

# General Description of State-Space Continuous-Time $G_m$ -C Filters

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*Otrzymano 2001.11.13  
Autoryzowano 2002.01.14*

A general approach to analysis continuous-time analog  $G_m$ -C filters and equalizers, based on matrix description, is presented. A general transfer function formula for any  $G_m$ -C filter structure are given. The considerations accompanying this approach lead to useful relationships between the passive network of the filter and its transfer function. Based on these considerations, a new definition of continuous-time state-space  $G_m$ -C filters is given. Moreover, connections between state matrices for voltage- and current-mode state-space  $G_m$ -C filters are formulated and two canonical transformations of state-space filters into direct state-space ones, i.e. those having grounded capacitors only, are defined. Another two classes of  $G_m$ -C filters, i.e. reducible and quasi-state-space structures are also considered.

*Keywords:* state-space filters,  $G_m$ -C filters, matrix description, state-space representation.

## 1. INTRODUCTION

In recent years there has been a growing interest in continuous-time (CT) filters using transconductance amplifiers and capacitors ( $G_m$ -C). A number of attractive designs of such filters that are fully compatible with current CMOS technology have been reported in the literature [1]–[21]. The low-complexity excellent high-frequency performance and easy tunability of  $G_m$ -C filters make them suitable solutions for many applications such as hard disc drives, video filters, wireless communications, computer systems and instrumentation systems [11], [27]–[31].

In this paper, a novel and general approach to continuous-time  $G_m$ -C filters and equalizers is developed. Special attention is focused on an interesting sub-class of  $G_m$ -C filters, so-called state-space filters. This class of filters is usually identified with filters

that have only grounded capacitors [6], [25]. Such topologies are very attractive for integrated circuit (IC) realizations. In this work a novel definition of state-space filters is proposed, which comprises much wider class of  $G_m$ -C structures. According to this definition, the filter structure is called a state-space one if and only if there exist the state matrices for this structure. Of course, filter structures that have grounded capacitors satisfy this condition, however, there are many filters containing floating capacitors that are also state-space ones according to the new definition. Due to this, it is possible to apply a very useful technique of state-space description to much broader class of structures. It is important, since contemporary CMOS technologies make it possible to implement floating capacitors without any problems.

The paper is organized as follows. In Section 2, a general structure of  $G_m$ -C filter is presented. We derive a matrix description of general  $G_m$ -C filter based on a separate treatment of passive and active network of the circuit. It follows that the transfer function of any  $G_m$ -C filter can be easily calculated by means of the presented formulas. Section 3 is devoted to studying relationships between the transfer function of  $G_m$ -C filter and its passive network. In this section we also describe practical criteria of estimating the order of the filter's transmittance that can be used in computer aided filter synthesis. In Section 4 we deal with state-space  $G_m$ -C filters. A new definition of state-space  $G_m$ -C filters, based on invertibility properties of matrix corresponding to the passive network of the filter, is introduced. Relationships between state matrices of voltage- and current-mode  $G_m$ -C filters are derived. Moreover, we discuss two canonical transformations which enable us to convert any state-space filter into structures containing only grounded capacitors. In Section 5 we introduce two interesting classes of  $G_m$ -C structures, i.e. reducible and quasi-state-space ones. The paper concludes with Section 6.

## 2. GENERAL STRUCTURE OF $G_m$ -C FILTER

Consider a general structure of a voltage-mode  $G_m$ -C filter shown in Fig. 1. The current-mode counterpart can be obtained by inverting all transconductors and interchanging input and output of the filter [23], [32]. The structure in Fig. 1 contains  $n$  internal nodes denoted as  $x_i$ ,  $i = 1, \dots, n$ ,  $n$  input transconductors  $G_{mbi}$ , an output summer consisting of transconductors  $c_i G_m$  and  $-G_m$  as well as a feedforward transconductor  $dG_m$ . All transconductors form *active network*, while input capacitors  $C_{bi}$ ,  $i = 1, \dots, n$  and capacitors  $C_{ij}$ ,  $1 \leq i \leq j \leq n$  form *passive network*. It is easily seen that any  $G_m$ -C filter is a particular case of the general structure in Fig. 1. Note also that  $n$  is not necessarily equal to the order of the filter transmittance. In the following, we will investigate the connection between the number  $n$  of internal nodes, the order of the filter and its particular structure.

Now we would like to derive an analytical description of the considered structure. In particular, we will calculate the transmittance  $H(s)$  of the filter and show that the voltage-to-voltage transmittance  $H_v(s)$  of the voltage-mode filter is the same as the current-to-current transmittance  $H_c(s)$  of the current-mode one. In the following con-

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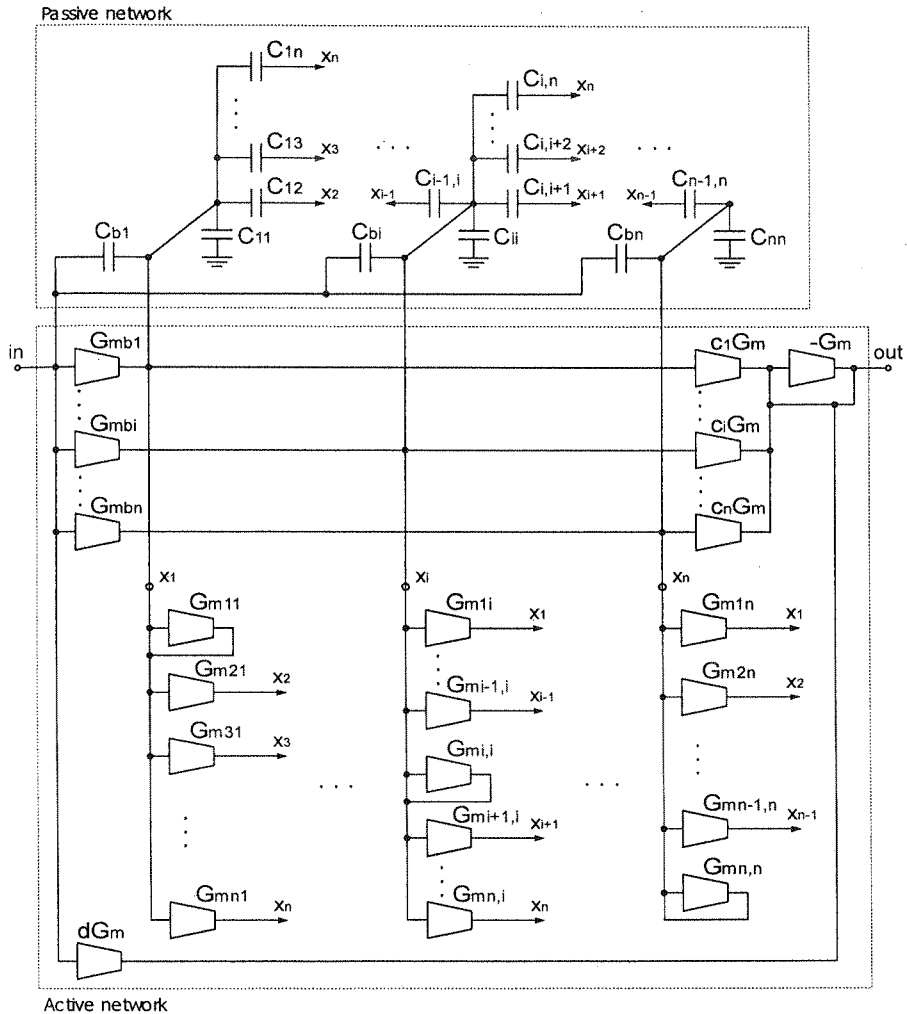


Fig. 1. General structure of voltage-mode  $G_m$ -C filter

siderations we will denote the voltage at the  $i$ -th node  $x_i$  also by  $x_i$ , which will not lead to confusion. A general structure of the voltage-mode  $G_m$ -C filter in Fig. 1 can be described by the following matrix equations:

$$s \begin{bmatrix} \sum_{j=1}^n C_{1j} & -C_{12} & \cdots & -C_{1n} \\ -C_{12} & \sum_{j=1}^n C_{2j} & \cdots & -C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -C_{1n} & -C_{2n} & \cdots & \sum_{j=1}^n C_{nj} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} u_i G_{mb1} + (u_i - x_1) s C_{b1} + \sum_{j=1}^n G_{m1j} x_j \\ u_i G_{mb2} + (u_i - x_2) s C_{b2} + \sum_{j=1}^n G_{m2j} x_j \\ \vdots \\ u_i G_{mbn} + (u_i - x_n) s C_{bn} + \sum_{j=1}^n G_{mnj} x_j \end{bmatrix} \quad (1)$$

and

$$u_0 = [c_1 \dots c_n] \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + d \cdot u_i \tag{2}$$

where  $u_i, u_o$  are the input and output voltages, respectively. To simplify the notation, in (1) we used symbols  $C_{kl}$  ( $k > l$ ) that denote the same elements as  $C_{lk}$ . Note that the vector on the right-hand side of (1) can be written in the form:

$$\begin{bmatrix} G_{m11} & G_{m12} & \dots & G_{m1n} \\ G_{m21} & G_{m22} & \dots & G_{m2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{mnn} & G_{mnn} & \dots & G_{mnn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} - s \begin{bmatrix} C_{b1} & 0 & \dots & 0 \\ 0 & C_{b2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_{bn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} G_{mb1} + sC_{b1} \\ G_{mb2} + sC_{b2} \\ \vdots \\ G_{mbn} + sC_{bn} \end{bmatrix} u_i \tag{3}$$

