Fractional Spectral and Fractional Finite Element Methods: A Comprehensive Review and Future Prospects

Muhammad Bilal Hafeez[1]*, Marek Krawczuk[1]

[1] *Gdansk University of Technology, Faculty of Mechanical Engineering and Ship Technology, Institute of Mechanics and Machine Design, Narutowicza 11/12, 80-233 Gdańsk, Poland.*

Abstract

In this article, we will discussed the applications of the Spectral element method (SEM) and Finite element Method (FEM) for fractional calculus. The so called fractional Spectral element method (f-SEM) and fractional Finite element method (f-FEM) is crucial in various branches of science plays a significant role. In this review, we discuss over the advantages and adaptability of FEM and SEM, which provide the simulations of fractional derivatives and integrals and are therefore appropriate for a broad range of applications in engineering, biology, and physics. We emphasize that they can be used to simulate a wide range of real-world phenomena because they can handle fractional differential equations that are both linear and nonlinear Although many researchers have already discussed applications of FEM in a variety of fractional differential equations (FDEs) and delivered very significant results, in this review article we aspire to enclose fundamental to advanced articles in this field which will guide the researchers through recent achievements and advancements for the further studies.

Keywords: Fractional Calculus, fractional Spectral element method, Science and Engineering, fractional finite element method

1. Introduction

1.1. Fractional Calculus

Fractional calculus is a branch of mathematics that deals with integrals and derivatives of non-integer order. Its roots can be found in the 1695 introduction of classical calculus by Newton and Leibniz. Recently, the fractional calculus is used in many applications in the field of science, engineering, chemistry and biochemistry [1] for example: viscoelastic materials modelling [2], beam theory [3, 4, 5, 6, 7, 8], physics [9, 10, 11, 12], life sciences [13], applied mathematics [14, 15, 16, 17, 18] finance [19, 20, 21] and geophysics [22, 23, 24]. For additional subtleties on this, see Podlubny [25], Hilfer [26], Ahmad Jafarian, Alireza Khalili Golmankhaneh and Dumitru Baleanu [27], Trujillo [28], Mainardi [29, 30, 31] and numerous mathematicians have been work on the advancement of fractional calculus, including Riemann Liouville, [32, 33, 34], Weyl [35], and Riesz [36, 37]. Fractional calculus briefly refers to fractional integration and fractional differentiation.Based on its nomenclature, fractional calculus usually refers to the Riemann-Liouville integral when addressing fractional integration. But fractional derivatives have more than one definition when it comes to fractional differentiation. Several definitions of this type will be explained in the exposition that follows.

$$D_{a,t}^{-\alpha} f(t) = {}^{(}RL)D_{a,t}^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} f(s)ds, \tag{1}$$

and

$$D_{t,b}^{-\alpha} f(t) = {}^{(}RL)D_{t,b}^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_t^b (t-s)^{\alpha-1} f(s)ds, \tag{2}$$

where, Γ is the Euler gamma function.

1.2. Numerical methods for fractional Calculus

Many Researchers use different numerical methods to solve fractional differential equations (FDEs). The recent development in the field of the heat equation for using fractional calculus [38, 39] develop a problem generated by a non-local operator. In [40] presents a class of fractional variational problems and offers a thorough finite element method to solve them. In [13] for the finite difference method and by using proper orthogonal decomposition (POD) technique for the fractional diffusion equation and high accuracy using Spectral element method (SEM) in two-dimensional by [41, 42]. The model described attributes of lower dimensions and higher accuracy, which resulted in a decrease in the amount of work that needed to be done computationally and a decrease in the amount of time that calculations performed. In [43], the mechanical characteristics of one-dimensional degraded non-local structures were investigated. This study considered the effects of scale effects

