THERMAL LENSES CAUSED BY ANY ACOUSTIC SOURCE

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The modern theory concerning to the heating cased by powerful sound source is presented. In contrast to the well-known approach allowing to calculate slowly varying heating due to periodic ultrasound, any acoustic sources may be treated. Subtle temporal structure of thermal lens forming may be traced. Formulae governing the forming of the thermal lens by arbitrary (including non-periodic) source are presented. The process is illustrated by some figures.

INTRODUCTION

The effect of heating of the surrounding cased by the powerful sound is well known and paid attention in many papers [1,2]. As the secondary effect heating nevertheless is much less studied then streaming (the rotational bulk movement of the fluid following ultrasound) probably due to small amplitude of the observed velocity relating to heating. In many applications of the ultrasound it is necessary to predict a rate at which heat is produced per unit volume. For example, it is extremely important in medical therapy to calculate elevation of temperature in tissues. An elevation of temperature is followed by a decrease of density. The change of density caused by absorption was not taken into account in many sources [3,4] since the ambient state was traditionally considered as a purely incompressible liquid. Recently, the distortion of density was proved to be important when studying the acoustic heating for the majority of fluids.

Acoustic heating may exist only in absorbing fluids. The reason of the heating is namely a dissipation of sound energy. The secondary processes in the sound field such as acoustic heating change the background of acoustic wave propagation and therefore influence on the primary wave itself. The novel phenomena as focusing of an acoustic beam are observed due to the strong ultrasound absorption [5]. Elevation of temperature being non-uniform across the sound beam leads to the non-uniformity of acoustic sound. This way the thermal lens forms.

Usually, the theory applies to the periodic acoustic wave caused by transducer (quasi-periodic meaning absorption). Since the heating is known as a slow process, the classic approach is to get quasi-stationary heating by the temporal averaging of the field, the interval of averaging should be integer number (enough large) of sound periods. So, in the frames of the general theory, forming of the heated area may be traced with the temporal step much longer then the period of the ultrasound. In an absorbing fluid, the basic relation to be averaged is the total energy conservation law: $\partial E/\partial t+\vec{\nabla}\vec{J}=0$. Here, $E=\rho e+\rho(\vec{v}\cdot\vec{v})/2$ is the total energy volume density, $\vec{J}=p\vec{v}+E\vec{v}$ is energy flux density vector, $e, \rho, \vec{v}, p$ are internal energy per mass unit, mass density, velocity, and pressure, correspondingly. All variables are thought as a sum of the acoustic (subscript a) and non-acoustic parts. Temporal averaging yields in the result:

$$\langle q\rangle=-\vec{\nabla}\langle\vec{J}_a\rangle=-\vec{\nabla}\langle p_a\vec{v}_a\rangle,$$

where q is instantaneous rate per unit volume which heat is produced in a medium by ultrasound with an acoustic source in the right-hand side [6].

This way to evaluate heat generation is suitable for the periodic acoustic waves. The temporal averaging fails when pulses or other non-periodic acoustic waves are the source of heating. Though important, quasi-periodic waves are not the single type of possible acoustic sources. In medical therapy, single short pulses are of great importance; actually almost all experimental data deal with wave packets. We present here a way to evaluate heating and corresponding thermal field caused by any acoustic field including non-periodic one. The basic idea is to separate modes on the level of the initial system of conservation laws, it comes to the papers [7,8]. Modes as all possible flows following from the linear conservation laws in fluid are thought as eigenvectors of this system of equations. Matrix projectors allow to separate every mode from the overall perturbation at any moment and to get dynamic equations for the interacting modes. The author has developed the idea accordingly to some applications to flows over inhomogeneous and stratified media [7,8]. The steps to get approximate solution are also pointed out there. The heat (entropy) mode is not certainly secondary mode caused by sound but may be even dominant in the initial field.

