Paper

Propagation in rectangular waveguides with a pseudochiral Ω slab

Jerzy Mazur and Krzysztof Dorko

Abstract — The transfer matrix approach is applied for analysis of waveguides loaded with a uniaxial pseudochiral Ω slab. In particular a pseudochiral parallel plate and rectangular guides are investigated. Based on the numerical analysis the influence of the pseudochirality on propagation characteristics and field distribution are examined. Other feature such as a field displacement phenomenon appearing in the both considered structures due to the pseudochirality is also discussed.

Keywords — waveguides, propagation, pseudochiral media.

1. Introduction

A thorough characterization of a wave propagation and scattering in complex media allows to investigate their potential usefulness in various applications. Among complex media we can find a new pseudochiral medium [1], called Ω media. The pseudochiral Ω material contains small omega shaped metal particles mostly ordered along a preferred direction. Although the concept of Ω media has arised from the chiral media and both materials are characterized by the similar constitutive relations their interactions with electromagnetic wave are considerable different. In particular the chiral medium exhibits the effect of optical activity that refers to the rotation of the wave polarization plane while the pseudochiral medium yields the field displacement effect. Recently, the propagation properties of the pseudochiral guides were intensively studied [2 – 4]. These behaviors in particular depend in particular on the arrangement of the Ω particles in the medium inserted to the structure. For example the direction of the field displacement in the cross-section of the guide is changed when the Ω particles in the medium are reversed [3, 4]. The electromagnetic wave interaction with pseudochiral media have recently suggested their potential applications, e.g.: phase shifter [2], nonradiative Ω guides [5] nonreflecting shields [6]. Moreover, the scattering properties of pseudochiral guides have been utilized in [7] to reconstruction of the Ω medium parameters. In this letter we consider the problem of the electromagnetic wave propagation in a rectangular guide containing a slab of Ω medium as shown in Fig. 1. This structure completes the class of pseudochiral guides presented in [4]. The problem is solved by using transfer matrix approach (TMA) proposed in [8] for ferrite filled guides. Here the TMA is modified to the cases where pseudochiral materials, such as Ω media, are involved. It is shown that except of hybrid modes the considered guides supports also the TE_n modes. The main feature of the guide is that the field displacement occurs along the x axis and the magnitude of the phenomenon depends on the Ω medium parameters. Considerable differences in field distribution are observed when the slab is reversed in the guide.

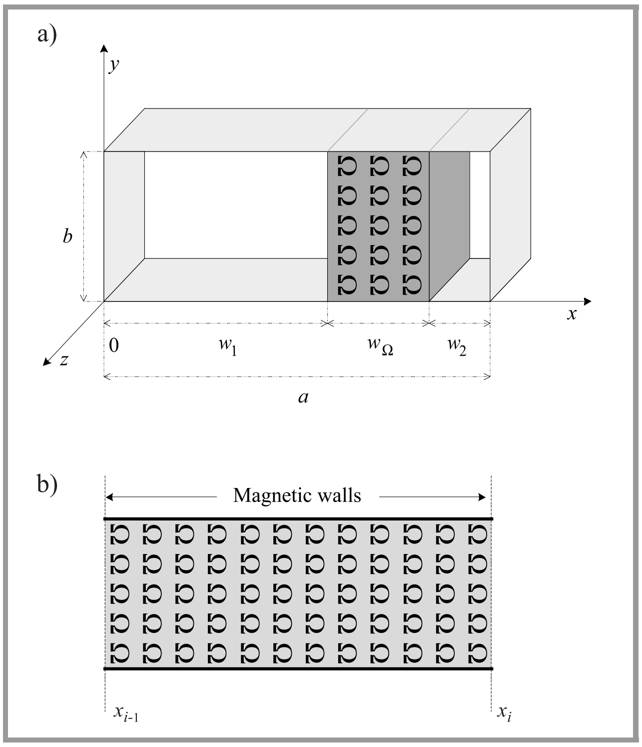

Fig. 1. Structures under examination (a) a rectangular waveguide containing pseudochiral Ω slab; (b) a parallel – plate pseudochiral guide.

