muscles, one can observe quasi-periodic lower and higher frequency oscillations in force and torque. These oscillations lead to variability in force and torque, which is also dependent on the level of maximal voluntary contraction [17]. According to paper [30], this force variability occurs across all examined levels of isometric contraction (5, 35, 65 and 95% of MVC) in the upper limb muscles over a specific time interval, with the most significant variability observed at 95% of MVC. Notably, the oscillations in force variability are minimized at the 35% MVC level. The authors of the aforementioned publication also pointed out that the global maximum power in the power spectrum is located at around 1.24 Hz (absolute power). In the case of proportional power, it exhibits slight variations based on the MVC level, with an additional local maximum at 7 Hz (at 35% of MVC). It's worth emphasizing that the results of this analysis were presented for the first 30 bands (spectrum in the range of 0–11.72 Hz, with a spectral resolution of 0.39 Hz, calculated as 100 Hz/256). This choice was based on the observation that most of the signal energy is concentrated within this frequency range. The authors also present simulation results obtained using two different types of signals, namely white Gaussian noise and sine waves. Additionally, they provide values for the approximate entropy parameter (ApEn).

The paper [8] presented experimental results of muscle examination during "steady contractions", including both qualitative and quantitative analyses. It should be emphasized that the authors conducted *in vivo* measurements by testing groups of muscles rather than isolated individual muscles.

From a mechanical perspective, the need to consider broadband noise and different harmonic components is evident from the results published in [12], [30]. This requirement arises due to the characteristics of force produced by contracting muscle groups under the control of the nervous system. At the motor unit level, it is hypothesized that higher-frequency noise is generated by "an unfused contraction and timelocked to each motor neuron spike", while lower-frequency noise results from the stochastic discharge of motor neurons [12]. The imposed influence of both higher and lower frequency noises can lead to variability in force production during voluntary muscle contraction. Additionally, it may reduce the precision and repeatability of movements. Some studies indicate that a stochastic component is present in force and surface electromyography (EMG) measurements during musculoskeletal system contractions. Furthermore, this stochastic component is significantly influenced by the individual physiological factors of the examined person.

In the literature, there are reports dedicated to numerical modelling of the muscle system using various methods, including the finite element method [23], [32], as well as rheological approaches [38] such as Hill-type muscle models (for fusiform muscles) and Hill–Zajac-type muscle models (for pennate muscles) [39], [42]. However, the reported rheological implementations do not take variable stochastic perturbations of the involved mechanical properties into account. Assuming that these variable stochastic perturbations reflect both internal and external influences, we hypothesize that incorporating these perturbations into mechanical properties may provide a more accurate description of the behaviour of musculoskeletal models [12], [20], [35], [39].

The aim of this study was to develop an enhanced rheological model of the musculoskeletal system by incorporating a stochastic model in the form of stationary processes. The scope of this study encompassed the modelling of quasi-isometric contractions and experimental verification.

2. Materials and methods

Numerical simulations

The rheological model was derived based on the assumptions presented in [39] and taken into consideration the non-linear damping and stiffness characteristics that described the mechanical properties of the musculoskeletal system (Fig. 1).

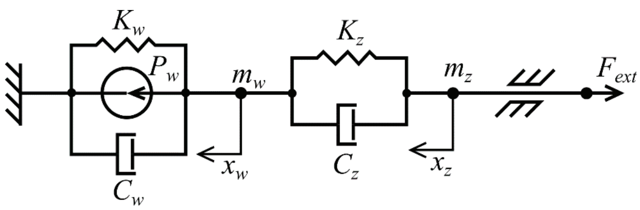

Fig. 1. Considered rheological model, where K_w, K_z – stiffness of active and passive tissues, C_w, C_z – damping of active and passive tissues, m_w, m_z – mass of active and passive tissues, x_w – displacement of active tissue, x_z – displacement of the system, P_w – force generated by active tissue, F_{ext} – external force