Introducing the following notation:

$$T_C = \begin{bmatrix} C_{b1} + \sum_{j=1}^n C_{1j} & -C_{12} & \dots & -C_{1n} \\ -C_{12} & C_{b2} + \sum_{j=1}^n C_{2j} & \dots & -C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -C_{1n} & -C_{2n} & \dots & C_{bn} + \sum_{j=1}^n C_{nj} \end{bmatrix}, \tag{4a}$$

$$G = \begin{bmatrix} G_{m11} & G_{m12} & \dots & G_{m1n} \\ G_{m21} & G_{m22} & \dots & G_{m2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{mnn} & G_{mnn} & \dots & G_{mnn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad \tilde{C}_v = [c_1 \dots c_n], \tag{4b}$$

$$\tilde{C}_c = [G_{mb1} + sC_{b1} \dots G_{mbn} + sC_{bn}], \quad D = d \tag{4c}$$

(note that  $T_C$  is symmetrical matrix, i.e.  $T_C^T = T_C$ ), one can rewrite (1) and (2) in the form of:

$$sT_C X = GX + \tilde{C}_c^T u_i \tag{5}$$

$$u_o = \tilde{C}_v X + D u_i$$

In a similar way one can write appropriate equations for the current-mode filter. Using notation (4) we get:

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$$\begin{aligned} s\mathbf{T}_C\mathbf{X} &= \mathbf{G}^T\mathbf{X} + \tilde{\mathbf{C}}_v^T i_i \\ i_o &= \tilde{\mathbf{C}}_c\mathbf{X} + \mathbf{D}i_i \end{aligned} \tag{6}$$

(2) where the entries  $x_i$  of the vector  $\mathbf{X}$  denote, as before, the intrinsic node voltages, and  $i_i$ ,  $i_o$  are the input and output current, respectively.

On the basis of (5) and (6) we can calculate the transmittances  $H_v$  and  $H_c$  which are:

$$H_v(s) = \frac{u_o(s)}{u_i(s)} = \tilde{\mathbf{C}}_v(s\mathbf{T}_C - \mathbf{G})^{-1}\tilde{\mathbf{C}}_c^T + \mathbf{D} \tag{7a}$$

$$H_c(s) = \frac{i_o(s)}{i_i(s)} = \tilde{\mathbf{C}}_c(s\mathbf{T}_C - \mathbf{G}^T)^{-1}\tilde{\mathbf{C}}_v^T + \mathbf{D} \tag{7b}$$

It is worth noticing that using (7a) and (7b), one can easily check the reciprocal behavior of voltage- and current-mode filters:

$$\begin{aligned} H_v(s) &= \frac{u_o(s)}{u_i(s)} = \mathbf{C}_v(s\mathbf{T}_C - \mathbf{G})^{-1}\mathbf{C}_c^T + \mathbf{D} = (\mathbf{C}_v(s\mathbf{T}_C - \mathbf{G})^{-1}\mathbf{C}_c^T + \mathbf{D})^T = \\ &= \mathbf{C}_c(s\mathbf{T}_C - \mathbf{G}^T)^{-1}\mathbf{C}_v^T + \mathbf{D} = \frac{i_o(s)}{i_i(s)} = H_c(s) \end{aligned} \tag{8}$$

(4a)

where we made use of the facts that  $H^T = H$  (since  $H$  is a scalar),  $(A^{-1})^T = (A^T)^{-1}$  for any invertible matrix  $A$  [22], and  $\mathbf{T}_C$  is symmetrical. Since  $H_v(s) = H_c(s)$ , we will use general symbol  $H(s)$  to denote a filter transmittance. Now, let us denote adjoint matrix of  $s\mathbf{T}_C - \mathbf{G}$  as  $\tilde{\mathbf{A}}$  where

$$\tilde{\mathbf{A}}(s) = \text{adj}(s\mathbf{T}_C - \mathbf{G}) = \text{adj}(s\mathbf{T}_C - \mathbf{G}^T)^T = \begin{bmatrix} \tilde{A}_{11}(s) & \tilde{A}_{12}(s) & \cdots & \tilde{A}_{1n}(s) \\ \tilde{A}_{21}(s) & \tilde{A}_{22}(s) & \cdots & \tilde{A}_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{n1}(s) & \tilde{A}_{n2}(s) & \cdots & \tilde{A}_{nn}(s) \end{bmatrix} \tag{9}$$

(4c)

This allows us to rewrite  $H$  in the form:

$$H(s) = \frac{1}{\det(s\mathbf{T}_C - \mathbf{G})} \sum_{i,j=1}^n c_i(G_{mbj} + sC_{bj})\tilde{A}_{ij}(s) + d \tag{10}$$

(5)

Note that many filter structures have only one input transconductor (i.e. no input signal distribution), a trivial output summer (i.e. one of the internal nodes is the output of the filter), and no input capacitors. This means that  $\tilde{\mathbf{C}}_c = [0 \cdots 0 \ G_{mbk} \ 0 \cdots 0]$ ,  $\tilde{\mathbf{C}}_v = [0 \cdots 0 \ 1 \ 0 \cdots 0] - 1$  at  $l$ -th position and  $C_{bi} = 0$  for  $i = 0, 1, \dots, n$ . In such case, expression (13) reduces to the form:

filter. Using

$$H(s) = \frac{G_{mbk} \tilde{A}_{ik}(s)}{\det(s\mathbf{T}_C - \mathbf{G})} \tag{11}$$

Similar expression can be written for two slightly more general cases. The first one is the case with no input capacitors, input signal distribution (i.e.  $\tilde{\mathbf{C}}_c = [G_{mb1} \dots G_{mbn}]$ ), and trivial output summer ( $\tilde{\mathbf{C}}_v = 0 \dots 0 \ 1 \ 0 \dots 0$ ) – 1 at  $l$ -th position). Then, the filter transmittance takes the form:

$$H(s) = \frac{1}{\det(s\mathbf{T}_C - \mathbf{G})} \sum_{i=1}^n G_{mbi} \tilde{A}_{li}(s) \tag{12}$$

Another case is when there are no input capacitors, no input signal distribution (i.e.  $\tilde{\mathbf{C}}_c = [0 \dots 0 \ G_{mbk} \ 0 \dots 0]$ ), however, there is non-trivial output summer (e.g. in follow-the-leader-like structures). Then, we have:

$$H(s) = \frac{G_{mbk}}{\det(s\mathbf{T}_C - \mathbf{G})} \sum_{i=1}^n c_i \tilde{A}_{ik}(s) \tag{13}$$

On the basis of the above expressions one can easily calculate the transmittance of any particular structure of  $G_m$ -C filter. In the next section we deal with the sensitivity of our general  $G_m$ -C structure.

We end this section with an example of a particular filter structure considered in the general setting presented above. Fig. 2 shows a structure of third order elliptic  $G_m$ -C filter based on a LC ladder simulation.

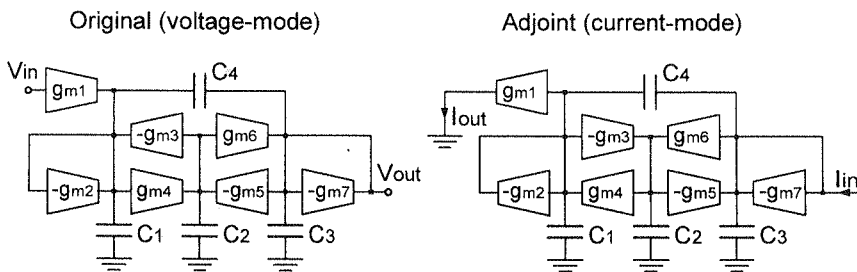


Fig. 2. Third order elliptic  $G_m$ -C filter in voltage- and current-mode

Matrices  $\mathbf{T}_C$ ,  $\mathbf{G}$ ,  $\tilde{\mathbf{C}}_c$ ,  $\tilde{\mathbf{C}}_v$  and  $\mathbf{D}$  for this circuit are the following (internal nodes are indexed from left to right):

$$\mathbf{T}_C = \begin{bmatrix} C_1 + C_4 & 0 & -C_4 \\ 0 & C_2 & 0 \\ -C_4 & 0 & C_3 + C_4 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} -g_{m2} & -g_{m3} & 0 \\ g_{m4} & 0 & -g_{m5} \\ 0 & g_{m6} & -g_{m7} \end{bmatrix}, \quad \begin{matrix} \tilde{\mathbf{C}}_c = [g_{m1} \ 0 \ 0] \\ \tilde{\mathbf{C}}_v = [0 \ 0 \ 1] \\ \mathbf{D} = 0 \end{matrix} \tag{14}$$

(11) Transmittance of the filter can be calculated using (11) with  $k = 1$  and  $l = 3$ . Assuming for simplicity that all transconductances are equal, i.e.  $g_{mi} = g_m$  for  $i = 1, \dots, 7$ , we obtain:

$$\det(sT_C - G) = C_2(C_1C_3 + C_1C_4 + C_3C_4)s^3 + C_2(C_1 + C_3 + 2C_4)g_m s^2 + (C_1 + C_2 + C_3)g_m^2 s + 2g_m^2 \quad (15)$$

$$\tilde{A}_{31}(s) = C_2C_4s^2 + g_m^2 \quad (16)$$

Let  $C_x^2 = C_1C_3 + C_1C_4 + C_3C_4$ . Then, filter transmittance  $H(s)$  has the form:

$$H(s) = \left( \frac{C_4g_m}{C_x^2} \right) \cdot \frac{s^2 + \frac{g_m^2}{C_2C_4}}{s^3 + \frac{(C_1 + C_3 + 2C_4)g_m}{C_x^2} s^2 + \frac{(C_1 + C_2 + C_3)g_m^2}{C_x^2} s + \frac{2g_m^3}{C_2C_x^2}} \quad (17)$$

The above example shows that the presented general approach provides as with a very elegant and easy way to calculate transfer function of any  $G_m$ -C filter structure.