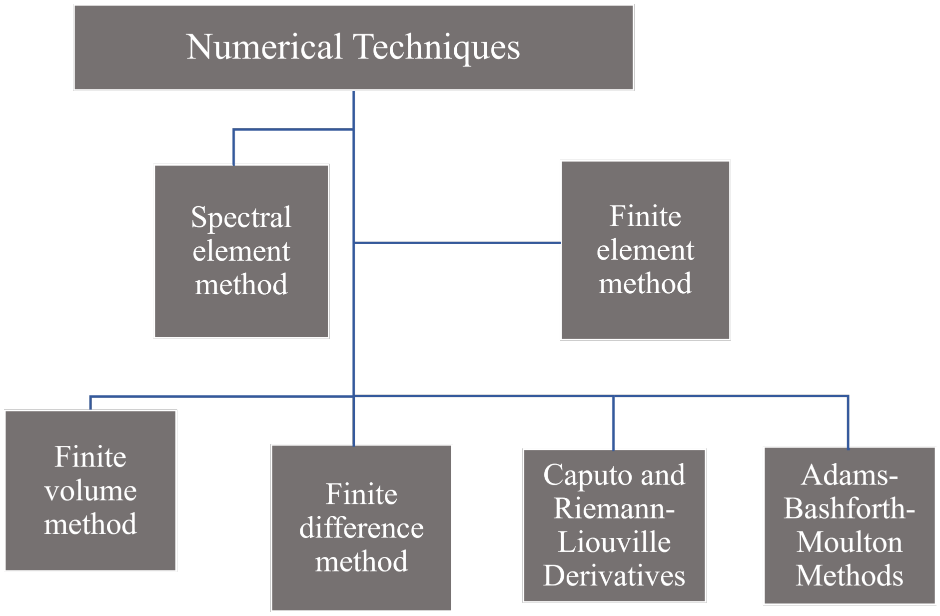

Figure 1: A flow chart presenting various numerical techniques.

while investigating fractional non-local materials using the finite difference method.

1.3. Finite element method for fractional calculus

By using the Finite Element Method (FEM) to solve fractional differential equations, one can achieve stronger stability standards and more flexibility when handling complex and inhomogeneous geometries than is possible with other numerical techniques. FEM is a numerical method for approximating solutions to differential equations where the domain of interest is divided into various elements. It is applied to a variety of complex physical phenomena, especially those displaying geometrical and material non-linearities (like those frequently found in the sciences and engineering) [44]. For solving traditional differential equations, the Finite Element Method (FEM) is a practical numerical technique. FEM is a useful and efficient tool for solving complicated problems when it comes to fractional differential equations. Recently, some significant papers were published concerning about the FEM for partial differential conditions [45, 46].

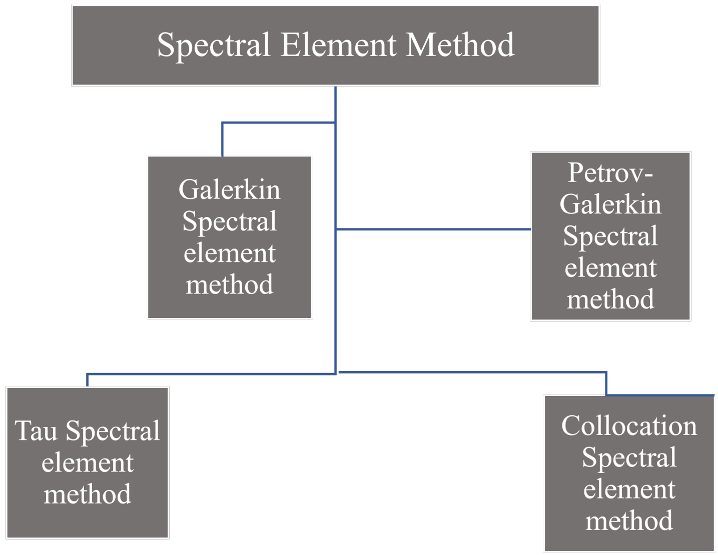

Figure 2: Flow charts for various spectral element methods.

1.4. Spectral Finite element method for fractional calculus

The combination of spectral element methods (SEM) and fractional calculus provides a powerful way to solve partial differential equations (PDEs) involving fractional order derivatives. The benefits of both spectrum and finite element methods are combined in spectral element methods, a high-order numerical methodology that works especially well for situations involving complicated geometries or irregular domains. They become an effective tool for solving fractional PDEs when combined with fractional calculus. Many researcher are already investigating the involvement of SEM in fractional calculus [47, 48, 49, 50].