The mathematical model of the system is described by the following dynamical equations:

$$m_w\ddot{x}_w + C_w\dot{x}_w + K_w x_w - C_z(\dot{x}_z - \dot{x}_w) - K_z(x_z - x_w) = P_w(t), \tag{1}$$
$$m_z\ddot{x}_z + C_z(\dot{x}_z - \dot{x}_w) + K_z(x_z - x_w) = -F_{\text{ext}}(t),$$

where x_w is mass m_w displacement, x_z – mass m_z displacement, C, K – damping and stiffness parameters, where w and z means internal and external respectively, $P_w(t)$ – internal force of contractile elements, $F_{\text{ext}}(t)$ – external force acting on/generated by the model.

The mass coefficients and the nonlinear relationships of stiffness and damping parameters were determined based on the data published in [2], [31].

To introduce variability in muscle stiffness and damping parameters, a stochastic stationary process was incorporated in the form of white noise. Based on the power spectra analysis presented in [30], it was determined that two components should be implemented in the model: a dominant frequency component (harmonic) and a noise component. The noise component was introduced in the form of a white noise signal applied to the damping (C) and stiffness (K) parameters through superposition using a pseudo-random MATLAB generator. The harmonic component of the displacement (x_z, as can be seen in Fig. 1) was obtained by assuming the following forms for the stiffness and damping parameters:

$$K_j(t) = k_j x_j^2 + N(t) + S(t), \quad j = w, z,$$
$$C_j(t) = c_j \dot{x}_j^2 + N(t), \quad j = w, z, \tag{2}$$

where k_j is correction factor for stiffness parameter, $N(t)$ – time dependent noise, $S(t)$ – time dependent sine-type function c_j – correction factor for damping parameter.

The numerical simulations of muscle systems presented in this study were based on data published in papers [30] and [12]. To match the frequency characteristics of the external force (F_{ext}) as presented in those works, the rheological model (Fig. 1) was tuned by adjusting its damping and stiffness parameters for both papers separately. However, this approach does not allow us to determine the exact nature of the noise signal whether it is white (Gaussian), pink or another type of noise. Additionally, the signal spectrum analysis in [30] indicates the presence of a single dominant frequency within a narrow frequency range (0–12 Hz).

To achieve results similar in terms of force production with a dominant frequency within the same range as reported in [30] to the biological systems, the presented model incorporated both single-frequency (harmonic) and noise components to adapt the characteristics of stiffness and damping coefficients. This modification was applied in the form of white noise, with an amplitude of 0.16% of the initial stiffness coefficient value and 0.01% of the initial damping coefficient value.

It is worth noting that the measurements in [30] were conducted for different force values expressed as a percentage of (MVC) force. That is why in the model was specifically fitted to the highest tested MVC-force level, which was 95%.

In this paper, we analysed two types of dynamic tasks and we tuned our model to fit results published in works [30] and [12]:

- Task 1: The input variables included the displacement of the system (x_z) and the external load (F_{ext}) in the form of the Heaviside function, while the output variable was the internal force generated by the muscle system (P_w).
- Task 2: The input variable was the internal force (P_w), and the output variable was the displacement of the insertion (x_z).

The initial conditions of displacement and velocity in the simulations were set to zero.

To make a comparative analysis of the model behaviour in both tasks possible, we applied the same signal analysis procedure as in [30]. It is important to note that changes in filter topologies can influence the results [22]. Therefore, following the method of work [30], we set the sampling rate for the simulation to 100 Hz. The signal was then passed through a low-pass filter with a cutoff frequency of 30 Hz (Butterworth, 9th order). To determine the power spectrum, we employed the Welch method with a 256-point Fourier transform, resulting in a real spectrum resolution of 0.39 Hz (sampling frequency of 100 Hz divided by 256 points). This procedure allows us to directly compare the frequency characteristics presented in this paper with those from publication [30].