2. Transfer matrices formulation

Let us consider a structure shown in Fig. 1b, where pseudochiral material is located between two parallel metal plates. The Ω particles lie in the guide cross-section and the stemps are perpendicular to the metal plates of the guide. For this medium, the relative electric permittivity and magnetic permeability tensors are of the diagonal form: $\overline{\varepsilon} = \varepsilon(\vec{i}_x\vec{i}_x + \vec{i}_z\vec{i}_z) + \varepsilon_y\vec{i}_y\vec{i}_y$ and $\overline{\mu}_c = \mu(\vec{i}_x\vec{i}_x + \vec{i}_y\vec{i}_y) + \mu_z\vec{i}_z\vec{i}_z$. The magnetoelectric coupling tensors have the following dyadic representation $\overline{\Omega}_{yz} = \Omega\vec{i}_y\vec{i}_z$ and $\overline{\Omega}_{zy} = \Omega\vec{i}_z\vec{i}_y$, where Ω is a coupling admittance between electric and magnetic field along y and z axis, respectively. In fact, the polarization vectors induced in Ω inclusions increase the values of ε_y and μ_z. Thus it follows that $\varepsilon < \varepsilon_y$ and $\mu < \mu_z$.

In general, the unbounded pseudochiral medium in Fig. 1 supports hybrid modes. However, if we assume that the field is uniform in the y direction, then the hybrid field is decoupled into TE and TM modes. Therefore the TE$_n$ modes appear in the guide. Hereafter, the propagation characteristics of these waves are determined applying TMA. We get a following transfer matrix equation that defines the relation between the tangential fields components $\underline{F} = (H_z, E_y)^T$ at the side inter faces x_i and x_{i-1}:

$$\begin{bmatrix} H_z \\ E_y \end{bmatrix}_{x_i} = \begin{vmatrix} T_{11}^{\Omega} & T_{12}^{\Omega} \\ T_{21}^{\Omega} & T_{22}^{\Omega} \end{vmatrix} \cdot \begin{bmatrix} H_z \\ E_y \end{bmatrix}_{x_{i-1}}, \qquad (1)$$

where

$$T_{11}^{\Omega} = \cosh(pw_{\Omega}) + k_0 \kappa \mu_z \frac{\sinh(pw_{\Omega})}{p},$$

$$T_{12}^{\Omega} = j \frac{\beta^2 - k_0^2 \varepsilon_y \mu \left(1 + \frac{\mu_z}{\varepsilon_y} \kappa^2\right)}{k_o \eta_0 \mu} \frac{\sinh(pw_{\Omega})}{p},$$

$$T_{21}^{\Omega} = -jk_o \eta_0 \mu_z \frac{\sinh(pw_{\Omega})}{p},$$

$$T_{22}^{\Omega} = \cosh(pw_{\Omega}) - k_o \kappa \mu_z \frac{\sinh(pw_{\Omega})}{p}.$$

Here, β is the propagation constant in z direction, the eigenvalue p is found from relation: $p = \sqrt{(\mu_z/\mu)\beta^2 - k^2 \mu_z \varepsilon_y}$, k_0 and η_0 are the wavenumber and intrinsic impedance of the free space, respectively, $\kappa = \eta_0 \Omega$ is the coupling coefficient. The structure containing n regions is defined by total transfer matrix relation determined as follow: $\underline{F_{x_i}} = T^n T^{n-1} T^1 F_{x_{i-1}}$.

Note that the TMA solution can be easily derived for any combination of the regions and boundary conditions at the side walls of the guide. If the transfer matrix for dielectric regions needed then it can be determined by introducing in Eq. (1) the $\kappa = 0$ and scalar values of the material electric permittivity and magnetic permeability.

3. Propagation in Ω waveguides

As a first example we consider a parallel plate pseudochiral guide shown in Fig. 1b. Using Eq. (1) together with magnetic wall conditions at the side planes x_i and x_{i-1}, the dispersion relation $T_{12}^{\Omega} = 0$ is determined.