### 3. PASSIVE NETWORK OF GENERAL $G_m$ -C FILTER

In this section we investigate how the passive network of general of  $G_m$ -C filter influences its properties, especially the order of filter transfer function  $H(s)$ . First, we would like to introduce necessary notation and give some general remarks.

Recall that matrix  $T_C$  representing the passive network of the filter is symmetrical. By construction of  $T_C$ , for any  $i = 1, \dots, n$  we have (see (4a)):

$$|(T_C)_{ij}| \geq \sum_{k=1, k \neq i}^n |(T_C)_{ik}|, \quad \text{and} \quad |(T_C)_{ii}| \geq \sum_{k=1, k \neq i}^n |(T_C)_{ki}| \quad (18)$$

i.e. matrix  $T_C$  is diagonally dominant. From linear algebra (Gershgorin's theorem),  $T_C$  is then positively semidefinite (that is  $\mathbf{x}^T T_C \mathbf{x} \geq 0$  for any nonzero  $n \times 1$  vector  $\mathbf{x}$ ). Moreover,  $T_C$  is invertible ( $\det(T_C) \neq 0$ ) if and only if  $T_C$  is positively definite (i.e.  $\mathbf{x}^T T_C \mathbf{x} > 0$  for any nonzero  $n \times 1$  vector  $\mathbf{x}$ ). Since  $T_C$  is symmetrical, there exists an orthogonal matrix  $T$  (i.e.  $TT^T = T^T T = I$ , which implies  $T^{-1} = T^T$ ) such that  $T^{-1} T_C T$  is diagonal (with non-negative elements, since  $T_C$  is positively semidefinite).

It is useful to introduce graphs representing passive network of the  $G_m$ -C filter. Let  $G_i(V_i, E_i)$  be a graph in which  $V_i$  is the set of nodes  $V_i = \{v_0, v_1, \dots, v_n\}$  and  $E_i$  is a set of edges. Node  $v_0$  represents both the common node  $x_0$  and input node  $x_{in}$  of the filter, while  $v_1, \dots, v_n$  represent internal nodes  $x_1, \dots, x_n$  of the filter. Edge  $e_{ij} \in E_i$  if and only if  $x_i$  and  $x_j$  are connected by the floating capacitor  $C_{ij}$  ( $j > i > 0$ ) or  $x_0$  and  $x_i$  are connected by

the grounded capacitor  $C_{ii} (i > 0)$ . Edge  $e_{in,j} \in E_t$  if and only if  $x_{in}$  and  $x_j$  are connected by input capacitor  $C_{bj}$ ,  $j = 1, \dots, n$ . We define also graph  $G_f(V_f, E_f)$  as the subgraph of  $G_t$ , in which  $V_f = \{v_1, \dots, v_n\}$  represents internal nodes of the filter and  $E_f \subset E_t$  such that  $e_{ij} \in E_f$  if and only if  $j > i > 0$ . Fig. 3 gives an example of the passive network and corresponding graphs  $G_t$  and  $G_f$ . It is seen from Fig. 3 that our example graph  $G_t$  has three disjoint components, while  $G_f$  has four disjoint components.

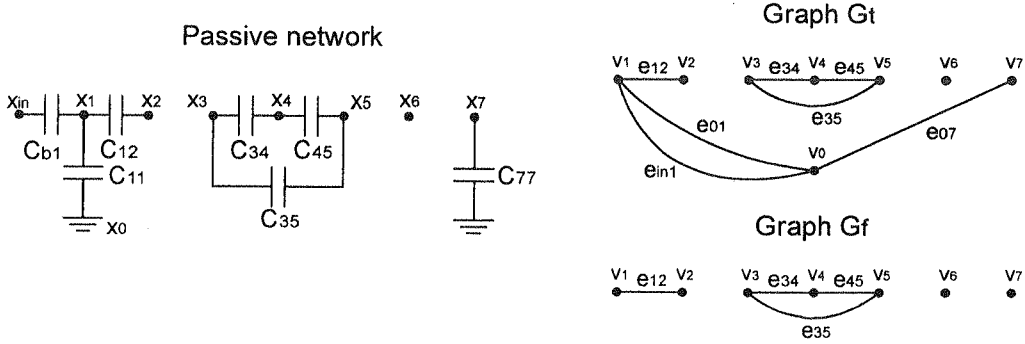


Fig. 3. Example of filter's passive network and related graphs  $G_t$  and  $G_f$

Having defined graphs  $G_t$  and  $G_f$ , we can consider the structure of the matrix  $T_C$ . It is seen from (4a) and the definition of graphs that  $T_C$  consists of square submatrices (*blocks*) situated along its diagonal (up to the renumbering of node indexes). The number of blocks is equal to the number of disjoint components of graph  $G_f$  reduced by the number of disjoint components of graph  $G_t$  having cardinality one and different from  $v_0$ . The latter represent just those internal nodes of the filter which are not connected to any other node by capacitance elements (we will call them *floating nodes*). The structure of matrix  $T_C$  for the passive network shown in Fig. 3 is the following:

$$T_C \approx \begin{bmatrix} X & X & 0 & 0 & 0 & 0 & 0 \\ X & X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X & X & X & 0 & 0 \\ 0 & 0 & X & X & X & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X \end{bmatrix} \quad (19)$$

It is evident that  $T_C$  cannot be invertible if filter has floating nodes. It is important for further applications to formulate necessary and sufficient conditions for invertibility of matrix  $T_C$ . In the subsequent considerations we will identify the block of matrix  $T_C$  with the corresponding sets of capacitors. We have the following proposition:

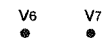
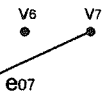
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**Proposition 1.** Let  $\tilde{T}_C$  be the block of matrix  $T_C$ . Then,  $\tilde{T}_C$  is invertible if and only if the corresponding set of capacitors contains at least one grounded capacitor or input capacitor.

*Proof.* Matrix  $\tilde{T}_C$  is symmetrical and diagonally dominant since it inherits these properties from  $T_C$ . Thus,  $\tilde{T}_C$  is invertible if and only if it is positively definite. Assume that  $\tilde{T}_C$  is  $k \times k$  matrix. The general form of  $\tilde{T}_C$  is then analogous to those of  $T_C$  given by (4a) (with possible renumbering of node indexes if necessary). Let  $y = (y_1, \dots, y_k)^T$  be any nonzero vector. Then, we have (recall that  $C_{ji} = C_{ij}$ ):

$$\begin{aligned}
 y^T \tilde{T}_C y &= y^T \begin{bmatrix} C_{b1} + C_{11} + \sum_{j \neq 1}^k C_{1j} & -C_{12} & \dots & -C_{1k} \\ -C_{21} & C_{b2} + C_{22} + \sum_{j \neq 2}^k C_{2j} & \dots & -C_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ -C_{k1} & -C_{k2} & \dots & C_{bk} + C_{kk} + \sum_{j \neq k}^n C_{kj} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} \\
 &= y^T \begin{bmatrix} y_1(C_{b1} + C_{11}) + \sum_{j \neq 1}^k C_{1j}(y_1 - y_j) \\ y_2(C_{b2} + C_{22}) + \sum_{j \neq 2}^k C_{2j}(y_2 - y_j) \\ \vdots \\ y_k(C_{bk} + C_{kk}) + \sum_{j \neq k}^k C_{kj}(y_k - y_j) \end{bmatrix} = \sum_{i=1}^k y_i^2(C_{bi} + C_{ii}) + \sum_{i=1}^k \sum_{j \neq i}^k C_{ij} y_i (y_i - y_j) \\
 &= \sum_{i=1}^k y_i^2(C_{bi} + C_{ii}) + \sum_{i=1}^k \left( \sum_{j>i}^k C_{ij} y_i (y_i - y_j) + \sum_{i>j}^k C_{ij} y_i (y_i - y_j) \right) \\
 &= \sum_{i=1}^k y_i^2(C_{bi} + C_{ii}) + \sum_{i=1}^k \left( \sum_{j>i}^k C_{ij} y_i (y_i - y_j) + \sum_{j>i}^k C_{ij} y_j (y_j - y_i) \right) \\
 &= \sum_{i=1}^k y_i^2(C_{bi} + C_{ii}) + \sum_{i=1}^k \sum_{j>i}^k C_{ij} (y_i - y_j)^2
 \end{aligned}$$

(19)

It is evident from (20) that  $y^T \tilde{T}_C y = 0$  if and only if  $y_i = y_j$  for any  $i, j = 1, \dots, k$  and  $C_{bi} + C_{ii} = 0$  for  $i = 1, \dots, k$ . This proves our proposition since if  $\tilde{T}_C$  has at least one grounded or input capacitor then it is positively definite (so it is invertible). Otherwise, there exists  $y$  for which  $y^T \tilde{T}_C y = 0$  so  $\tilde{T}_C$  is not invertible.