The mechanical properties of the model, represented by stiffness and damping coefficients, were tuned to achieve similar nonlinear characteristics as reported in the work [31]. Additionally, we introduced time--dependent perturbations (harmonic and stochastic) to those coefficients as described in Eq. (2). All numerical testing, data post-processing and analysis were carried out within the MATLAB-Simulink (R2020b) environment.

Experimental verification

In the next step, we performed an experimental verification of the results obtained in numerical simulations. In the referenced and aforementioned publication [30], authors focused on examining the variability of force exerted by patients during conducted tests (force in index finger flexion, where the finger is pressed on a force sensor). Test subjects were provided with real-time feedback on the current force values displayed on a monitor, allowing them to make continuous adjustments. Based on the obtained time-course profiles of force and different force levels (with the assumption that the force should be maintained at a constant level by the patient), it was observed that the frequency of force fluctuations, regardless of the force level, exhibited a strong harmonic component around 1.24 Hz.

The objective was to verify whether the muscle vibration frequency aligns with the results obtained and published in [30]. The authors of this publication conducted their experiment under different conditions and with following method. The experiment involved three male volunteers, aged 22 ± 1, body mass 70 ± 5 kg, body height 178 ± 6 cm, without overweight, all of them had not been diagnosed with musculoskeletal or neurological conditions and were, as per their declaration, in good psychophysical condition. The experiment was conducted by applicable laws and regulations, with the approval of the local ethics committee.

During the experiment, each volunteer was seated with their back supported, the upper limb flexed in the saggital plane and the index finger pointing straight ahead while keeping it stationary at the level of a marker permanently attached to a stand. Another marker was attached to the finger and changes in the position of both markers (thus, finger movement) were recorded by the OptiTrack optoelectronic system (6 cameras, 120 Hz). Using data of displacements of those markers we performed frequency analyses according to the recommendations published in [30].

3. Results

Numerical simulations

To solve Task 1, we fine-tuned the stiffness and damping coefficients to match the numerical outcomes with the experimental results presented in [30]. Task 2 was performed with the same initial conditions as Task 1 to calculate the displacement of the system over time.

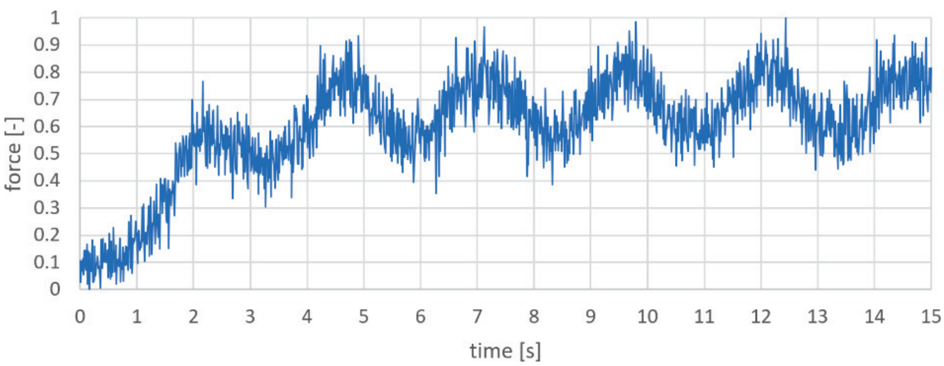

Fig. 2. Chosen normalised internal force P_w values in time for model tuned to the data from [30]

By solving Task 1, we obtained normalized values of force P_w as a function of displacement (x_z), using parameters tuned according to [30]. The numerical results are presented in Figs. 2–4.

In Figure 2, a normalized force over time is presented. Two distinct frequencies are observable: a lower frequency (sine-type) and a higher frequency (stochastic perturbation). The analysis of the results was conducted from the 5th to the 15th second, using a 10-second interval as in [30]. Both force and displacement results are presented in a normalized form to facilitate direct comparison with the results from [2].

The absolute power spectrum of the muscle system force for Task 1. simulation is plotted in Fig. 3. Maximum power is observed at 0.391 Hz, with an additional local peak at 4.251 Hz.