2. Applications

2.1. Introduction

TBy combining the advantages of both spectral methods and finite element methods (FEM), the spectral element method (SEM) is a method for solving complicated partial differential equations. SEM is particularly useful when attempting to solve complex problems related to fractional calculus, a branch of mathematics that deals with derivatives of non-integer order. Key steps in SEM for fractional calculus include the use of orthogonal spectral basis functions, the substitution of fractional derivatives for integer-order derivatives, the division of the domain into elements, the formulation of the problem as a weak form, the assembly of global equations, the solution, the consideration of boundary conditions, and the use of numerical quadrature for fractional derivatives. When modeling systems with memory effects or anomalous diffusion, this approach is useful because it offers high accuracy for problems involving singularities and irregular behavior. The basic idea of f-SEM is not so different from the classical to divide the domain (geometry in the sense of solid mechanics) into small but finite sized elements. The collection of elements is called the finite element mesh. By using FEM, a semidiscrete semigroup is obtained [51, 52, 53].In f-SEM, the domain of equation is divided leads to a set of equations by the numerical scheme.

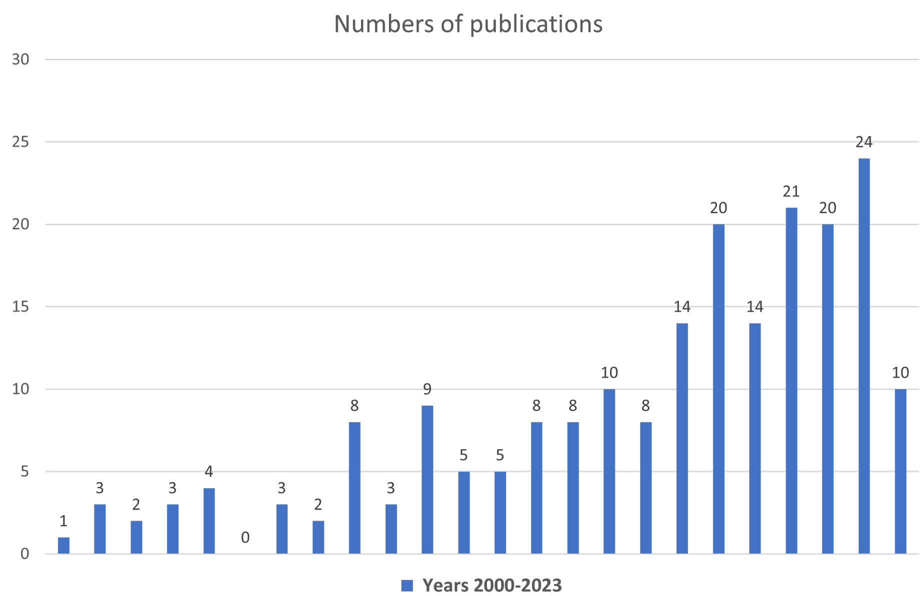

Figure 3: List of publications flow chart for fractional finite element methods.