It can be shown that the dominant wave in the guide is the TEM mode. This mode propagates in the guide with the propagation constant $\beta = \sqrt{k_0^2 \mu \mu_z \kappa^2 + \mu \varepsilon_y k_0^2}$ and distribution of the transverse field components E_y, $H_x \sim \exp(-px)$, where $p = k_0 \mu_z \kappa$. Note that the eigenvalue p is a real quantity so the dominant mode is a surface wave. The coupling coefficient κ is small, in a typical Ω material $\kappa \leq 1$. Hence, β slightly depends on coupling coefficient variation. In spite of this, the coupling coefficient strongly affects the field distribution inside the waveguide. The effect observed is similar to the field displacement phenomena appearing in a ferrite parallel plate guide with perpendicular magnetization [9]. Contrary to the ferrite guide, the phenomenon in the pseudochiral structure is reciprocal. It means that the field distribution in the Ω guide does not depend on the propagation direction. However, the field energy is concentrated at the one edge of the guide in dependence on the κ sign. Note that the change of the sign occurs when the Ω particles are reversed. It relates to the π turn of the pseudochiral sample in the guide. This operation causes the change of the propagation properties when the Ω sample is asymmetrically placed in the guide.

Therefore, we now consider the pseudochiral rectangular waveguide which cross-section is subdivided into three regions (one pseudochiral and two isotropic) as shown in Fig. 1a. The TMA is now used to determine the total transfer matrix. At this point it only remains to apply the electric wall boundary conditions at the edge planes of the guide to obtain the dispersion relation:

$$\mu \mu_z k_0 \kappa \tanh(p_{\Omega} w_{\Omega}) \left[p_1 \tanh(p_2 w_2) - p_2 \tanh(p_1 w_1) \right] + \\ + p_{\Omega} p_2 \mu \tanh(p_1 w_1) + p_1 p_{\Omega} \mu \tanh(p_2 w_2) + \\ + \left[\beta^2 - k_0^2 \mu \varepsilon_y \left(1 + \frac{\mu_z}{\varepsilon_y} \kappa^2\right) \right] \tanh(p_1 w_1) \times \\ \times \tanh(p_2 w_2) \tanh(p_{\Omega} w_{\Omega}) = 0. \qquad (2)$$

Let us consider two Ω guides depicted as $R^{+\Omega}$ and $L^{+\Omega}$ in Fig. 2a. The $L^{+\Omega}$ guide arises from the $R^{+\Omega}$ structure when the Ω slab in this guide is displaced symmetrically to the opposite side. We can infer from Eq. (2) that the propagation properties of these two guides are different. The another situation is shown in Fig. 2b where $R^{+\Omega}$ and $L^{-\Omega}$ guides are considered. The $L^{-\Omega}$ structure appears when the Ω slab is turn of π in the $L^{+\Omega}$ guide. Now we can note from Eq. (2) that both guides shown in Fig. 2b preserve invariant wave properties. The rotation of the guide by π with respect the y or z axis leads to the $L^{-\Omega}$ structure as shown in Fig. 3 while the identical structure appears after rotation of $R^{+\Omega}$ with respect the x axis. It means [10]

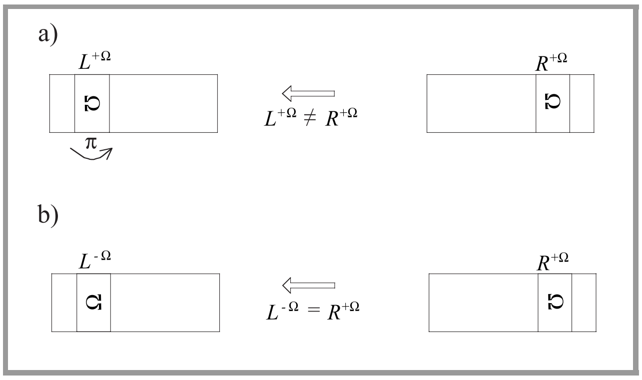

Fig. 2. Examples of pseudochiral guides after symmetrical displacement of the Ω slab in the basis structure.