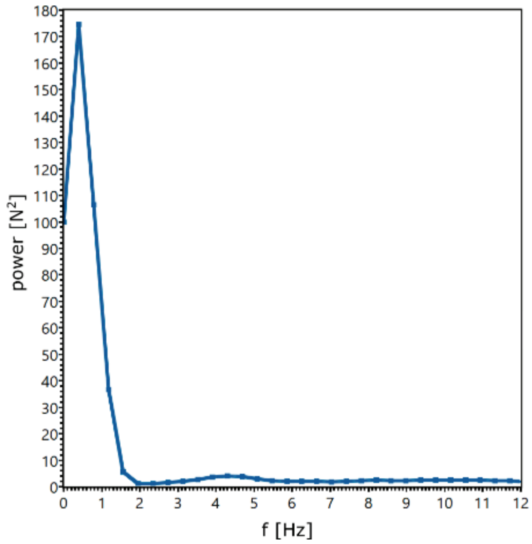

Fig. 3. Absolute power of force

In Figure 4a, the proportional power as a function of spectral frequency is plotted, and in Fig. 4b, the power spectrum is presented in the form of log power as a function of log frequency. The proportional power (Fig. 4a) was obtained by dividing the power in each frequency bin by the total power in the respective power spectrum, also the format of presenting results has been prepared in the same manner as in [30] to ensure comparability. Notably, the maximum power spectrum is observed at 0.391 Hz, with an additional local maximum at 4.251 Hz (Fig. 4a), which aligns with the results published in the aforementioned paper.

Using the same coefficients and initial conditions, Task 2 was solved, and its results are presented in Fig. 5. The plot suggests that for a constant level of muscle system force (P_w), assumed for this particular simulation, oscillations in the displacement of the system can be observed. This phenomenon can be interpreted as micro-scale vibrations or displacements of the body segment, which are often observable in living systems. For example, when pointing a finger at a fixed point, small but noticeable vibrations are observed. This phenomenon arises because in biological systems, perfect stability is not achievable.

In the next phase of our research, we tuned our model by adjusting the stiffness and damping coefficients to achieve a qualitative similarity in the force characteristics (shape) to those presented in work [12], where torque measurements were conducted in an isometric configuration of the upper limb. The results of Task 1 are displayed in Fig. 6a. Notably, in this case, a smaller amplitude of sine-type force oscillation was obtained in comparison with the results tuned to match those in [30] (see and compare with Fig. 2). Furthermore, the displacement of the system, as shown in Fig. 6b, exhibits a different characteristic compared to Fig. 5, highlighting the model's adaptability and ease of tuning.

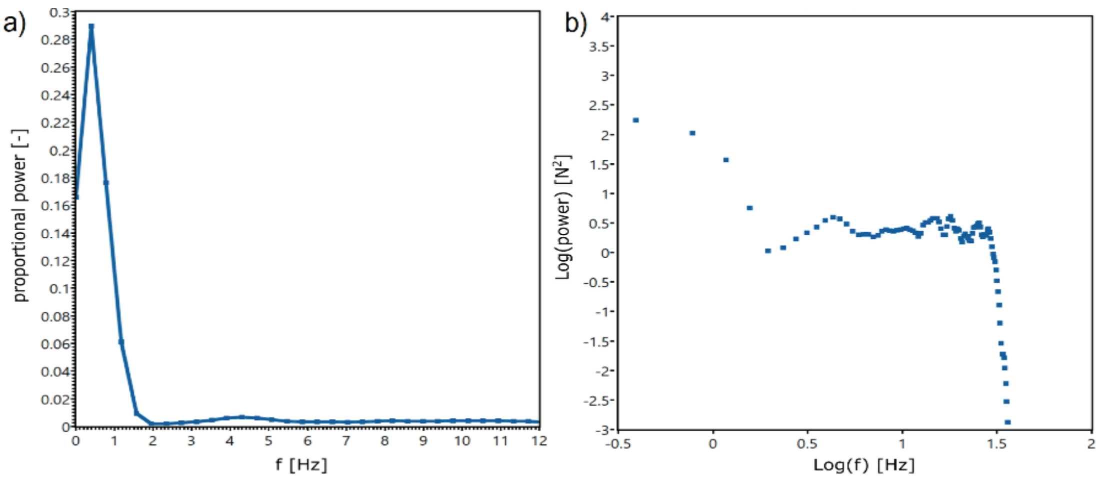

Fig. 4. Power spectrum of force: a) proportional power as a function of spectral frequency, b) log power as a function of log frequency