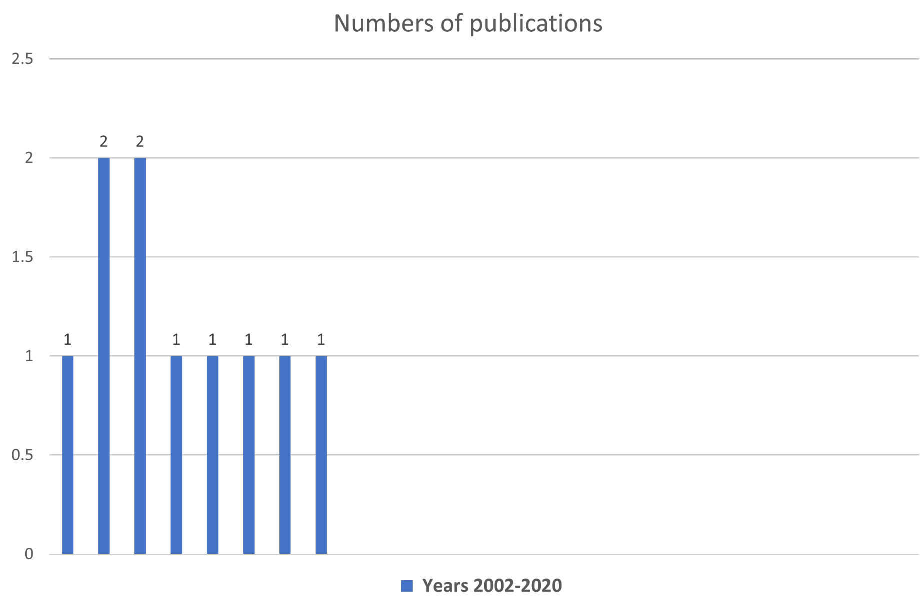

Figure 4: List of publiations flow chart for fractional spectral element methods

3. Fractional dynamic by using FEM and SEM

The Spectral Element Method (SEM) is a suitable approach for analyzing systems governed by fractional order differential equations, a crucial component of fractional dynamics. SEM, as a numerical method, proves effective in addressing challenges within the realm of fractional dynamics by harnessing the advantages of both spectral methods and the finite element method (FEM). In [39] studies the fractional-spectral approach for vibration of damped space structures. A dynamic analysis for FEM [54] in a structural system with fractional derivative models by using finite element formulations is presented. High-frequency dynamics is used for a structural and complex engineering system. High-frequency phenomena provide a link between vibration theory and thermodynamics, emphasizing that high-frequency dynamics can be thought of as both the low-frequency limit of thermodynamics and the high-frequency extreme of vibration theory. High-frequency dynamic properties use in many problems like in polymeric system and polymer films [55]. Nokhbatolfoghahai [56] investigated the use of the Finite Element Method (FEM) for the dynamic simulation of high-frequency

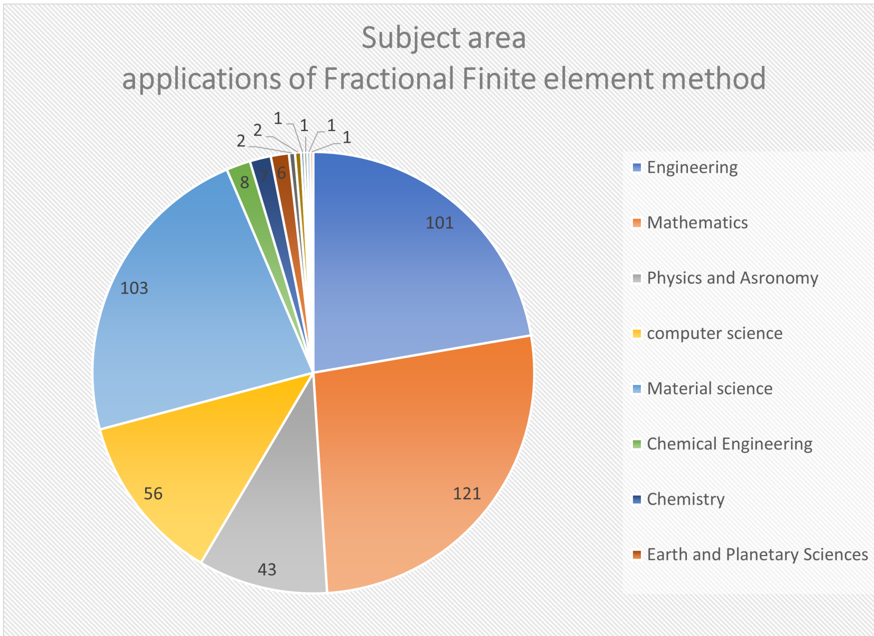

Figure 5: Subject area flow chart for fractional finite element methods.