**Corollary 1.** Let  $\tilde{T}_C$  be the block of matrix  $T_C$ . Then,  $\tilde{T}_C$  is invertible if and only if the sum of its elements is positive.

*Proof.* It is apparent from (20) that the sum of elements of  $\tilde{T}_C$  equals  $\sum_{i=1}^k (C_{bi} + C_{ii})$  and is positive if and only if  $\tilde{T}_C$  contains at least one grounded or input capacitor. Our assertion follows now from Proposition 1.

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**Corollary 2.** Let  $\tilde{T}_C$  be the block of matrix  $T_C$ . Suppose that the size of  $\tilde{T}_C$  is  $k \times k$ . Then,  $rank(\tilde{T}_C) = k$  if and only if at least one of the capacitors corresponding to  $\tilde{T}_C$  is grounded or input one. Otherwise,  $rank(\tilde{T}_C) = k - 1$ .

*Proof.* If at least one of the capacitors corresponding to  $\tilde{T}_C$  is grounded or input one, then it is invertible by Proposition 1, hence  $rank(\tilde{T}_C) = k$ . Suppose now that  $\tilde{T}_C$  contains only floating capacitors  $C_{ij}(j > i > 0)$ . Then the sum of elements of  $\tilde{T}_C$  is zero by definition (see (20)). Consider any submatrix  $(\tilde{T}_C)_i$  of  $\tilde{T}_C$  defined by removing  $i$ -th row and  $i$ -th column of  $\tilde{T}_C$ ,  $i = 1, \dots, k$ . We assert that  $(\tilde{T}_C)_i$  is invertible. Obviously,  $(\tilde{T}_C)_i$  is symmetrical and diagonally dominant. Thus, it is enough to show that  $(\tilde{T}_C)_i$  is positively definite. Without loss of generality we can assume that  $i = k$ . Proceeding similarly as in the proof of Proposition 1 we obtain for any nonzero vector  $y = (y_1, \dots, y_{k-1})^T$ :

$$y^T(\tilde{T}_C)_k y = \sum_{i=1}^{k-1} y_i^2 C_{ik} + \sum_{i=1}^{k-1} \sum_{j>1}^{k-1} C_{ij}(y_i - y_j)^2 \tag{21}$$

This sum is positive since  $C_{ik} > 0$  for at least one  $i = 1, \dots, k-1$  by connectivity of the component of graph  $G_f$  corresponding to  $\tilde{T}_C$ . We have proved that the  $(k-1) \times (k-1)$  submatrix  $(\tilde{T}_C)_k$  of  $\tilde{T}_C$  is invertible so  $rank(\tilde{T}_C) = k - 1$ .

**Corollary 3.** Let  $T_C$  be  $n \times n$  matrix. Then,  $rank(T_C) = n + 1 - k$ , where  $k$  is the number of disjoint components of the corresponding graph  $G_f$ .

*Proof.* It is apparent that  $rank(T_C) = \sum_{i=1}^n rank((\tilde{T}_C)_i)$ , where  $(\tilde{T}_C)_i$  is  $i$ -th block of  $T_C$  and

$N$  is the total number of blocks. Denote the number of floating nodes of the filter as  $n_f$  and number of blocks connected to the common node or to the input node (by at least one capacitor) as  $N_c$ . It is seen from the definition of graphs  $G_f$  and  $G_r$  that the number of disjoint components of graph  $G_f$  equals  $k = N - N_c + 1 + n_f$ . Indeed, components of  $G_f$  are the following: blocks not connected to the common node ( $N - N_c$ ), one component consisting of all blocks connected to the common node and  $n_f$  floating nodes. Now,  $rank(T_C) = n - n_f - (N - N_c)$  since each floating node ‘produces’ zero row and zero column in matrix  $T_C$  and, by Corollary 2, each block of  $T_C$  not connected to the common node reduces  $rank(T_C)$  by one. Thus  $rank(T_C) = n - n_f - (N - N_c) = n + 1 - (N - N_c + 1 + n_f) = n + 1 - k$  which concludes the proof.

It is easy to observe that matrix  $T_C$  is invertible if and only if the filter has no floating nodes and all blocks of  $T_C$  are invertible. Now, we formulate it in terms of graph  $G_f$ .

**Proposition 2.** Matrix  $T_C$  is invertible if and only if the corresponding graph  $G_f$  is connected.

*Proof.* Suppose that  $T_C$  is invertible. From the preceding paragraph there are no floating nodes and each block of  $T_C$  contains a grounded capacitor or input capacitor. This means that each component of graph  $G_f$  is connected as a subgraph of  $G_r$  to node  $v_0$  by at least one edge. Thus,  $G_f$  is connected. Suppose now that  $G_f$  is connected. This implies in particular that the filter has no floating nodes. Moreover, if there exists a block without a grounded (or input) capacitor then  $G_f$  cannot be connected, which leads to contradiction. Thus, all blocks are invertible and so is  $T_C$ .

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Let us now turn to relationships between the passive network of the  $G_m$ - $C$  filter and the order of the filter transmittance which we will denote as  $n_H$ . Recall that  $H(s)$  is given by (11). Let  $T$  be any invertible  $n \times n$  matrix (so  $TT^{-1} = T^{-1}T = I$  — identity matrix). Then, we have:

$$\begin{aligned} H(s) &= \tilde{C}_v(sT_C - G)^{-1}\tilde{C}_c^T + D = \tilde{C}_v TT^{-1}(sT - G)^{-1}TT^{-1}\tilde{C}_c^T + D = \\ &= \tilde{C}_v T(sT^{-1}T_C T - T^{-1}GT)^{-1}T^{-1}\tilde{C}_c^T + D \end{aligned} \quad (22)$$

This means that  $H(s)$  is invariant under the transformation:

$$\tilde{C}_v \rightarrow \tilde{C}_v T, \quad \tilde{C}_c^T \rightarrow T^{-1}\tilde{C}_c^T, \quad T_C \rightarrow T^{-1}T_C T, \quad G \rightarrow T^{-1}GT \quad (23)$$

Using this fact we can prove the proposition which is the main goal of the section:

**Proposition 3.** Order  $n_H$  of the transmittance of a general  $G_m$ - $C$  filter is not larger than the rank of matrix  $T_C$ .

*Proof.* Transmittance of the filter is given by  $H(s) = \tilde{C}_v(sT_C - G)^{-1}\tilde{C}_c^T + D$ . It follows from the preceding paragraph that  $H(s)$  is invariant under the transformation given by (23) for any invertible matrix  $T$ . However, matrix  $T_C$  is symmetrical, hence there exists such  $T$  that  $T^{-1}T_C T$  is diagonal. Since  $\text{rank}(T_C)$  is invariant under similarity transformation, the number of nonzero (diagonal) elements of  $T^{-1}T_C T$  is exactly equal to  $\text{rank}(T_C)$ . It follows that  $\det(sT_C - G)$  is a polynomial in  $s$  of order  $\text{rank}(T_C)$ . This means that  $n_H \leq \text{rank}(T_C)$  (a strict inequality may occur when  $H(s)$  has common poles and zeros).

Using Corollary 3 we can reformulate assertion of Proposition 3 in terms of graph  $G_i$  corresponding to matrix  $T_C$ . Such a formulation may be more convenient for practical applications.

**Proposition 4.** Order  $n_H$  of the transmittance of a general  $G_m$ - $C$  filter is not larger than  $n+1-k$ , where  $n$  is the number of internal nodes of the filter and  $k$  is the number of disjoint components of graph  $G_i$  corresponding to matrix  $T_C$ .

*Proof.* The proof is straightforward and emerges from Proposition 3 and Corollary 3.

Note that the estimate similar to that given in Proposition 4 is known in literature (e.g. [33]), however here it is formulated in terms of matrix  $T_C$ , hence more convenient for practical applications.

The results formulated in this section have both theoretical and practical significance. For example, Proposition 2 can be used to define some important class of  $G_m$ - $C$  filters, which is done in the next section. Proposition 4 can be useful from the point of view of computer aided synthesis of  $G_m$ - $C$  filters. This is because it allows us to estimate a priori the possible order of transmittance of a filter generated by any design algorithm without calculating this transmittance (which is usually a time consuming task) so it can be a convenient criterion while choosing of a particular filter structure within some design strategies.