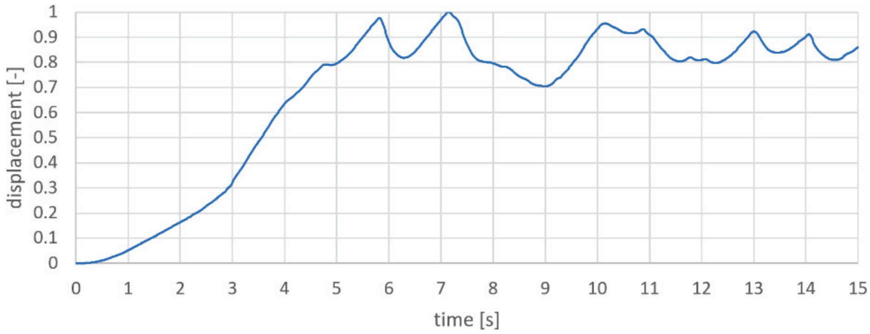

Fig. 5. Normalised values of displacement in time for model tuned to the data from [30]

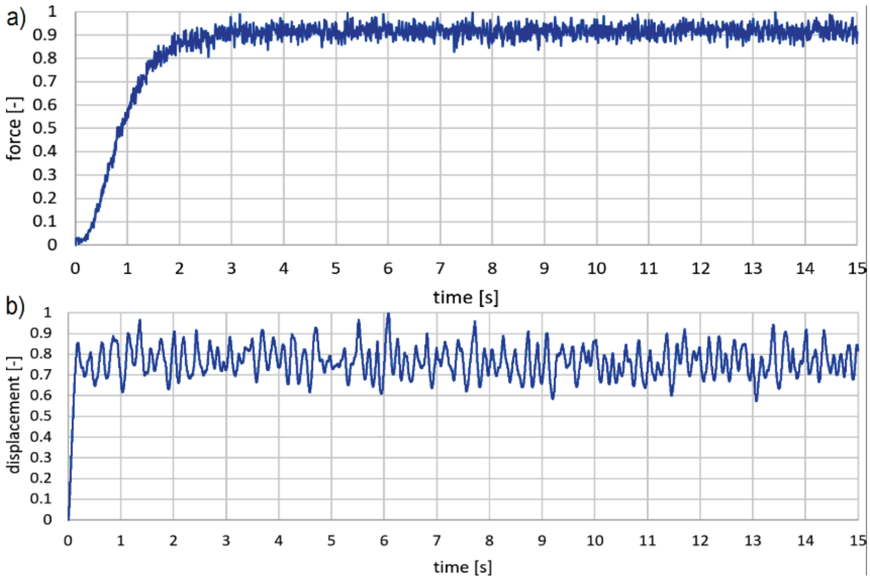

Fig. 6. Normalised values of (a) force in time for model tuned to the data from [12],
(b) displacement in time for model tuned to the data from [12]

Experimental verification

The results obtained for one of the chosen volunteers are presented in Figs. 7a, and 7b. In Figure 7a, the results of the Fast Fourier Transform (FFT) are displayed, while in Fig. 7b, the results obtained using the same method as in Hamilton's work [30] are exhibited – frequency analyses were conducted using identical methods (Proportional Power calculated using Welch's method (like in [30]) based on a 256-point Fourier transform, resulting in a real spectrum resolution of 0.39 Hz).