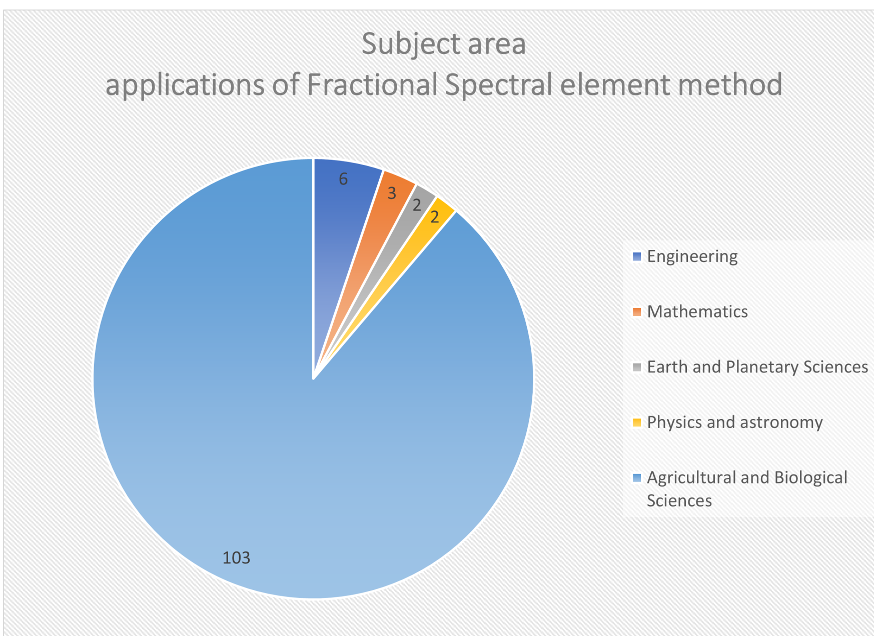

Figure 6: Subject area flow chart for fractional spectral element methods.

vibrations in extended complex structures. Furthermore, SHOYAMA's 2018 study [57] examined the high-frequency dynamic properties of a compressed O-ring that was utilized to support a bearing. FEM modeling is widely used in modern practice to predict the vibration and noise behavior of entire engines or their subsystems.Finite Element Method (FEM) simulations often leave out elastomeric components found in engines or subsystems because of some challenges such as the absence of material properties at higher frequencies. As mentioned by Lu in 2007 [58], who investigated elastomeric properties through fractional calculus in the framework of FEM, viscoelastic properties have been utilized to address this problem. Experimental measurements were used to validate this approach. Engine covers with elastomer seals were modeled as an example of the use of fractional FEM. To solve the partial differential equations with fractional derivatives, the Fractional Spectral Element Method (f-SEM) blends fractional calculus and the highly accurate Spectral Element Method (SEM). f-SEM ensures stability and physical relevance in the modeling of systems with fractional damping when combined with limited damping treatment. f-FEM is a versatile method that allows us to solve more complex problems like beams constrained [59] damping treatment by f-FEM. They observed the behaviour of the damping material is described using the fractional derivative model of viscoelasticity. In this model, f-FEM developed is a one-dimensional beam element with three degrees of freedom per node. The accuracy of the modal properties obtained with the beam model is compared with those calculated from a more elaborate plane stress finite element model. In [60] presents a nano-scale Timoshenko beam using the integral model of nonlocal elasticity with Finite element analysis. Sandwich radiates with implanted viscoelastic material utilizing partial fractional equation by using FEM investigated by [61]. Development of the strategy is represented by assessing the second-order measurements of the redirection of a beam whose unbending nature changes arbitrarily along its axis by using the FEM in [62].