1.THEORY

The mass, momentum and energy conservation equations read:

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}(\rho\vec{v})=0 \tag{1}$$

$$\rho\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\vec{\nabla})\vec{v}\right]=-\vec{\nabla}p+\eta\Delta\vec{v}+\left(\varsigma+\frac{\eta}{3}\right)\vec{\nabla}(\vec{\nabla}\vec{v})$$

$$\rho\left[\frac{\partial e}{\partial t}+(\vec{v}\vec{\nabla})e\right]+p\vec{\nabla}\vec{v}-\chi\Delta T=\varsigma\left(\vec{\nabla}\vec{v}\right)^2+\frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)^2$$

Here, among already mentioned variables, η,ς are shear and bulk viscosities, respectively (both supposed to be constants), χ is heat conductivity, x_i - space coordinates. Except of the dynamical equations (1), the two thermodynamic relations are necessary: $e(p,\rho), T(p,\rho)$. To treat a wide variety of fluids (both gases and liquids), let us use the most general form of these relations as expansion in the Fourier series:

$$\rho_0 e'=E_1 p'+\frac{E_2 p_0}{\rho_0}\rho'+\frac{E_3}{p_0}p'^2+\frac{E_4 p_0}{\rho_0^2}\rho'^2+\frac{E_5}{\rho_0}p'\rho'+\ldots$$
$$T'=\frac{\Theta_1}{\rho_0 C_v}p'+\frac{\Theta_2 p_0}{\rho_0^2 C_v}\rho'+\ldots \tag{2}$$

The background values are marked by zero, perturbations are primed, C_v means specific heat per unit mass at constant volume, $E_1,\ldots\Theta_1,\ldots$ are dimensionless coefficients:

$$\Theta_1=\frac{\rho_0 C_v \kappa}{b},\ \Theta_2=-\frac{\rho_0 C_v}{p_0 b}, \tag{3}$$

where $\kappa=\frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial p}\right)_{T=T(p_0,\rho_0)}$, $b=-\frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial T}\right)_{\rho=\rho_0}$

The equivalent system in non-dimensional variables marked by asterisks that will be omitted everywhere later, $\vec{v}=c\vec{v}_*, p'=c^2\rho_0 p_*$, $\rho'=\rho_0\rho_*$, $\vec{x}=\lambda(x_*/\sqrt{\mu}\quad y_*\quad z_*/\sqrt{\mu})$, $t=t_*\lambda/c$ (c is adiabatic sound velocity, $c=\sqrt{\frac{p_0(1-E_2)}{\rho_0 E_1}}$, λ means characteristic scale of disturbance) looks as follows

$$\frac{\partial}{\partial t}\psi+L\psi=\hat{\psi} \tag{5}$$

Here, introducing of the small parameter $\sqrt{\mu}$ relates to the quasi-plane geometry. Vector $\psi=\begin{pmatrix}v_x & v_y & v_z & p' & \rho'\end{pmatrix}^T$ is variables column, vector $\hat{\psi}$ notes nonlinear terms of the second order that are of the most importance in acoustics, and

$$
L = \begin{pmatrix}
-\delta_1^1\mu\frac{\partial^2}{\partial x^2}-\delta_1^2\Delta & -\delta_1^1\sqrt{\mu}\frac{\partial^2}{\partial x\partial y} & -\delta_1^1\mu\frac{\partial^2}{\partial x\partial z} & \sqrt{\mu}\frac{\partial}{\partial x} & 0 \\
-\delta_1^1\sqrt{\mu}\frac{\partial^2}{\partial x\partial y} & -\delta_1^1\frac{\partial^2}{\partial y^2}-\delta_1^2\Delta & -\delta_1^1\sqrt{\mu}\frac{\partial^2}{\partial y\partial z} & \frac{\partial}{\partial y} & 0 \\
-\delta_1^1\mu\frac{\partial^2}{\partial x\partial z} & -\delta_1^1\sqrt{\mu}\frac{\partial^2}{\partial z\partial y} & -\delta_1^1\mu\frac{\partial^2}{\partial z^2}-\delta_1^2\Delta & \sqrt{\mu}\frac{\partial}{\partial z} & 0 \\
\sqrt{\mu}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \sqrt{\mu}\frac{\partial}{\partial z} & -\delta_2^1\Delta & -\delta_2^2\Delta \\
\sqrt{\mu}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \sqrt{\mu}\frac{\partial}{\partial z} & 0 & 0
\end{pmatrix} \tag{6}
$$