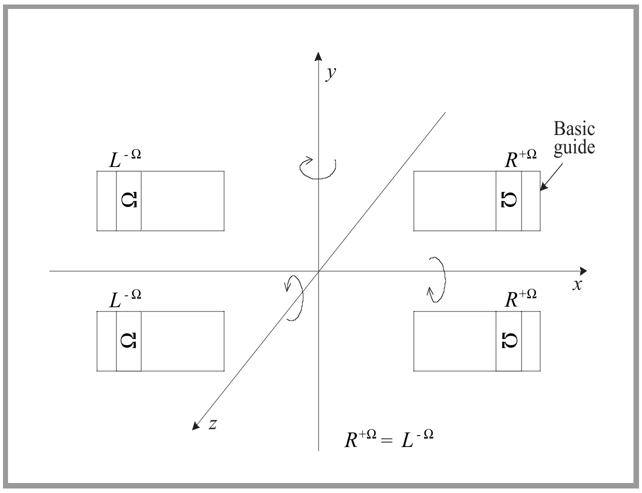

Fig. 3. Complementary guides arising from rotation of the basis structure by π about any axis of the rectangular coordinate system.

that the considered pseudochiral guide has symmetry with respect to the rotation by π about any x, y, z axis.

To discuss the influence of the reversing of Ω slab on the guide propagation properties we have displayed in Fig. 4 dispersion characteristics of the dominant mode, TE mode calculated from Eq. (2) against the change of κ sign. One of the most important features shown in Fig. 4 is that the difference between the characteristics depends only on parameter κ. The variation of the dominant mode cutoff frequency with κ parameter is shown in Fig. 5 for different localizations of the pseudochiral sample in the guide cross-section. As one can see, the cutoff frequency of the dominant mode varies with the displacement of the pseudochiral slab. However if the slab reaches the symmetrical position

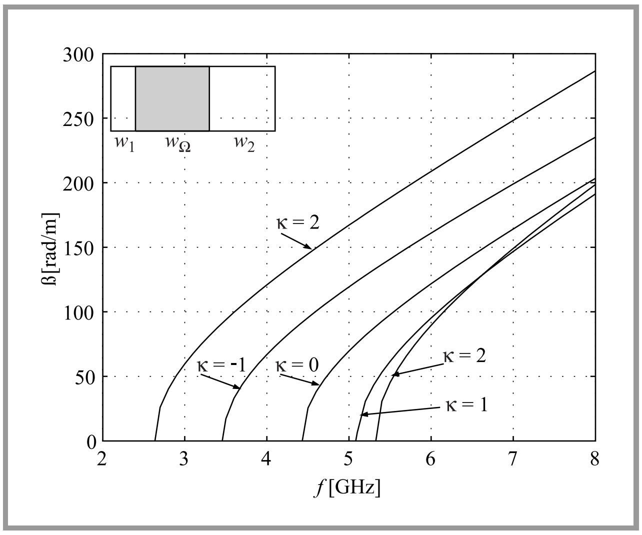

Fig. 4. Dispersion characteristics of the dominant mode in the pseudochiral waveguide shown in Fig. 1. Rectangular waveguide is WR-90. Dimensions in millimeters are, $w_1 = 2$, $w_\Omega = 10$, $w_2 = 10.86$, Ω slab; $\varepsilon = 2$, $\varepsilon_y = 2.5$, $\mu = 1$, $\mu_z = 1.5$, κ is a parameter.

with respect to first one and the slab is reversed then the cutoff frequencies in the both cases are equal. It confirms the predicted symmetry properties of such Ω guide. Finally, Fig. 6 shows the power density distribution of dominant mode in the cross-section of the guide with symmetrical localization of the pseudochiral slab. Note that the field displacement phenomenon can be observed when the pseudochirality is introduced. The intensity of the effect becomes greater due to a greater value of the κ parameter. The field energy is concentrated at the one or the other side of the slab in depending on the sign of κ. This phenomenon can be recognized as an effect complementary to this one appearing in the ferrite guide when the magneti-