3.1. Spectral element method and Finite element method on fractional viscoelasticity

Bagley (1983), [63], discusses using fractional calculus to solve viscoelasticity in his paper. Moreover, it clarifies a relationship between the macroscopic behavior of certain viscoelastic materials and molecular theories explaining their microscopic behavior. The Spectral Element Method (SEM) applied to fractional viscoelasticity is a computational approach used to study and simulate materials or systems with viscoelastic behavior involving fractional calculus. There are many methods for addressing viscoelasticity in materials. The use of fractional calculus in the field of viscoelasticity is noteworthy because it can precisely represent, using experimental parameters, the constitutive relationships of some viscoelastic materials. Crucially, some problems can be produced by incorporating fractional calculus into finite element formulations. The application of fractional calculus to viscoelasticity is thoroughly examined in a review published in [64]. The non-local forces as viscoelastic long-range interactions present in [65]. The expression of the elastic and viscoelastic matrices obtained when applied formulation of the FEM. The 3D fractional viscoelastic model [66] with the implementation of the FEM. Viscoelastic structures uses in many engineering problems and [67] presented FEM on the viscoelastic frame. Application of fractional calculus to viscoelasticity and also explain a link between molecular theories and macroscopic behaviour of certain viscoelasticity in [63]. In [68] give overviews on fractional derivative viscoelasticity.

3.2. Control

There are several approaches available to solve fractional optimal control problems. Zhou et al. (2018) [69] focuses on optimal control problems involving space fractional diffusion equations and uses the finite element and spectral element methods to solve them. Furthermore, control system-related problems can be addressed with the Spectral Element Method (SEM). SEM is a numerical method that combines aspects of finite element and spectral methods, making it an effective tool for partial differential equation solutions. Spectral element method (SEM) is used to study in [70] the vibration suppression and dynamic responses of frame structures considering shear deformation. A nonuniform elliptic operators which consider as a state equation by using the finite element to solve mesh points in [71]. Consider second-order partial differential equations with Dirichlet boundary conditions to solve an elliptic optimal control problem with FEM in [72]. By using an approximation technique [73] of optimal control problems for the fractional dynamic system in separable Hilbert space.

In [74] presents a study that presents a method for solving fractional optimal control problems. With this approach, the problem is discretized using the discrete method, which works by applying finite differences. The fractional order parabolic equations and investigation on two semidiscrete approximation schemes the FEM and establish optimal concerning the data regularity error estimates for a semidiscrete FEM in [75]. Zhou (2016) [76] examines the application of finite element approximation to time-fractional optimal control problems. Additionally, Zhou uses the Finite Element Method (FEM) in his 2020 work [77] to study space fractional optimal control problems. Furthermore, Dohr studied finite element approximation in the context of optimal control problems governed by the fractional Laplacian in his 2018 and 2019 studies [78, 79]. These investigations include the use of finite element analysis to compute an approximation for the state equation through spatial discretization. Additionally, as discussed in [80], a piecewise linear FEM approach is used for optimal control problems involving fractional operators.

3.3. SEM and FEM approximations of fractional cable equation

A numerical method for simulating and modeling the behavior of natural neurons or electrical cables with fractional calculus is the Spectral Element Method (SEM) applied to the fractional cable equation. The cable equation accounts for resistivity and capacitance when describing the propagation of electrical signals in a structure that resembles a cable, such as the axon of a neuron or an electrical transmission line. The cable equation takes into consideration derivatives of non-integer order when fractional calculus is introduced. For the fractional cable condition, a few numerical models are available, such as finite differences orthogonal spline collocation method and FEM[81, 82, 83]. The fractional cable condition which was inferred from the fractional Nernst-Planck conditions was presented to show electrotonic properties of spiked neuronal dendrites[84]. Numerical Recognizable proof of the fractional derivatives within the two-Dimensional fractional cable equation is present by [85]. An effective calculation for fathoming the one-dimensional cable condition within the Laplace (recurrence) space for a self-assertive straight film is presented by [86]. A two-grid finite element approximation is used to solve a nonlinear time-fractional Cable equation that is introduced in Wang's 2016 paper [87]. The paper investigates multiple second-order time-discretization schemes using Galerkin finite element (GFE) analysis and varying parameters. According to Liu's 2018 discussion [88], these schemes are intended to offer a numerical solution for the nonlinear cable equation with time-fractional derivatives. An analysis of the fractional Cable equation's numerical solution, as it appears in Lin's 2010 [89] work, is also included in this study. Develop a numerical technique use for Riemann–Liouville fractional derivatives in time-fractional cable equation and explore a semidiscrete scheme based on the lumped mass Galerkin FEM, utilizing piecewise linear capacities in [90].