We end this section with the following remark. It has been mentioned in the discussion preceding Proposition 3 that the transformation given by (23) does not influence transmittance of the filter for any invertible matrix  $T$ . It is worth noticing that

such a transformation may determine a new realization of  $H(s)$ . Suppose, in particular, that  $T$  is similarity matrix for which  $T^{-1}T_cT$  is diagonal, and that there are no input capacitors in the filter (i.e.  $C_{bi} = 0$  for  $i = 1, \dots, n$ ). Then, transformed matrices  $\tilde{C}_vT$ ,  $T^{-1}\tilde{C}_c^T$ ,  $T^{-1}T_cT$ ,  $T^{-1}GT$  determine a new structure that realizes the same transmittance (of course, the considered transformation is nontrivial if  $T_c$  is not diagonal). This new structure has only grounded capacitors with values equal to the values of nonzero diagonal elements of  $T^{-1}T_cT$ .

#### 4. STATE-SPACE $G_m$ -C FILTERS

In this section we consider a certain class of filters which we call state-space  $G_m$ -C filters. Recall that a general description of linear filtering circuits can be obtained using the state-space representation. The appropriate state equations can be written as follows [7], [25]:

$$sX(s) = AX(s) + BU(s) \quad (24)$$

$$Y(s) = CX(s) + DU(s)$$

where  $X$ ,  $U$ ,  $Y$  are, respectively: state vector consisting of output signals of  $n$  internal integrators (thus,  $X \in M_{n \times 1}$ , where  $M_{n \times m}$  denotes the set of  $n \times m$  matrices), input signal and output signal;  $A \in M_{n \times n}$ ,  $B \in M_{n \times 1}$ ,  $C \in M_{1 \times n}$ ,  $D \in M_{1 \times 1}$ .

Using (50), the overall transfer function of the filter can be expressed in the form:

$$H(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \quad (25)$$

where  $I \in M_{n \times n}$  stands for the identity matrix. It is useful to introduce internal transfer functions  $f_i$  and  $g_i$ , i.e. the transfer function from the input of the filter to the output of integrator  $i$  (i.e. to  $x_i$ ) and the transfer function from the input of integrator  $i$  to the output of the filter, respectively [7]. Using the following notation:

$$\mathbf{F} = (f_1 \ f_2 \ \dots \ f_n)^T, \quad \mathbf{G} = (g_1 \ g_2 \ \dots \ g_n) \quad (26)$$

one can define the so-called observability and controllability Gramian matrices  $\mathbf{W}$  and  $\mathbf{K}$  [7]:

$$\mathbf{W} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{G} * \mathbf{G} d\omega, \quad \mathbf{K} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{F} \mathbf{F} * d\omega \quad (27)$$

One can also prove that the matrices  $\mathbf{W}$  and  $\mathbf{K}$  satisfy the following equations:

$$A^T \mathbf{W} + \mathbf{W} A = -C^T C, \quad (28a)$$

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$$AK + KA^T = -BB^T \tag{28b}$$

Now, we turn to the general description of a  $G_m$ -C filter presented in Section 2. Assume that matrix  $T_c$ , corresponding to the passive network of the filter, is invertible, and that there are no input capacitors in the filter (i.e.  $C_{bi} = 0$  for  $i = 1, \dots, n$  and  $\tilde{C}_c = [G_{mb1} \dots G_{mbn}]$ ). Then, we can rewrite (5) and (6) in the following form:

$$\begin{aligned} sX &= T_c^{-1}GX + T_c^{-1}\tilde{C}_c^T u_i \\ u_o &= \tilde{C}_c X + Du_i \end{aligned} \tag{29}$$

$$\begin{aligned} sX &= T_c^{-1}G^T X + T_c^{-1}\tilde{C}_v^T i_i \\ i_o &= \tilde{C}_c X + Di_i \end{aligned} \tag{30}$$

We compare (29) and (30) to (24). It is apparent that all those equations are the same provided that  $A = T_c^{-1}G$ ,  $B = T_c^{-1}\tilde{C}_c^T$ ,  $C = \tilde{C}_v$  in (29),  $A = T_c^{-1}G^T$ ,  $B = T_c^{-1}\tilde{C}_v^T$ ,  $C = \tilde{C}_c$  in (30), and that intrinsic node voltages are chosen as state variables. Note that absence of input capacitors is crucial, since otherwise we would have a complex variable  $s$  in the right-hand sides of (29) and (30); in such a case (29) and (30) could not be compared to (24). Due to this fact we can introduce the following definition:

**Definition 1.** Any  $G_m$ -C filter without input capacitors and such that corresponding matrix  $T_c$  is invertible is called a *state-space  $G_m$ -C filter*. The state variables of such a filter, relating to equations (29)–(30), are just internal node voltages.

Thus, invertibility of matrix  $T_c$  and absence of input capacitors are the necessary and sufficient conditions for the existence of the state matrices. From the discussion preceding Definition 1 we obtain that the state matrices of voltage-mode filter denoted as  $A_v, B_v, C_v, D_v$  and the state matrices of current-mode filter denoted as  $A_c, B_c, C_c, D_c$  are the following:

$$A_v = T_c^{-1}G \quad B_v = T_c^{-1}\tilde{C}_c^T \quad C_v = \tilde{C}_v \quad D_v = D \tag{31}$$

$$A_c = T_c^{-1}G^T \quad B_c = T_c^{-1}\tilde{C}_v^T \quad C_c = \tilde{C}_c \quad D_c = D \tag{32}$$

Note that if the  $G_m$ -C filter has only grounded capacitors and no floating nodes, then the corresponding matrix  $T_c$  is diagonal and, of course, invertible. In literature (e.g. [7], [26]), only such filters are usually considered as state-space ones. We will refer to them as *direct state-space filters*. Our definition comprises a much wider class of filters. Using Proposition 2 (Section 4) one can easily reformulate Definition 1 to the form: any  $G_m$ -C filter for which graph  $G_r$ , corresponding to matrix  $T_c$  is connected, is called a state-space filter.

Using (31) and (32) we can obtain the following relationships between the state matrices for voltage- and current-mode filter:

$$A_c = T_c^{-1}A_v^T T_c \quad B_c = T_c^{-1}C_v^T \quad C_c = B_v^T T_c \quad D_c = D_v \tag{28a}$$

Having (33) we can also derive analogous relations for the observability and controllability matrices  $W$  and  $K$ . Rewriting (28a) for the voltage-mode filter and using (33) we obtain:

$$A_v^T W_v + W_v A_v = -C_v^T C_v \tag{34}$$

which yields:

$$(T_c A_c T_c^{-1}) W_v + W_v (T_c^{-1} A_c^T T_c) = -T_c B_c B_c^T T_c \tag{35}$$

$$A_c (T_c^{-1} W_v T_c^{-1}) + (T_c^{-1} W_v T_c^{-1}) A_c^T = -B_c B_c^T \tag{36}$$

However, rewriting (28b) for the current-mode filter we have:

$$A_c K_c + K_c A_c^T = -B_c B_c^T \tag{37}$$

Thus, we get the following equality:

$$K_c = T_c^{-1} W_v T_c^{-1} \tag{38}$$

A similar reasoning that starts from (54b), yields the following for voltage-mode filter:

$$W_c = T_c K_v T_c \tag{39}$$

Relations (33), (38) and (39) are important in applications. For example, in [32] the authors considered dynamic range ( $DR$ ) of voltage- and current-mode state-space  $G_m$ - $C$  filters. It turned out that the general  $DR$  formulas for a current-mode filter can be (due to relations (33), (38) and (39)) easily expressed in terms of a voltage-mode filter, which significantly facilitates further comparison of both modes of the filter.

It is well known (e.g. [2]) that structures containing only grounded capacitors and no floating nodes (i.e. direct state-space filters in our terminology) are most suitable for monolithic implementation. The reason is that parasitic capacitances can be absorbed into the circuit capacitances in these structures and that grounded capacitors need smaller chip areas than floating ones. General description of  $G_m$ - $C$  filters introduced in Section 2 gives an easy way to transform any state-space filter (i.e. containing, in general, also floating capacitors) into direct state-space structure. Below, we describe two 'canonical' transformations from a state-space structure into a direct one.