is the linear matrix operator with parameters

$$
\delta_1^1 = \frac{\left(\varsigma+\frac{\eta}{3}\right)}{\rho_0 c\lambda}, \delta_1^2 = \frac{\eta}{\rho_0 c\lambda}, \ \delta_2^1 = \frac{\chi\theta_1}{\rho_0 c\lambda C_v E_1}, \delta_2^2 = \frac{\chi\theta_2}{\rho_0 c\lambda C_v(1-E_2)}. \tag{7}
$$

Dropping the details, the linear analogue of (5) yields in five roots of the dispersion relation $\omega(\vec{k})$, and to the five basic types of the motion in fluid differing by the relations of components of velocity and perturbations of pressure and density. The first two roots relate to progressive (acoustic) modes of different direction of propagation, the third one relates to the heat (or entropy) mode, and the last two - to the rotational one. Any field of the linear flow may be presented as a sum of independent modes.

The next step is to get projectors that decompose a concrete mode from the overall field. Since calculations are rather complicated in the three-dimensional geometry of the flow, a reader is referred for more details to [7,8]. Finally, five matrix projectors follow that decompose the overall field into the concrete type of the motion.

The achievement of the modern theory is an expansion of the method into the area of nonlinear flow. All possible interactions between modes are predictable by acting of the corresponding projector at the initial nonlinear system of equations (5). To investigate a dynamics of the thermal layer, it is enough to act by the matrix projector relating to the entropy mode at the system when in the nonlinear vectors inputs of the progressive acoustic mode are given rise. As a result, an equation for the rate of density decrease in the thermal lens ρ_t looks:

$$
\frac{\partial\rho_t}{\partial t}+\delta_2^2\frac{\partial^2\rho_t}{\partial y^2} =
$$

$$-(N_1+N_2+1)p_1\frac{\partial p_1}{\partial y}+\frac{\beta(N_1+N_2+1)}{2}p_1\frac{\partial^2 p_1}{\partial y^2}+(\delta_2(N_1+1)-\delta_1/E_1)\left(\frac{\partial p_1}{\partial y}\right)^2 \qquad (8)$$

Here, β means a sum of attenuation coefficients $\beta=\delta_1^1+\delta_1^2+\delta_2^1+\delta_2^2$, constants N_1, N_2 are as follows:

$$N_1=\frac{1}{E_1}\left(-1+2\frac{1-E_2}{E_1}E_3+E_5\right), N_2=\frac{1}{1-E_2}\left(1+E_2+2E_4+\frac{1-E_2}{E_1}E_5\right).$$

In the case of the perfect gas that will be considered for simplicity, $E_1=(\gamma-1)^{-1}, N_1=-\gamma, N_2=0$, and (8) goes to the following:

$$\frac{\partial\rho_t}{\partial t}-\frac{\delta_2}{\gamma-1}\frac{\partial^2\rho_t}{\partial y^2}=(\gamma-1)\left[p_1\frac{\partial p_1}{\partial y}-\frac{\beta}{2}p_1\frac{\partial^2 p_1}{\partial y^2}-\beta\left(\frac{\partial p_1}{\partial y}\right)^2\right]. \qquad (9)$$

In the equation (9), only the main terms are given rise, nonlinear ones multiplied by viscous coefficient though there are cross-nonlinear terms of order $\beta\sqrt{\mu}$. It may be easily shown that for the plane periodic acoustic source formula goes to the well-known one discussed in introduction.