4. Comparison of Fractional Derivatives Over Time and Space

4.1. Finite and Spectral element method for Time differential equation

A powerful numerical method for modeling and researching dynamic systems and transient phenomena in a variety of scientific and engineering domains is the spectral element method for time-dependent differential equations. It is a useful tool for precisely and effectively solving time-dependent problems due to its high-order accuracy in both space and time. In the fractional time model, it means there is a memory which is the past state can affect the present state. Many scholars have been studying fractional differential equations recently, and they frequently use the fractional Finite Element Method (f-FEM) for their research. Several significant works in this field have been produced by Deng in 2009 [91], Liu in 2014 [51], Ford in 2011 [92], Liu in 2014 [93], Huang in 2020 [94], and Jin in 2014 [95]. Furthermore, Manimaran's work in 2019 [96] explores the uniqueness of a weak solution using the Finite Element Method to solve nonlocal diffusion operators for the time-fractional cancer invasion system. Furthermore, as Esen's 2013 study [97] discusses, the diffusion wave equation and time-fractional diffusion equations are numerically solved using the Galerkin Finite Element Method. In Jin's work from 2013 [98], the Galerkin Finite Element Method is used to obtain numerical solutions of multiple time-fractional derivatives. For solving time-fractional equations, Zeng's 2017 study [99] presents a novel Crank–Nicolson Finite Element Method. This procedure uses a modified L1 method to discretize the Riemann-Liouville fractional derivative. The Finite Element Method is applied to two-dimensional time-fractional Tricomi-type equations in Zheng's 2010 study [100]. Jiang (2011) focused his work [101] on the development of high-order techniques for the Finite Element Method-based solution of time-fractional partial differential equations. A Finite Element Method (FEM) approach for solving time-fractional partial differential equations is presented in Jiang's 2013 study [102]. The Finite Element Method is used in Esen's work from 2015 [103] to provide a numerical solution for the time-fractional Burgers Equation. A number of works, including those by Zhao in 2015 [104], Zhao in 2013 [105], Sun in 2013 [106], and Zhao in 2016 [107], investigate solutions for time-fractional diffusion equations.

This is accomplished using a semi-discrete FEM methodology. A numerical approximation for a time-fractional cable equation that includes two Riemann-Liouville fractional derivatives is developed in Al Maskari's 2018 work [90]. Piecewise linear programming in a semidiscrete scheme based on the mass Galerkin FEM is utilized.

4.2. Finite element method for space differential equation

In fractional space, model studied one point can affect to another point. Present the fractional-order non local continuum for 2D model [108, 109] and 1D Euler-Bernoulli beam by using fractional FEM (f-FEM).The space fractional optimal control problem with integral state constraints was the subject of a study by Liu in 2021 [110]. The problem was approached using a finite element approximation. Using the Finite Element Method (FEM), Zhao et al. (2017) [111] investigated optimal control problems governed by the space fractional diffusion equation. For the space-fractional advection-diffusion equation with non-homogeneous boundary condition solve by FEM proposed [100]. Consider a Riesz fractional operator for space-fractional partial differential equations to solve by FEM [45, 112]. In [113] present a convergence analysis of moving FEMs for space fractional differential equations. In [114] used a Space-Fractional Diffusion Equations with Dirichlet Boundary-Value Problems by FEM. In [115, 116] build up a quick and exact finite element technique for space-fractional equation in two space measurements, which are communicated regarding fractional directional subordinates in all the ways that are coordinated concerning a likelihood measure on the unit circle. Space fractional diffusion equation for finite element solutions with a nonlinear source term is presented by [117].