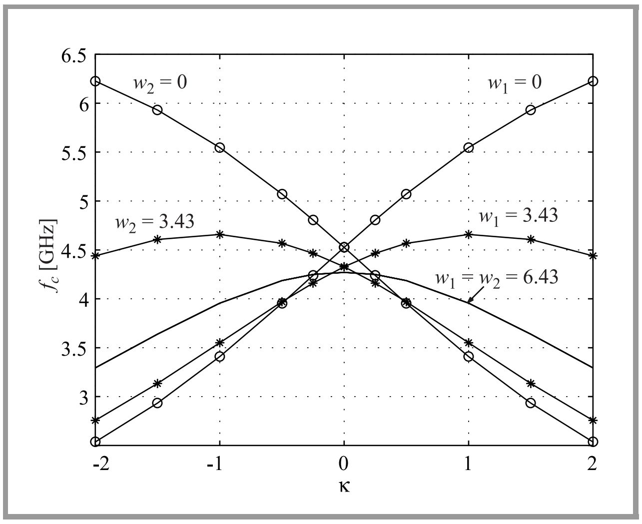

Fig. 5. Cutoff frequency characteristics of the dominant mode in the Ω guide shown in Fig. 1a. Parameters follow from Fig. 4.

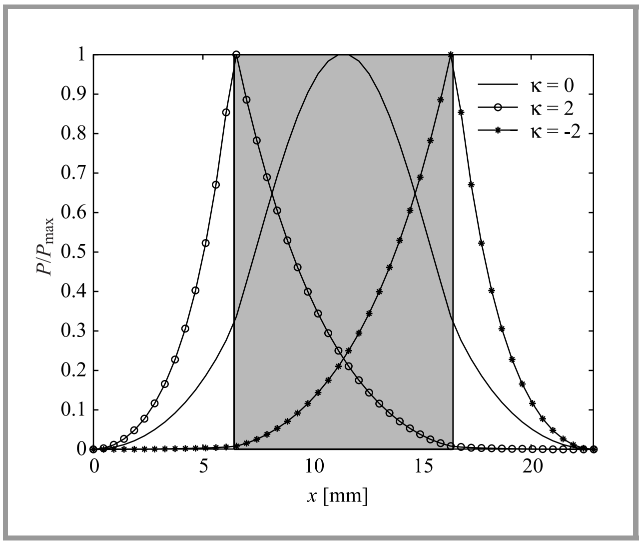

Fig. 6. The change of n power density distribution of dominant mode in the symmetrical Ω guide with respect to the sign of κ. The guide dimensions and parameters of the Ω slab are as in Fig. 4.

zation direction is reversed. However, in contrary to the ferrite structure the field distribution in the pseudochiral guide does not depend on the propagation direction.

4. Conclusion

We have examined a rectangular guides with a slab of pseudochiral Ω medium. Several unique and notable features associated with the considered structure were presented. It has been shown that the TE_0 wave can propagate in the guide as a dominant mode. The significant influence of the pseudochiral medium on the propagation characteristics and field distribution in the guide was observed. It has been shown that in such structure the field displacement phenomenon occurs. This effect depends on the coupling parameter κ. The direction of the field displacement is changed when the Ω slab in the guide is reversed. The similar phenomenon appearing in the ferrite guide depends on the magnetization or propagation directions. However, contrary to the ferrite structure the field displacement phenomenon in the pseudochiral guide is reciprocal. Future studies will focus on examination of scattering characteristics of the Ω guides to determine their prospective application.

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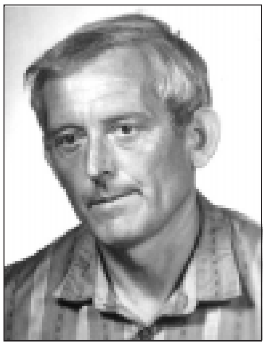

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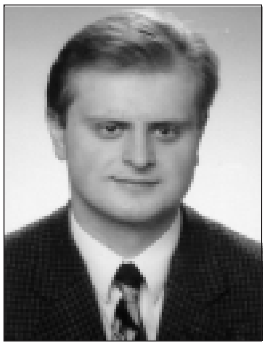

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