Let  $\hat{T}_c$  be any diagonal matrix with positive elements, i.e.  $(\hat{T}_c)_{ii} = \hat{C}_i$ ,  $\hat{C}_i > 0$ ,  $i = 1, \dots, n$ . Multiplying the first equation in (29) by  $\hat{T}_c$ , one gets:

$$s\hat{T}_c X = \hat{T}_c T_c^{-1} G X + \hat{T}_c T_c^{-1} \tilde{C}_c^T u_i \tag{40}$$

$$u_o = \tilde{C}_v X + D u_i$$

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Introducing the following notation:

$$\hat{G} = \hat{T}_c T_c^{-1} G, \quad \hat{C}_c = \tilde{C}_c T_c^{-1} \hat{T}_c, \quad \hat{C}_v = \tilde{C}_v, \quad \hat{D} = D \quad (41)$$

(34) we can rewrite (40) in the form

$$s\hat{T}_c X = \hat{G}X + \hat{C}_c^T u_i \quad (42)$$

$$u_o = \hat{C}_v X + \hat{D} u_i \quad (35)$$

(36) which is the same as the equations (5) describing the general structure of a voltage-mode  $G_m$ -C filter. This means that matrices  $\hat{T}_c$ ,  $\hat{G}$ ,  $\hat{C}_c$ ,  $\hat{C}_v$  and  $\hat{D}$  determine the new filter structure. Due to the assumptions concerning matrix  $\hat{T}_c$ , this structure is a direct state-space one and has the same transfer function as the original filter. Indeed:

$$\begin{aligned} (37) \quad \hat{H}(s) &= \hat{C}_v (s\hat{T}_c - \hat{G})^{-1} \hat{C}_c^T + \hat{D} = \tilde{C}_v (s\hat{T}_c - \hat{T}_c T_c^{-1} G)^{-1} \hat{T}_c T_c^{-1} \tilde{C}_c^T + D = \\ &= \tilde{C}_v (T_c \hat{T}_c^{-1} (s\hat{T}_c - \hat{T}_c T_c^{-1} G))^{-1} \tilde{C}_c^T + D = \tilde{C}_v (sT_c - G)^{-1} \tilde{C}_c^T + D = H(s) \end{aligned} \quad (43)$$

(38) Note that invertibility of matrix  $\hat{T}_c$  is crucial in proving above equality. All elements of a new filter structure are given by the appropriate matrix elements, namely:  $C_i = (\hat{T}_c)_{ij}$ ,  $G_{mij} = (\hat{G})_{ij}$ ,  $G_{mbi} = (\hat{C}_c)$  and  $c_i = (\hat{C}_v)$ . It is seen that changing elements  $\hat{C}_i$  of matrix  $\hat{T}_c$  means changing the values of filter elements without changing its structure. This is very convenient since it makes possible to optimize the filter, e.g. with respect to the sensitivity or the dynamic range.

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Now we describe another transformation of a state-space  $G_m$ -C filter to direct state-space one. Selecting any diagonal matrix  $\check{T}_c$  with positive elements  $(\check{T}_c)_{ii} = \check{C}_i$ ,  $\check{C}_i > 0, i = 1, \dots, n$ , we can repeat, for the current-mode filter, the procedure presented in the preceding paragraph. Then, matrices  $\check{G}$ ,  $\check{C}_c$ ,  $\check{C}_v$  and  $\check{D}$ , defined as:

$$\check{G} = G T_c^{-1} \check{T}_c, \quad \check{C}_c = \tilde{C}_c, \quad \check{C}_v = \tilde{C}_v T_c^{-1} \check{T}_c, \quad \check{D} = D \quad (44)$$

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as well as matrix  $\check{T}_c$ , determine another structure of the  $G_m$ -C filter which is, of course, a direct state-space one. Filter elements are defined similarly as in the previous case.

$= \hat{C}_i, \hat{C}_i > 0,$   
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In order to see better the difference between filter structures generated by the two described above transformation procedures, consider the case when the original filter has only one input transconductor (no input signal distribution) and a trivial output summer, i.e.  $\tilde{C}_c = [0 \dots 0 G_{mbk} 0 \dots 0]$  and  $\tilde{C}_v = [0 \dots 0 1 0 \dots 0] - 1$  at  $l$ -th position. Comparing (41) and (44) one can see that the filter structure determined by matrices  $\hat{T}_c$ ,  $\hat{G}$ ,  $\hat{C}_c$ ,  $\hat{C}_v$  and  $\hat{D}$  has (in general) input signal distribution and a trivial output summer, however, the structure generated by matrices  $\check{T}_c$ ,  $\check{G}$ ,  $\check{C}_c$ ,  $\check{C}_v$  and  $\check{D}$  has no input signal distribution and (in general) a non-trivial output summer. This is very convenient, since it gives us an additional degree of freedom while designing the filter for particular application.

It is worth noticing that the described above transformation procedures can be easily generalized. Suppose that  $\hat{T}_C$  is any invertible matrix, which is symmetrical, diagonally dominant, with positive diagonal and negative non-diagonal elements. Then, the transformation generated by  $\hat{T}_C$  determines a new filter structure, since any  $n \times n$  matrix satisfying the above conditions represents the passive network of some state-space filter (see Section 3). Indeed, any non-diagonal element  $(\hat{T}_C)_{ij}, j > i$ , can be interpreted as a floating capacitor and the element  $(\hat{T}_C)_{ij} - \sum_{j=1, j \neq i}^n (\hat{T}_C)_{ij}, i = 1, \dots, n$  as a grounded capacitor. Other filter elements are given as usual:  $G_{mij} = (\hat{G})_{ij}, G_{mbi} = (\hat{C}_C)_i$  and  $c_i = (\hat{C}_v)_i$ . The same is true for the transformation generated by matrix  $T_C$ . Thus, we have proved the following result:

**Proposition 5.** Any state-space Gm-C filter can be transformed (without changing its transfer function) to the structure that has any passive network, provided that the corresponding matrix  $T_C$  is invertible. For each such passive network there exist two canonical transformations that yields different state-space filter structures.

The above result creates new possibilities of filter optimization. For instance, given any structure realizing the desired transfer function, by changing the passive network of the filter we can look for another structure with as small number of transconductors as possible. It is also possible to look for design trade-offs concerning complexity of passive and active network.

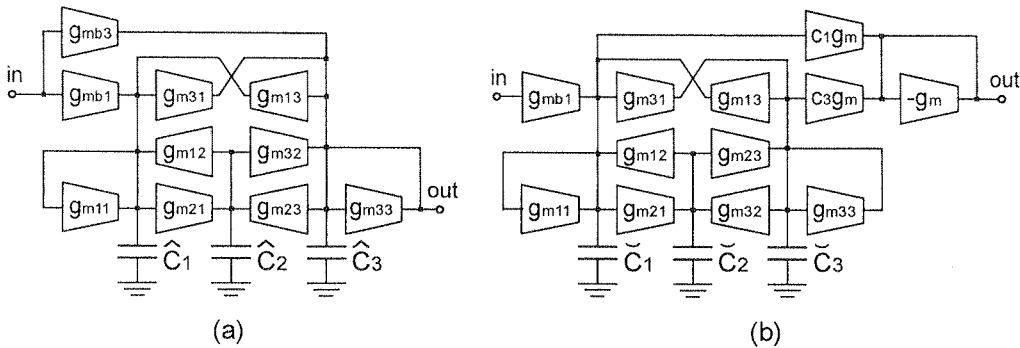


Fig. 4. Third order elliptic filter of Fig. 2 transformed to direct state-space structure by the transformation generated by matrix  $\hat{T}_C$  (a) and  $T_C$  (b)

Now we would like to give an example of the state-space filter transformation into the direct state-space structure. Consider the third order elliptic filter shown in Fig. 2 (Section 2). Fig. 4a shows the (voltage-mode) filter structure obtained by means of transformation generated by matrix  $\hat{T}_C$ . Filter elements are the following:  $\hat{C}_i = (\hat{T}_C)_{ii}, g_{mij} = (\hat{G})_{ij}$  and  $g_{mbi} = (\hat{T}_C)_i$  with matrices  $\hat{G}$  and  $\hat{C}_C$  calculated according to (41), where matrices  $T_C, G$  and  $C_C$  of the original filter are given by (14). Fig. 4b shows the (voltage-mode) filter structure obtained using the transformation generated by matrix  $T_C$ . Filter elements are the following:  $\tilde{C}_i = (T_C)_{ii}, g_{mij} = (G)_{ij}, g_{mbi} = (C_C)_i$  and  $c_i = (C_v)_i$  with

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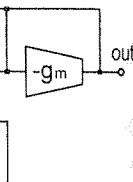
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matrices  $\check{G}$ ,  $\check{C}_c$ ,  $\check{C}_v$  calculated according to (44). In particular, matrices  $\hat{T}_c$  and  $\check{T}_c$  can be identical, which means that both resulting direct state-space structures have the same passive network.

Note that the structure in Fig. 4a has two input transconductors and no output summer (output signal is taken directly from one of the internal nodes), while structure in Fig. 4b has one input transconductor and nontrivial output summer consisting of two transconductors.

### 5. REDUCIBLE AND QUASI-STATE-SPACE STRUCTURES

In this section we focus our attention on certain interesting classes of  $G_m$ -C structures which we call reducible structures and quasi-state-space structures. Suppose that we have  $G_m$ -C filter with  $n$  internal nodes  $x_1, x_2, \dots, x_n$  and corresponding matrices  $T_c, G, \check{C}_c, \check{C}_v$  and  $D$  defined by (4a)-(4c). Moreover, let the filter has a floating node, say  $x_k$  (recall that floating node is the node not connected to any other node by capacitance elements). It follows that if this  $k$ -th node has inner loop (i.e.  $G_{mkk} \neq 0$ ) then node  $x_k$  can be removed, i.e. the filter structure can be reduced to the structure with  $n-1$  internal nodes. The reduction does not influence the transmittance of the filter. Below, we give the detailed description of the reduction procedure.