2. ILLUSTRATIONS ON THE THERMAL LENS CAUSED BY DIFFRACTING BEAM

As an acoustic source, let us take a pressure wave in the diffracting beam as follows:

$$p_1(x,y,z,t)=\frac{P_0}{1-iZ}\exp\left(-\frac{r^2}{1+Z^2}-\frac{\beta}{2}y-i\tau\right)+cc, \qquad (10)$$

where $r=\sqrt{x^2+z^2}, L=(2\mu)^{-1}, Z=y/L$, L being dimensionless length of diffraction. That is the well-known analytical solution for the linear diffracting beam [3].

After numerical integration of (9) (the term $-\frac{\delta_2}{\gamma-1}\frac{\partial^2\rho_t}{\partial y^2}$ is usually omitted), the temperature field may be reconstructed due to the formula:

$$\frac{\Delta T(x,y,z,t)}{T_0}=-\int_0^t\frac{\partial\rho_t}{\partial\tau}d\tau. \qquad (11)$$

Illustrations on the delicate temporal structure of the thermal lens are presented by the figures 1-4 for the chosen set of constants: $\mu=0.1, \beta=0.1, \gamma=1.4, P_0=0.1$.

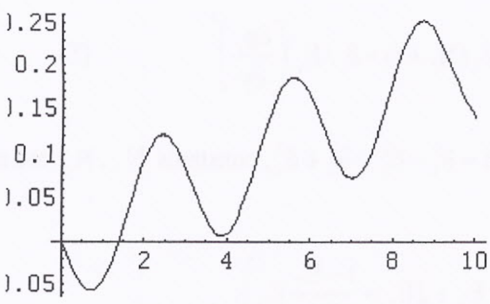

Fig.1 $10 \cdot \Delta T / T_0$ via t at y=1, r=0.

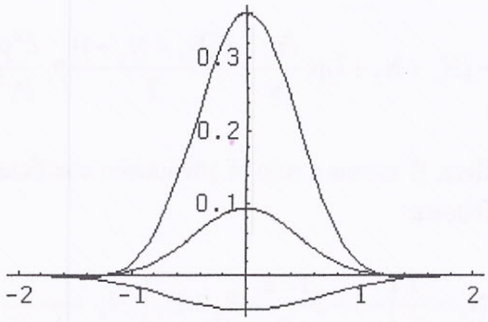

Fig.2 $10 \cdot \Delta T / T_0$ via r at y=1, t=20 (upper curve), t=2 (middle curve), t=1

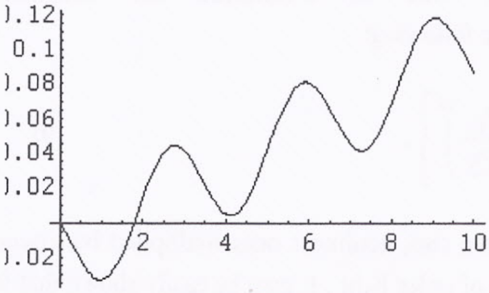

Fig.3 $10 \cdot \Delta T / T_0$ via t at y=5 (focal length), r=0.

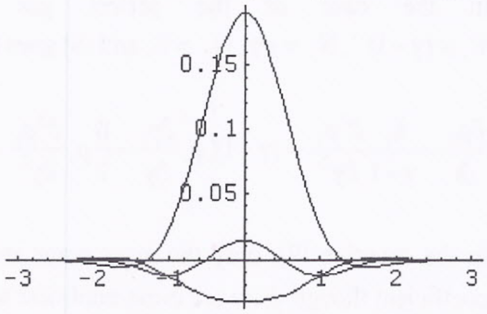

Fig.4 $10 \cdot \Delta T / T_0$ via r at y=5, t=20 (upper curve), t=2 (middle curve), t=1.

As a conclusion, it should be noted that a wide variety of acoustic sources could be treated including impulse ones that are of special important in some applications of medicine and techniques. Forming of the thermal lenses both in gases and liquids may be calculated.

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