Analysing these results, a significant difference in the frequency distribution in the spectra can be observed (it is important to note that making direct amplitude com-

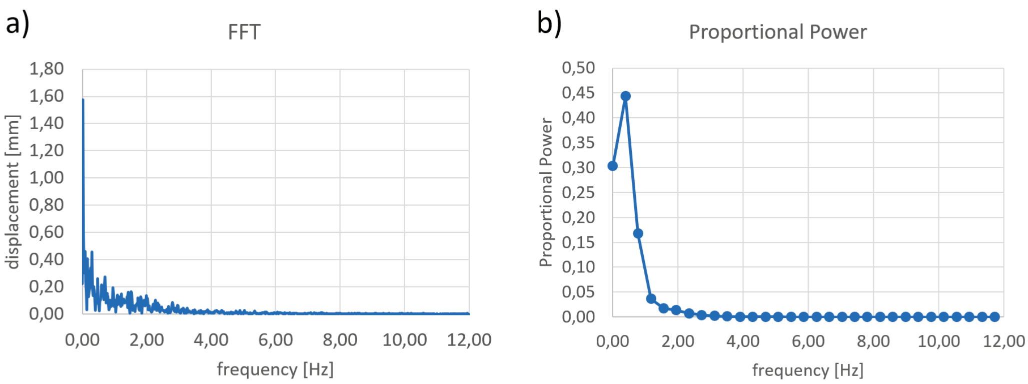

Fig. 7. Frequency analysis of finger displacement: (a) FFT, (b) Proportional power

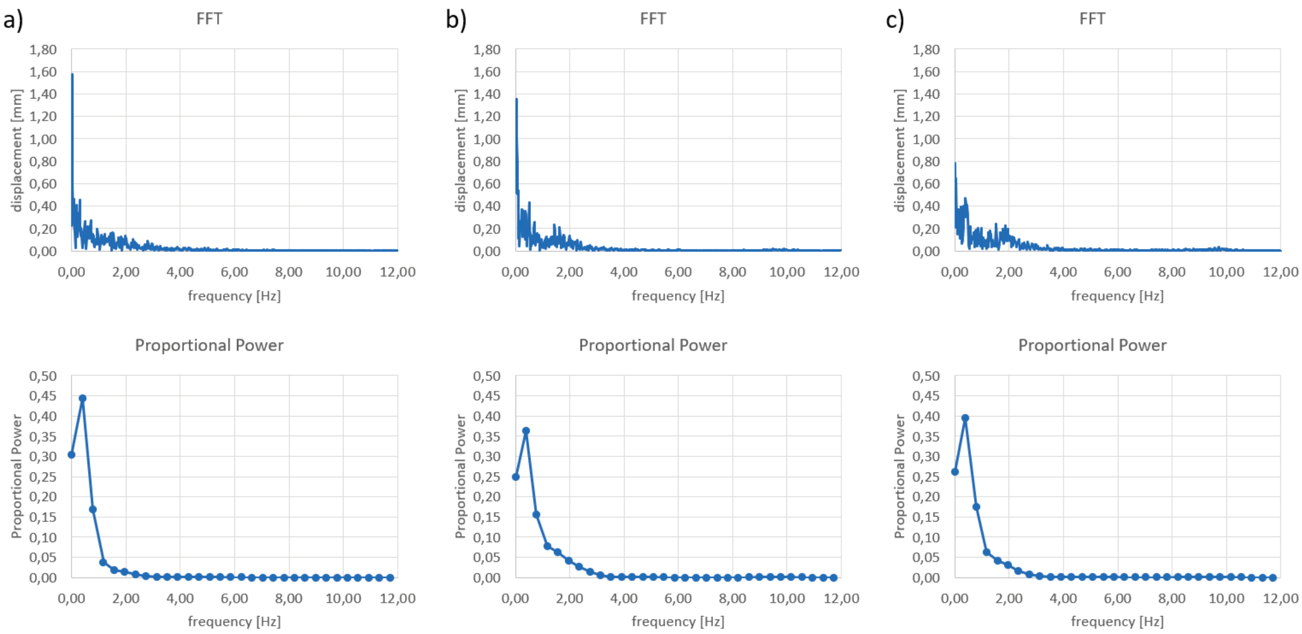

Fig. 8. Comparison of frequency analysis for: (a) volunteer 1, (b) volunteer 2, (c) volunteer 3