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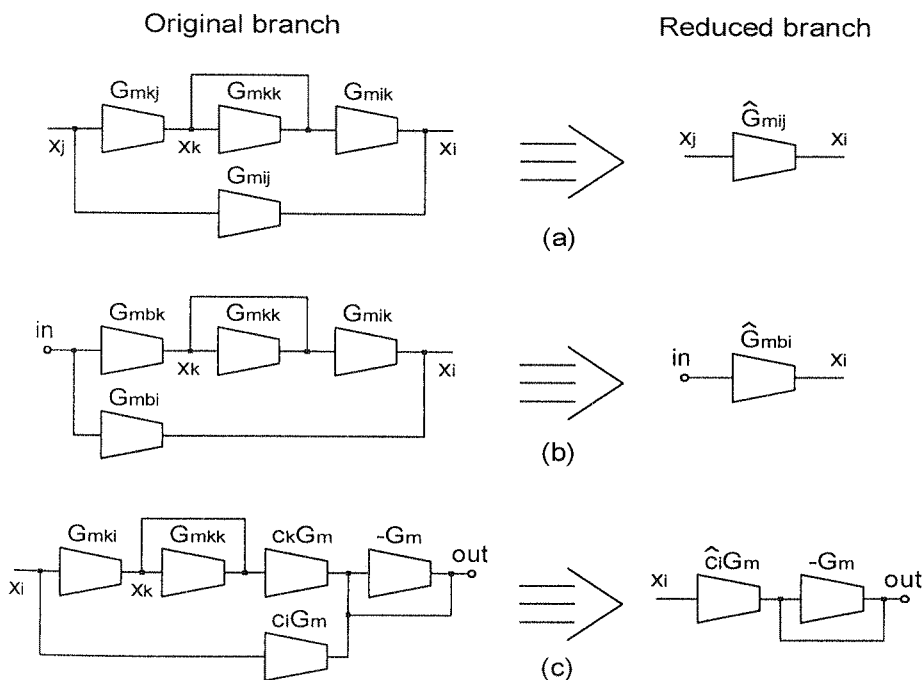


Fig. 5. Filter structure reduction: connection between input node and  $i$ -th internal node (a),  $j$ -th and  $i$ -th internal nodes (b),  $i$ -th internal node and output of the filter (c)

Fig. 5a shows the fragment of the active network of the filter concerning connections between input node and  $i$ -th internal node ( $i \neq k$ ). It is seen that the current at  $i$ -th node due to the voltage  $u_i$  at the input of  $G_{mbi}$  and  $G_{mbk}$  is  $u_i(G_{mbk} - G_{mbk}G_{mik}/G_{mkk})$ . Thus, the two branches shown in Fig. 5a can be replaced by one transconductor  $\hat{G}_{mbi}$  such that:

$$\hat{G}_{mbi} = G_{mbi} - \frac{G_{mbk}G_{mik}}{G_{mkk}} \tag{45}$$

Similar reasoning concerning connections between  $j$ -th and  $i$ -th internal nodes ( $j \neq i \neq k$ ) and between  $i$ -th internal node and output of the filter ( $i \neq k$ ) results in the reduction of appropriate branches shown in Figs. 5b and 5c. The new transconductance value  $\hat{G}_{mij}$  and new coefficient  $\hat{c}_i$  are the following:

$$\hat{G}_{mij} = G_{mij} - \frac{G_{mkj}G_{mik}}{G_{mkk}} \tag{46}$$

$$\hat{c}_i = c_i - \frac{c_k G_{mik}}{G_{mkk}} \tag{47}$$

Described procedure enables us to eliminate  $k$ -th internal node from the filter structure. This results in a new structure with  $n-1$  internal nodes and corresponding matrices  $\hat{T}_C$ ,  $\hat{G}$ ,  $\hat{C}_c$ ,  $\hat{C}_v$  and  $\hat{D}$ . Appropriate matrix elements are given below:

$$(\hat{T}_C)_{ij} = \begin{cases} (T_C)_{ij} & i, j < k \\ (T_C)_{i, j-1} & i < k, j \geq k \\ (T_C)_{i-1, j} & i \geq k, j < k \\ (T_C)_{i-1, j-1} & i, j \geq k \end{cases} \quad (\hat{G})_{ij} = \begin{cases} \hat{G}_{m_{ij}} & i, j < k \\ \hat{G}_{m_{i, j-1}} & i < k, j \geq k \\ \hat{G}_{m_{i-1, j}} & i \geq k, j < k \\ \hat{G}_{m_{i-1, j-1}} & i, j \geq k \end{cases} \tag{48a}$$

$$(\hat{C}_c)_i = \begin{cases} \hat{G}_{mb_i} + sC_{b_i} & i < k \\ \hat{G}_{mb_{i-1}} + sC_{b_{i-1}} & i \geq k \end{cases} \quad (\hat{C}_v)_i = \begin{cases} \hat{c}_i & i < k \\ \hat{c}_{i-1} & i \geq k \end{cases} \quad \hat{D} = d \tag{48b}$$

It is seen that due to removing  $k$ -th node, some shift of indexes is necessary. Obviously, if the original filter structure has more than one floating node with inner loop, then all such nodes can be removed by iteration of described procedure.

On the basis of the above considerations we can formulate the formal definitions:

**Definition 2.** The  $G_m$ -C filter structure is called *reducible* if it has at least one floating node  $x_k$  with inner loop (i.e.  $G_{mkk} \neq 0$ ).

**Definition 3.** The  $G_m$ -C filter structure is called *completely reducible* if each of its floating nodes has inner loop.

Reduction procedure can be helpful in computer aided synthesis of  $G_m$ -C filters since it allows to remove some redundancy in filter structure. Note, however that reduction is not always desirable — more complex structures are often used for their

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flexibility (there exist universal structures making possible independent control of filter poles and zeros, e.g. [2]). Note also that some design techniques, e.g. Bruton Transformation [2], lead to the structures which are not reducible.

Now, we would like to introduce another class of  $G_m$ -C structures.

**Definition 4.** The  $G_m$ -C filter structure is called *quasi-state-space* structure if it has no input capacitors (i.e.  $C_{bi} = 0, i = 1, \dots, n$ ) and matrix  $T_C$  corresponding to filter's passive network is not invertible but each block of  $T_C$  is invertible.

Although for quasi-state-space filters do not exist state matrices, it follows that such structures exhibit similar properties as state-space structures. In particular, any quasi-state-space filter can be transformed to the direct structure (i.e. having only grounded capacitors). To show this, suppose that  $n \times n$  matrix  $T_C$  has the following structure:

$$T_C = \begin{bmatrix} (\tilde{T}_C)_1 & & & & 0 \\ & \ddots & & & \\ & & (\tilde{T}_C)_k & & \\ & & & 0 & \\ 0 & & & & 0 \end{bmatrix} \quad (49)$$

where  $(\tilde{T}_C)_i, i = 1, \dots, k$  are invertible blocks of sizes  $n_i \times n_i$  and the rest of elements are zeros. Let  $n^\# = n_1 + n_2 + \dots + n_k$ . Note, that matrix  $T_C$  of any quasi-state-space filter has this structure up to renumbering of indexes. Let  $T_C^\#$  be the matrix defined as below:

$$T_C^\# = \begin{bmatrix} (\tilde{T}_C)_1 & & & & 0 \\ & \ddots & & & \\ & & (\tilde{T}_C)_k & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} \quad (50)$$

Then,  $T_C^\#$  is invertible and  $(T_C^\#)^{-1} \cdot T_C = T_C \cdot (T_C^\#)^{-1} = I^\#$ , where  $I^\#$  is diagonal matrix such that  $I_{ii}^\# = 1$  for  $1 < i \leq n^\#$  and  $I_{ii}^\# = 0$  for  $n^\# < i \leq n$ . Now, multiplying equation (5) from the left by  $(T_C^\#)^{-1}$  we obtain an analog of equation (29) for quasi-state-space filters:

$$sI^\# X = (T_C^\#)^{-1} G X + (T_C^\#)^{-1} \check{C}_c^T u_i \quad (51)$$

$$u_o = \check{C}_v X + D u_i$$

Let  $\hat{T}_C^\#$  be any diagonal matrix such that, i.e.  $(\hat{T}_C^\#)_{ii} = \hat{C}_i, \hat{C}_i > 0$ , for  $1 \leq i \leq n^\#$  and  $(\hat{T}_C^\#)_{ii} = 1$  for  $n^\# < i \leq n$ . Multiplying first equation in (51) by  $\hat{T}_C^\#$  one gets:

$$s\hat{T}_C^\#X = \hat{T}_C^\#(T_C^\#)^{-1}GX + \hat{T}_C^\#(T_C^\#)^{-1}\check{C}_c^T u_i$$

$$u_o = \check{C}_v X + D u_i$$
(52)

where  $\hat{T}_C^{\#\#} = \hat{T}_C^\# I^\#$  is diagonal matrix such that  $(\hat{T}_C^{\#\#})_{ii} = \hat{C}_i, \hat{C}_i > 0$ , for  $1 \leq i \leq n^\#$  and  $(\hat{T}_C^{\#\#})_{ii} = 0$  for  $n^\# < i \leq n$ . It is seen, when comparing (52) to (5), that  $\hat{T}_C^{\#\#}$  and matrices  $\hat{G}^\#, \hat{C}_c^\#, \hat{C}_v^\#$  and  $\hat{D}^\#$  defined as:

$$\hat{G}^\# = \hat{T}_C^\#(T_C^\#)^{-1}G, \quad \hat{C}_c^\# = \check{C}_c(T_C^\#)^{-1}\hat{T}_C^\#, \quad \hat{C}_v^\# = \check{C}_v, \quad \hat{D}^\# = D$$
(53)

determine new structure which is a direct (quasi-state-space) one. Of course, elements  $\hat{C}_i$  are new grounded capacitances of the filter. Note that conclusions comprised in the discussion following (42) remain valid in this case. In particular, transmittance of the original and transformed filters are the same, since  $\hat{T}_C^\#$  is invertible matrix.