4.3. Finite element method for time-space differential equation

A time-space finite element method for solving time-space fractional diffusion equations has been developed. This method, which was put forth by Feng in 2015 [118] and Bu in 2019 [119], uses the Finite Element Method (FEM) to solve numerical problems. FEM is used to solve the space and time-fractional Fokker–Planck equation, which is a useful tool for analyzing processes involving both flights and traps. Deng and Li are credited for this development in their respective works [**?** 120]. The Finite Element Method (FEM) is applied to a multi-term time-space fractional diffusion equation with a Riesz fractional operator, and its convergence and stability are examined. The works of Liu in 2019 [121] and Li in 2017 [120] both propose this method. Lai's work from 2021 [122] presents a numerical solution for linear Riesz space fractional partial differential equations using a space-time finite element method.The Finite Element Method (FEM) has been utilized to a multi-term time-space fractional diffusion equation with a Riesz fractional operator, and its convergence and stability are examined. The works of Liu in 2019 [121] and Li in 2017 [120] both propose this method.

Lai's work from 2021 [122] presents a numerical solution for linear Riesz space fractional partial differential equations using a space-time finite element method. In the study by Gorenflo in 2002 [123], a discrete random walk approach is employed to solve the time and space fractional diffusion equation.

In the 2020 study by Gao [124], the nonhomogeneous two-dimensional distributed order time-fractional Cable equation on complex convex spaces is solved using the Galerkin Finite Element Method (FEM) with a weighted and shifted Grünwald contrast estimation and Composite Trapezoid formula. Numerical models for signal degradation in underwater or submarine transmission cables are created using this cable equation. Because it can explain non-local fading memory, the Atangana-Baleanu fractional derivative is used in this analysis [125], as suggested by Karaagac in 2018. Wang (2016) [87] investigated the use of the Galerkin Finite Element Method (FEM) for the numerical solution of the nonlinear time-fractional cable equation.

5. Error Estimation

5.1. Error estimates finite element method for fractional order

They are many methods to error estimate for fractional differential equations. Such as the collocation method In [126, 101] studied the optimal order error estimates by using high-order FEM for time-fractional PDEs. In [127, 128] studied the error analysis of PDEs by using the finite element method. Error estimate for a two-dimensional weakly singular integral-PDEs with time and space fractional derivatives by using FEM proposed [129]. For error, analysis [130] using FEM in the time-fractional biharmonic equation. Many researchers use different fractional differentials equations for error analysis with the help of FEM like error estimate with fractional diffusion equation [131, 132, 133, 134], fractional stochastic Navier–Stokes equations [135], and error estimate of fractional stochastic differential equations [136].

Li (2011) [137] and Li (2018) [138] discuss the development of time-step conditions for common linearized semi-implicit schemes for nonlinear parabolic equations, combined with Galerkin finite element approximations. In particular, the time-dependent nonlinear Joule heating equations are taken into account in these studies. The study presents optimal error estimates for the semi-implicit Euler scheme, suggesting that this method has no time-step boundaries. The error analysis of semilinear parabolic equations is performed out using a two-grid method with a backward Euler scheme. Unlike in traditional finite element analysis, temporal and spatial errors make up the discrepancy between the exact solution and the finite element solution. As suggested by Shi in 2017 [139] and Gunzburger in 2019 [140], this division is accomplished by introducing a corresponding time-discrete framework.

6. Conclusion

Based on fractional Finite Element Method (FEM) and Spectral Element Method (SEM), this review paper provides an extensive overview of the noteworthy developments in engineering and scientific modeling. Researchers interest in fractional FEM and SEM has increased significantly as a result of recent developments. A few crucial things to think about for your closing remarks are:

- The growing significance of fractional FEM and SEM in solving intricate engineering and scientific problems is acknowledged.
- Researchers can effectively simulate real-world phenomena that exhibit fractional-order behavior by using FEM and SEM, which have the advantage of providing high accuracy in approximating fractional derivatives and integrals.
- The both methods FEM and SEM can capture the non-local aspects of fractional calculus. Compared to conventional numerical methods, this allows for a more accurate modeling of phenomena involving long-range interactions and memory effects.
- In order to ensure the accuracy and dependability of results, mesh generation, numerical stability, and error analysis must be properly taken into account when using FEM and SEM for fractional calculus problems
- The advancement of FEM and SEM applications in fractional calculus depends on the cooperation of mathematicians, engineers, and domain

experts. These cross-disciplinary partnerships may result in creative fixes and breakthroughs across a range of industries.

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