parisons should not be performed). This difference arises, among other factors, from the resolution of the FFT, which, for a signal with a length of 60 seconds, is (1/60 Hz = 0.017 Hz). This substantial difference in resolutions, in practice, makes it challenging to conduct a proper signal analysis. As an example, in Fig. 8, the results of spectra for three examined individuals are presented using both the FFT method and the Proportional Power calculated using Welch's method (like in [30]).

While there is a noticeable difference in the frequency distribution among the examined individuals when using FFT, this difference is not apparent in the Proportional Power plots (Fig. 8). The reason for this is the low-frequency resolution of the spectrum. This is confirmed by a simple correlation test (Spearman) performed on the Proportional Power charts for any two individuals, where the correlation value consistently exceeded 0.98. In the case of a correlation analysis of FFT charts, a high correlation was also achieved, but not less than 0.74. However, as observed, this correlation significantly differs from the result obtained for Proportional Power.

4. Discussion

As mentioned before, this study aimed to develop an enhanced rheological model of the musculoskeletal system by incorporating a stochastic model in the form of stationary processes and to prove that such an improved model can simulate the variability of force production by a skeletal muscle and musculo-skeletal system.

To demonstrate the frequency characteristics and displacement of the proposed model, it was necessary to address two distinct dynamic tasks, Task 1 and Task 2. In both cases, the model was fine-tuned using data from works [12] and [30]. During the creation and tuning of the model, we chose to base it on the assumption of nonlinear characteristics of muscle tissue, as outlined in [39]. In Task 1, we simulated the value of the force generated by the model, in task 2 – the displacement of the system, both for two different datasets. As can be observed, finally we solved 4 (two pairs) tasks for two different datasets. Our model provides a good fit of the model to real systems in terms of force stability production and ability to maintain a constant position. This does not affect the frequency characteristics and, assuming a constant or slow change of muscle force acting at the joint during muscle contraction, allows for a proper comparison of the obtained results. All of our findings are consistent with those published in [12] and [30], particularly in terms of frequency characteristics and the presence of non-constant, disrupted force/torque production in biological systems. Our frequency characteristics were derived following the procedure described in [30] (Figs. 3, 4). Subsequently, the proposed model was further adjusted by modifying the components of mechanical characteristics that describe stochastic noise, to achieve results similar to those published in [12] (Fig. 6a). Moreover, our approach yields realistic behaviour in the mathematical muscle model and enables the recreation of various conditions under which biological systems operate, which results in non-constant force production [33], leads to movement inaccuracies [6], [12] and disables our musculo-neuro-skeletal system to attempt perfectly repetitive movement trajectory during even simple, repetitive tasks [6], [9], [20], [40]. In conclusion, as a result of solving two tasks with a model tuned to two different datasets, we obtained four sets of results, described below with study limitations addressed:

- normalized force values as a function of time for a muscle tuned to the data from [30] (Fig. 2) show a relatively higher level of sine-type signal and more periodic force oscillations compared to those observed in work [30]. Moreover, despite achieving consistency in frequency characteristics, some differences are noticeable in force production;
- results of normalized displacement of the system tuned to the data from [30] (Fig. 5), cannot be directly compared with this publication due to the lack of data presenting muscle insertion/body segment displacement. However, the presented force-time series can be considered representative of a concentric-isometric contraction of a real-life muscle system and can be compared with real-life observations, where we obtained good agreement of results with our experimental verification;
- results of normalized values of force as a function of time for a muscle model tuned to the data from [12] (Fig. 6a) (in this case, the muscle model was re-tuned) are very similar to that presented in [12]. In contrast to the case, when model was fitted to the data from [30], we observed smoother time series with a lower level of sine-type disturbance components. This phenomenon is a consequence of changes in the amplitude of the sine-type signal during simulation;
- results of normalized displacement as a function of time for the model tuned to the data from [12] (Fig. 6b) are similar to the case when the model was fitted to the data from [30], however, these results cannot be compared directly with the publication [12] due to the lack of published data presenting muscle insertion/body segment displacement. Nevertheless, these results represent the behaviour of the re-tuned model and the change in length during contraction. In comparison with the results obtained for the model tuned to the publication [30], we observed a higher level of signal increase in the concentric phase and changes in the characteristics of the isometric phase, including visible components of higher frequencies. Based on the spectrum analysis, one dominant frequency (0.391 Hz) can be observed, with other neighbouring values resulting from spectral leakage.

The final part of this work involved experimental verification done with 3 volunteers, which demonstrated a good agreement of our model with the measurements (Section 2, Experimental verification). From the analysis of the experimental data, a fundamental conclusion can be drawn that analyses of finger displacements for three different individuals show a similar frequency character of motion (correlation > 0.74). There is a slight difference in the amplitudes of oscillations, but it is minimal. Therefore, it can be assumed that the character of finger motion for the examined individuals was similar. This conclusion stays in accordance with research outcomes published in [12] and [30]. Moreover, despite some simplifications of the method used in work [30], it has been indicated that the results show a similarity between simulation and verification.

5. Conclusions

This study aimed to introduce a novel approach to modelling force production in the musculoskeletal system, by taking into account the real-life behaviour of biological systems and improving the rheological model. Despite certain technical limitations inherent in the source publications, the authors proposed a method for enhancing existing rheological models. These models had their mechanical characteristics tuned based solely on frequency characteristics presented in graphical form in works [12], [30].

Additionally, a procedure for adjusting spectral characteristics was provided, allowing any existing rheological model to be tailored to real-data measurements. From a practical standpoint, this procedure involved the inclusion of the required harmonic and low-amplitude noise components. To achieve the desired force level, the mechanical characteristics were adjusted by modifying damping and stiffness (C, K) coefficients (Eq. (2)), as well as noise and harmonic components.

To obtain results of force production that are biologically similar and exhibit a dominant frequency (harmonic) within the same range as those published in [30], the presented model in Fig. 1 utilized a single-frequency, harmonic component and noise (stochastic) to alter the characteristics of stiffness and damping coefficients in the form of white noise, at levels of 0.16% of the initial stiffness and 0.01% of the initial damping values.

Furthermore, we have demonstrated that even relatively simple mathematical simulations of muscles can yield numerical results that closely approximate real experiments when proper adjustments of the coefficients in the mathematical model are made.

The approach presented here can be applied to simulate the behaviour of the musculoskeletal system as well as that of individual muscles.

The uniqueness of this study is linked to the application of stochastic disturbances to the rheological muscle model. This approach enables the existing nonlinear model to simulate variations in muscle force production and to simulate more natural biological system behaviour. Also, our approach allows, after some modifications to add the stochastic disturbances to other, existing mathematical models of the muscles.

Study limitations

We point out the following main study limitations:

- mathematical model have nonlinear characteristics and adding stochastic perturbations to it increases the time of numerical simulations. Also, an additional effort has to be made for its proper fitting;
- the data analysis techniques employed were selected to mirror the methods used by the authors of the compared articles. Currently, both the measurement technique and numerical calculations allow for analyses with greater accuracy, such as increased sampling frequency and frequency resolution;
- for verification we decided to use only healthy volunteers. It will be valuable, to check the behaviour of the model for patients with musculo-neuro-skeletal problems.

Acknowledgements

This work was supported by the AGH University of Science and Technology, Poland (project number 16.16.130.942/KMiW) and Young Scientist Grant at the Mechanical Department of Łódź University of Technology.

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