Similarly as in Section 5, we can define second transformation starting from description of the current-mode filter. If  $\check{T}_C^\#$  is any diagonal matrix such that, i.e.  $(\check{T}_C^\#)_{ii} = \check{C}_i, \check{C}_i > 0$ , for  $1 \leq i \leq n^\#$  and  $(\check{T}_C^\#)_{ii} = 1$  for  $n^\# < i \leq n$ , then a new filter structure is determined by the following matrices:

$$\check{T}_C^{\#\#} = \check{T}_C^\# I^\#, \quad \check{G}^\# = G(T_C^\#)^{-1}\check{T}_C^\#, \quad \check{C}_v^\# = \check{C}_v, \quad \check{C}_c^\# = \check{C}_c(T_C^\#)^{-1}\check{T}_C^\#, \quad \check{D}^\# = D$$
(54)

The above transformations can be generalized similarly as in the state-space filters case, i.e. diagonal matrix  $\hat{T}_C^\#$  can be replaced by any matrix  $\hat{T}_C^\#$  such that its  $n^\# \times n^\#$  submatrix is symmetrical, diagonally dominant, having positive diagonal and negative non-diagonal elements and  $(\hat{T}_C^\#)_{ij} = 1$  for  $i, j > n^\#$ . By the argument similar to that given in paragraph preceding Proposition 5, transformation generated by  $\hat{T}_C^\#$  determine new filter structure with passive network corresponding to matrix  $\hat{T}_C^{\#\#} = \hat{T}_C^\# I^\#$  and the rest of elements given by  $G_{mij} = (\hat{G}^\#)_{ij}, G_{mbi} = (\hat{C}_c^\#)_i$  and  $c_i = (\hat{C}_v^\#)$ . We have also the result analogous to Proposition 5:

**Proposition 6.** Any quasi-state-space  $G_m$ -C filter can be transformed (without changing its transfer function) to the structure having any passive network provided that corresponding matrix  $T_C$  has the structure such as described in the preceding paragraph. For each such passive network there exist two canonical transformations that give different quasi-state-space filter structures.

We end this section with the remark that every quasi-state-space  $G_m$ -C filter that is completely reducible can be reduced to the state-space structure. Indeed, let the filter has  $k$  floating nodes, i.e. graph  $G_f$  corresponding to its passive network has  $k+1$  connected components. Removing all floating nodes causes that graph  $\check{G}_f$  representing passive network of the reduced filter becomes connected, hence corresponding matrix  $\check{T}_C$  is invertible. This means that the reduced filter is a state-space one.

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## 6. CONCLUSION

In the paper, a novel and general approach to  $G_m$ -C filters based on matrix description has been presented. There have been shown general transfer function formulas for any  $G_m$ -C filter structure. We have investigated relationships between the passive network of the filter and its transfer function as well as conditions of invertibility of matrix corresponding to the filter's passive network. Based on these considerations we have given a new definition of state-space  $G_m$ -C filters and investigated their properties. In particular, we formulated relations between state matrices for voltage- and current-mode state-space  $G_m$ -C filters and defined two canonical transformations of state-space filters into direct state-space ones, i.e. those having only grounded capacitors. It has been shown that any state-space filter can be realized with any passive network (provided that the corresponding matrix is invertible). All the results presented in the paper can be used in computer aided filter synthesis and optimization. The reason is that matrix description can be easily handled by computer. Moreover, based on matrix description of  $G_m$ -C filter one can derive general formulas for sensitivity function of any  $G_m$ -C filter structure. The reduction procedures discussed in Section 5 can be used in automated filter design to simplify filter structures generated by computer programs within some design strategies.

It seems that the presented consistent approach to  $G_m$ -C filters can be interesting not only from a theoretical point of view but also it may be useful for filter designers. The matter of future work is to construct tools for automatic filter design based on the results drawn in this paper, with special care to the sensitivity optimization.

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Słowa klucz

S. KOZIEL, S. SZCZEPAŃSKI

ANALIZA OGÓLNYCH STRUKTUR FILTRÓW  $G_m$ -C  
ZMIENNYCH STANU CZASU CIĄGŁEGO

## Streszczenie

W pracy przedstawiono nowe podejście do ważnej i szeroko stosowanej klasy filtrów aktywnych czasu ciągłego, wykorzystujących wzmacniacze transkonduktancyjne i pojemności (tzw. filtry  $G_m$ -C). Zaproponowano ogólny opis macierzowy filtrów  $G_m$ -C, który obejmuje wszystkie możliwe realizacje filtrów w rozważanej klasie elementów. Zaletą tego opisu jest między innymi to, że wszystkie macierze występujące w równaniach mogą być tworzone bezpośrednio przez wgląd w schemat układu. Podano ogólne wzory określające transmitancję filtru, a także wykazano wzajemną odwracalność struktur napięciowych i prądowych. Jakkolwiek jest to cecha powszechnie znana, w ramach przedstawianego podejścia otrzymuje się ją jako prosty i bezpośredni wniosek z ogólnych równań opisujących filtr. W dalszej kolejności, przedstawiono zależności pomiędzy strukturą pasywną (tj. pojemnościową) filtru a rzędem transmitancji filtru. Sformułowano szereg twierdzeń ustalających warunki konieczne i dostateczne odwracalności macierzy odpowiadającej sieci pasywnej filtru, a także górne ograniczenia rzędu transmitancji filtru w zależności od spójności grafu opisującego sieć pasywną.

W drugiej części pracy wprowadzono nową definicję filtrów zmiennych stanu, zgodnie z którą filtrem zmiennych stanu jest taki filtr, dla którego macierz odpowiadająca sieci pasywnej jest odwracalna, co jest równoważne istnieniu macierzy zmiennych stanu dla takiej struktury. Jest to istotne uogólnienie istniejących definicji, ponieważ dotychczas, za filtry zmiennych stanu uważano te, które posiadają wyłącznie uziemione pojemności. Zgodnie z nową definicją do podklasy filtrów zmiennych stanu należy również szereg struktur zawierających pojemności nieuziemione. Pozwala to na użycie aparatu opisu zmiennych stanu w odniesieniu do szerszej klasy filtrów. Zaproponowano również dwa kanoniczne przekształcenia umożliwiające transformację dowolnego filtru zmiennych stanu do struktury zawierającej wyłącznie pojemności uziemione, co może mieć znaczenie w przypadku realizacji monolitycznych. Wykazano również, że każdy filtr zmiennych stanu może być przetransformowany do struktury zawierającej dowolną sieć pasywną (o ile tylko odpowiadająca macierz jest odwracalna).

W pracy wprowadzono również kolejne dwie klasy filtrów, tzw. filtry redukowalne oraz filtry pseudo — zmiennych stanu. Pierwsza z nich obejmuje struktury zawierające węzły wewnętrzne nie dołączone do żadnej pojemności w układzie ale zawierające wewnętrzną pętlę transkonduktancyjną. Struktury tego typu można zredukować poprzez usunięcie takich węzłów bez zmiany charakterystyk filtru, modyfikując jednocześnie wartości elementów układu. Druga klasa filtrów, to struktury o nieodwracalnej macierzy podukładu pasywnego, która ma tę własność, że jej dowolna kwadratowa podmacierz o niezerowych elementach diagonalnych jest odwracalna. Pokazano, że struktury takie mają w pewnej mierze własności analogiczne do filtrów zmiennych stanu. Należy także zauważyć, że podklasa struktur pseudo — zmiennych stanu zawiera nietrywialne przykłady filtrów, np. realizacje układowe powstające przy użyciu transformacji Brutona.

Wyniki zawarte w omawianej pracy mogą być one zastosowane do automatycznego projektowania i analizy filtrów  $G_m$ -C. W szczególności, należy zauważyć, że opis macierzowy może być łatwo implementowany w postaci programu komputerowego, podobnie jak wszelkie transformacje i przekształcenia realizowane z jego pomocą.

**Słowa kluczowe:** filtry zmiennych stanu, filtry  $G_m$ -C, opis macierzowy, reprezentacja zmiennych stanu